J. Geophys. 41, 201–213, 1975

An Evaluation Method Combining the Differential Doppler Measurements from Two Stations that Enables the Calculation of the Electron Content of the Ionosphere

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Received August 18, 1974

Abstract. Differential Doppler measurements of signals from NNSS Navigational Satellites can be used to give the electron content of the ionosphere. Measurements carried out up to now using data from one station provide limited information about the structure of the ionosphere, since the method relies on an assumption being made about the prevailing ionospheric conditions. If these conditions are not fulfilled, this method can lead to large errors in the predicted electron content. In the method described in this paper, Differential Doppler data from two stations are combined, resulting in considerably more reliable results, particularly when there is strong horizontal structure in the ionosphere, as is often the case in Polar regions. Examples of model calculations and experimental measurements are also included.

Key words: Ionospheric Electron Content – Differential Doppler Effect – Two Station Evaluation - Horizontal Structure.

1. Outline of the Evaluation Method

The satellites of the Navy Navigational Satellite System (NNSS) allow Differential Doppler observations using two coherent signals (nominal frequencies 150 MHz and 400 MHz). The receiving system produces the phase difference between the 150 MHz-signal phase divided by 3 and the 400 MHz-signal phase divided by 8.

For the frequencies of the NNSS beacons, the slant electron content I_s can be considered as a linear function of the measured phase difference ψ (Ebel *et al.*, 1969):

$$\psi + \phi_0 = C_D I_s$$

where ϕ_0 is an unknown initial value, C_D combines all constants, $I_s = \int_B^S N\,ds$, N is the number density of free electrons, ds is the differential of the straight line B—S from the receiving station B to the satellite S (Fig. 1).

If it is assumed that the ionosphere does not change during the pass of an NNSS satellite, the time can be used as a parameter for the position of the satellite. Then I_s and ψ can be considered to be functions of time. To be able to convert $\psi(t)$ into electron content one first has to find a value for ϕ_0. For this purpose it is necessary to make assumptions on the state of the ionosphere or to use additional measurements.

In both cases one has to use the vertical electron content $I_v = \int_0^{h_s} N\,dh$ (dh: small increment in height above ground; h_s: height of the satellite). Let i be the angle between B—S and the vertical height h (Fig. 1). Then one can write

$$I_s = \int_0^{h_s} \frac{N}{\cos i}\,dh, \text{ where } i = i(h)\,.$$

To convert slant content into vertical content, one has to divide I_s by a factor

$$D' = \frac{\int_0^{h_s} (N/\cos i)\,dh}{\int_0^{h_s} N\,dh}$$

D' can be calculated only if the distribution of electrons along B—S is known. Generally this is not the case and one has to make a reasonable guess at a value for D'. This is done by choosing a "mean ionospheric height" h_i, some tens of kilometers above the height of the maximum of the electron density, h_m (compare among others, references [2], [3]). It can be shown by means of model calculations, that for various realistic electron distributions $(h_i - h_m) = 50$ km is a good choice. In practice it is not necessary to change h_i as a fixed value is good enough for most purposes. The guessed geometrical factor is denoted by the letter D, to distinguish it from the "true" conversion factor D'. D is given by the expression $D = 1/\cos\chi$, where $\chi = i(h_i)$. (Fig. 1).

Once a mean ionospheric height has been assumed, D is calculated for each measured ψ-value, using only the geometrical relations involving the position of the observing station and the position of the satellite.

For the vertical electron content obtained from D, the letter I is used to distinguish it from the "true" electron content I_v over P'; $I_v = \left(\int_0^{h_s} N\,dh\right)_{P'}\cdot P'$ is the "subionospheric point" (projection of the ionospheric point P onto the ground; P is found at the height h_i, on the line B—S. For an undisturbed ionosphere during the daytime, I and I_v will differ by not more than a few percent if the zenithal distance of the satellite, ξ, is not too large. The best agreement will be reached when the shape of the height distribution of electrons is nearly independent of both latitude and longitude, as can easily be shown by model calculations.

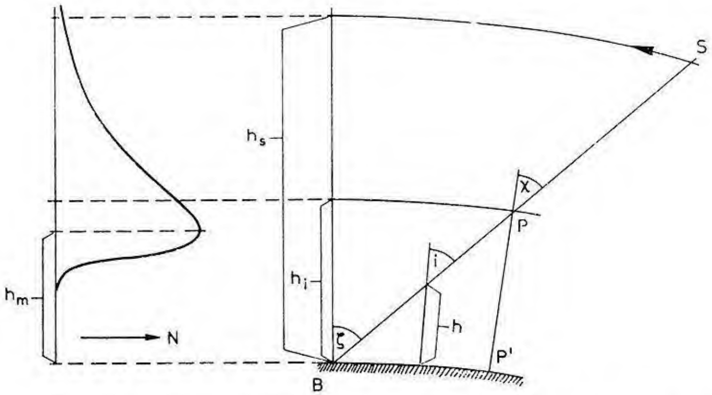

Fig. 1. Geometry receiving station (B) — position of the satellite (S) — ionospheric point (P) — subionospheric point (P'). (h_i: mean ionospheric height, h_m: height of the maximum of the electron density, h_s: height of the satellite)

If no other independent measurements are available then the assumption of a nearly linear "time" — dependence of vertical electron content is the only practicable way of obtaining a solution (time as a parameter for the position of the satellite!). As the NNSS satellites have orbits with a nearly constant longitude (except in the polar region) this means a nearly linear latitude dependence on the (vertical) electron content: $I_v = I_{v0}\ (1 + a\ (\phi - \phi_0))$ (ϕ: geogr. latitude, I_{v0}, a and ϕ_0 are constant). As no physical reasoning can be applied, it makes no sense to base the calculation of ϕ_0 upon a more complicated state of the ionosphere. Some of the restrictions imposed by our assumption can be overcome by using curve fitting methods that include all measured ψ-values in the calculation. By this means, reasonable values can still be obtained provided there is some mean linearity in the time dependence of the electron content in the latitude region under consideration. Several different methods of fitting should be applied simultaneously to check the validity of the basic assumption: if the difference between the results exceeds a few percent, then the ϕ_0-values obtained are certainly not good enough for some purposes, e. g. for the calibration of electron content measurements made with geostationary satellites. If the differences are in excess of 30%, one cannot contribute any significance to the results except for statistical purposes. During nighttime, the evaluation can even give negative values of electron content, which have no meaning at all.

It is impossible to distinguish which fitting method gives the best results if the assumption of a nearly linear latitude dependence of vertical electron content is not fulfilled: no criteria exist to qualify the results. Model calculations show that the "true" results can even be outside the range given by the lowest and the highest values obtained by different fitting methods. In one peculiar situation, all methods give identical wrong results, that is if $I_v = I_{v0}\ (1 + a\ (\phi - \phi_0) + b \cos\chi))$ ($b =$ const., all other symbols used as above). All fitting methods then give $I_{v0}\ (1 + a\ (\phi - \phi_0))$ neglecting the term $b \cos \chi$. This case can only be recognized when $I_{v0}\ (1 + a\ (\phi - \phi_0))$ reaches negative values in the observed interval of latitudes (s. Fig. 9). One has to draw the conclusion that even results obtained by several fitting methods which show a linear latitude dependence are not always reliable.

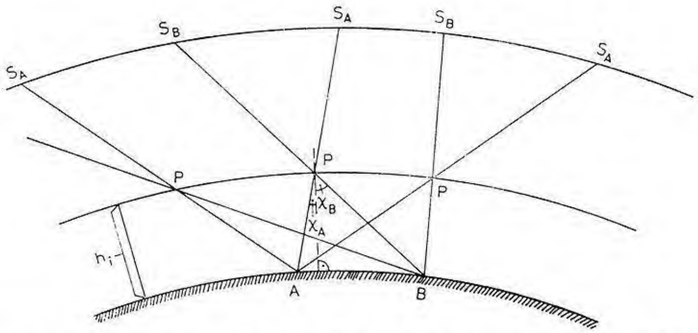

Fig. 2. Geometry for two stations. For simplicity of drawing, it is assumed that the orbit of the satellite and the centre of the earth define a plane, and that the stations *A* and *B* are in this plane

The best way to improve the situation is to use additional measurements. As in general no values of vertical electron content from the ground to the height of the satellite, derived from other observations than satellite beacon measurements are available, this can be done by operating a second Differential Doppler station. The combination of the observations is possible if a region in the mean ionospheric height can be seen from both stations (region of overlap).

The method we will describe also has to rely upon the conversion of slant into vertical electron content, but no assumptions are necessary concerning the state of the ionosphere.

In a given subionospheric latitude within the region of overlap, the ionospheric points of the two stations should be as close together as possible. For NNSS satellites this means that the longitude difference of the stations should be as small as possible.

The geometry for the combined evaluation is shown in Fig. 2. For simplicity in the figure, it is assumed that the orbit of the satellite and the centre of the earth define a plane, and that both stations are in this plane. If rays are drawn from the two stations *A* and *B*, one can see that a latitude dependence of the electron content introduces an error since

$$\left(\int_A^{S_A} N\,ds_1\right)\cos\chi_1 \neq \left(\int_0^{h_s} N\,dh\right)_{P'} \neq \left(\int_B^{S_B} N\,ds_2\right)\cos\chi_2 .$$

It can be shown that the differences between the three values are small if the latitude dependence of $\int_0^{h_s} N\,dh$ is not too large and if the shape of the height distribution of electrons is nearly independent of latitude (compare ch. 2). The results of the evaluations will show any serious differences between the three values quite clearly.

For the purpose of outlining our method, it is assumed that both stations give the same electron content. Then for a given subionospheric latitude (index i) the following equations hold

$$\phi_{01} + \psi_{1i} = C_D \, D_{1i} \, I_i$$
$$\phi_{02} + \psi_{2i} = C_D \, D_{2i} \, I_i$$

The indices 1 and 2 refer to the two stations A and B respectively. For another subionospheric latitude (index j) one has

$$\phi_{01} + \psi_{1j} = C_D \, D_{1j} \, I_j$$
$$\phi_{02} + \psi_{2j} = C_D \, D_{2j} \, I_j$$

The system of four equations can be solved to give the unknowns ϕ_{01}, ϕ_{02}, I_i, I_j.

Since it cannot be expected that using other pairs of subionospheric latitudes would give exactly the same ϕ_{01} — and ϕ_{02} — values it is advisable to use n subionospheric latitudes simultaneously. One gets n equations for each station:

$$\begin{aligned} \phi_{01} + \psi_{1k} &= C_D \, D_{1k} \, I_k \\ \phi_{02} + \psi_{2k} &= C_D \, D_{2k} \, I_k \end{aligned} \qquad k = 1, 2, \ldots n$$

A combination of the two equations with index k leads to

$$\frac{\phi_{01}}{D_{1k}} + \frac{\psi_{1k}}{D_{1k}} - \frac{\phi_{02}}{D_{2k}} - \frac{\psi_{2k}}{D_{2k}} = 0$$

The use of all n equations allows to calculate ϕ_{01} and ϕ_{02} when the expression

$$\sum_{k=1}^{n} \left(\frac{\phi_{01}}{D_{1k}} + \frac{\psi_{1}}{D_{1k}} - \frac{\phi_{02}}{D_{2k}} - \frac{\psi_{2k}}{D_{2k}} \right)^2 \text{ has a minimum value.}$$

This gives a best fit (in the sense of the method of least squares) for ϕ_{01} and ϕ_{02} in the region of overlap. The inevitable scatter of data and some of the effects which arise from the differences in slant electron contents for the two stations are compensated for.

The vertical electron content is calculated with ϕ_{01} and the ψ-values of station A and with ϕ_{02} and the ψ-values of station B for a set of subionospheric latitudes. A comparison of the I-curves for the two stations provides a check on the accuracy of the results.

2. Check by Model Calculations

Model calculations can be used to construct ψ-values for a pass of an NNSS satellite from a given three dimensional distribution of electrons. First the coordinates of the satellite are calculated for a set of n subionospheric latitudes (the difference in latitudes between adjacent subionospheric points is chosen to be 0.5°) and for two observing stations. Then the slant electron content I_s for each position is calculated by means of numerical integration. I_s is converted into ψ-values using the expressions

$$\begin{aligned} \psi_{1k} &= C_D\, I_{sk} - \bar{\phi}_{01} \\ \psi_{2k} &= C_D\, I_{sk} - \bar{\phi}_{02} \end{aligned} \qquad k = 1, 2, \ldots n$$

Appropriate values for $\bar{\phi}_{01}$ and $\bar{\phi}_{02}$ are assumed. The index k designates the subionospheric latitude. The two sets of ψ-values are then used to calculate ϕ_{01} and ϕ_{02}, using the combination method described above.

With ϕ_{01} and ϕ_{02} and the two sets of ψ-values, one can easily express the results in terms of slant electron content I_s' or of projected vertical electron content $I' = I_s'/D$. (I_s' and I' are used to distinguish the results of the evaluation from the given model values I_s and I).

In the following, only results for one type of model are shown: A Chapman-Elias profile of constant shape was used for the height distribution of electrons, a sinusoidal latitude dependence of electron density was assumed and the longitude dependence was neglected. This gave the following expression for the electron number density

$$N = N_0\,(1 - a\cos[b(\phi - \phi_0]) \exp(^1/_2[1 - z - \exp(-z)])$$

with $z = (h - h_m)/H$.

N_0 : undisturbed value of the maximal electron density

ϕ : geographical latitude

ϕ_0 : reference latitude

a : amplitude of the disturbance

b : scale factor for the spatial frequency of the disturbance

h : height

h_m : height of the maximum of the electron density

H : scale height

For the examples given in Fig. 3 and in Fig. 4 the following values have been used: $N_0 = 1.0 \times 10^{11}$ m^{-3}, $h_m = 350$ km, $H = 50$ km, $\phi_0 = 51.75°$ N, $a = 0.5$, $b = 100$ (Fig. 3), $b = 25$ (Fig. 4).
Height of the satellite: 1097 km. Position of the stations: station 1: 55.5° N, station 2: 40.5° N.

The orbit of an NNSS satellite was assumed but the excentricity was set to zero (circular orbit). The longitude of the satellite was chosen to be very close to the longitude of the stations. Since the stations have the same longitude, the neglection of any longitude dependence of electron density does not matter. For the model calculations, a mean ionospheric height $h_i = 400$ km was used. The amplitude of the sinusoidal disturbance was exaggerated to demonstrate the following effect: in the projected vertical electron content I, the disturbance is seen nearly unchanged only in the vicinity of the point of closest approach of the satellite (PCA). If one goes away from PCA, the amplitude of the disturbance appears attenuated and the phase of the disturbance appears shifted. This is an effect resulting from the integration along the line B—S from the receiver to the satellite, which leads to I_s (remember: $I = I_s/D$).

The characteristic scale for this effect is the spatial period of the disturbance. In the region of latitudes seen from the receiving stations, the integration effect is important only for disturbances of small scale. This can be seen clearly by comparing Fig. 3 and Fig. 4: in Fig. 3, a disturbance of comparatively small scale has been used (spatial period: 3.6° of latitude), for Fig. 4, the scale was four times larger (spatial period 14.4° of latitude). The figures show the projected vertical content I vs. subionospheric latitude. In Fig. 3 one period of the given sinusoidal I_v is shown on the right margin. The minima of the given I_v are indicated in Figs. 3 and 4 by vertical bars below the I-curves. The location of each station is also indicated.

The comparison of more material from model calculations leads to the conclusion that the large scale component of the latitude dependence of true vertical electron content is seen in the converted slant electron content to be nearly unaffected by the integration effect. Therefore, the large scale structure of the vertical electron content from Differential Doppler observations is realistic, if the evaluation process produces good ϕ_0-values. The small scale structure can be seen only in the vicinity of the PCA. The results from two stations should show the same large scale structure, but will have differences in the small and medium scale structure, when such a structure exists.

Some of the results of our model calculations in respect to the accuracy of ϕ_0-values obtained by the combination method are briefly summarized: In the case of Fig. 3 (sinusoidal structure of the ionosphere, spatial period 3.6°, relative amplitude of the disturbance: 0.5) our method of combined evaluation for two stations gives for the PCA, a relative error $(I' - I)/I$ of 5.4% for the minimum, and of 1.8% for the maximum of the disturbance. In the case of Fig. 4 (sinusoidal structure of the ionosphere, spatial period 14.4°, relative amplitude of the disturbance 0.5) the values are 5.9% and 2.9%. The relative error is reduced if the amplitude of the disturbance is smaller.

3. Some Results

To illustrate the use of our method of combined evaluation for two receiving stations, some results are shown in the following figures. The stations are Graz (47.08° N, 15.49° E), Lindau (51.62° N, 10.09° E) and Oulu (65.11° N, 25.48° E). There is a difference in longitudes between the stations and in interpreting the results this fact must be taken into account. If a significant longitude dependence of electron content exists, the values along the traces of the subionospheric points of the two stations are different. Our method will then give average values.

Fig. 5 shows very good agreement between the I-curves obtained by the combination method north of 48° N. A small scale structure exists, and can be seen a bit south of the PCA of the corresponding station. The large scale structure is nearly linear and therefore the results for single station evaluations (indicated by vertical bars in the figure) agree very well, and are very close to the results of the combined evaluation. Fig. 6 shows a map with the projection of the orbit of the satellite and

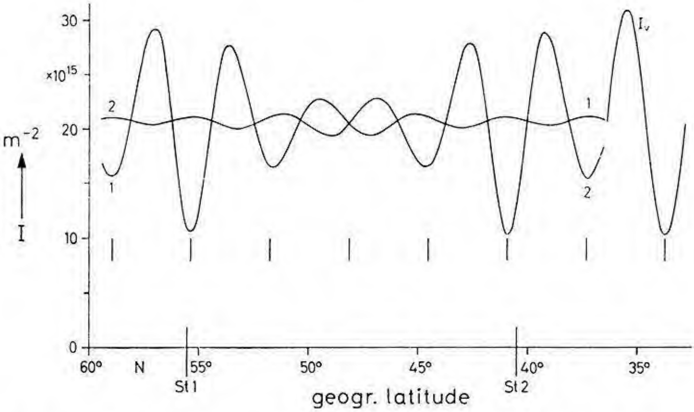

Fig. 3. Results of model calculations: vertical electron content I converted from slant content vs. subionospheric latitude for a sinusoidal disturbance of the ionosphere. Spatial period of the disturbance: 3.6°. On the right margin one period of the given (true) vertical content I_v is shown. The minima of I_v are indicated by vertical bars below the I — curves. The location of the two receiving stations is indicated (St 1, St 2). Curve 1 belongs to station 1, curve 2 to station 2

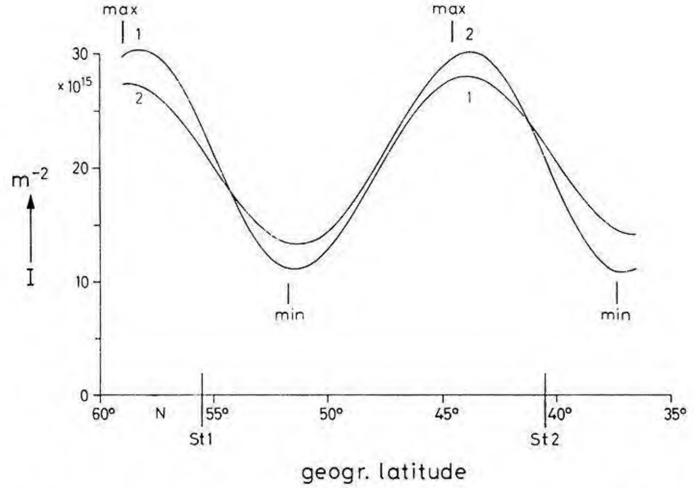

Fig. 4. Results of model calculations as in Fig. 3 but spatial period of the disturbance 14.4°. Again, the minima of the given vertical content I_v are indicated by vertical bars

with the traces for the subionospheric points for Graz (IG) and for Lindau (IL). Another example with a clear small scale structure is shown in Fig. 7. The deviation from a nearly linear large scale structure leads to greater differences between the single station evaluations for Lindau, but the agreement is still quite good.

Figure 8 shows the results obtained from 5 passes of NNSS satellites on the same day. It should be noted that during nighttime, the single station evaluations for Lindau cannot give reliable values: there is a strong deviation from a nearly linear latitude dependence. (For simplicity of drawing, no single station results are shown).

A very interesting case is shown in Fig. 9: there is no significant small scale structure, but the large scale structure has a distinct maximum in 47.5° N and a distinct minimum in 64° N. For Graz, the formula $I = I_0\ (1 + a\ (\phi - \phi_0) + b \cdot \cos\chi)$ would fit the results very well. Therefore, the single station evaluations give completely wrong values, since they neglect the term $b \cdot \cos\chi$. In this case, the situation can be guessed because the (wrong) straight line fitted by the single station evaluations gives negative values of electron content north of 55° N. For Lindau too, the large scale structure does not allow good single station evaluations.

In Fig. 10, an example is shown for the combination of measurements from Oulu, Lindau and Graz. Agreement between Lindau and Oulu and between Graz and Lindau is quite good but the combination Graz—Oulu gives significantly higher values. This could be an effect of the difference in longitudes (c. Fig. 11) but one should also bear in mind that the region of overlap between Graz and Oulu is only short and that the combination Lindau—Oulu shows medium scale differences of considerable amplitude. These differences could again be the result of the different subionospheric longitudes. The single station evaluations do not help in the interpretation of results: the differences between the results for Oulu are by far too large to allow any conclusions to be made, and the single station evaluations for Graz and for Lindau agree very well with the results of the combinations Lindau—Oulu and Graz—Lindau. It can be seen from the map in Fig. 11 that the geographical situation for the three stations is complex.

4. *Conclusion*

In many cases the accuracy of Differential Doppler evaluations can be considerably improved if the measurements from two stations are combined. This method does not depend on the assumption of a nearly linear latitude dependence of the vertical electron content (or on the a priori assumption of another special structure of the ionosphere). To give the best results, the difference in the longitudes of the two stations should be as small as possible in the case where NNSS satellites, which have polar orbits, are used. The difference in latitudes can range from about 2 to 20 degrees. If it is too small, there is some danger of numerical instability. If it is too large, the region of overlap may be too small to provide a good fit.

One important advantage of NNSS observations should be mentioned: the Differential Doppler results can give a very good representation of the latitude dependence of the electron content in the large scale, if a good value for the constant ϕ_0 can be found. In general we have found that the described combination method gives results which are much better than the results which can be achieved when the data from only one station are used.

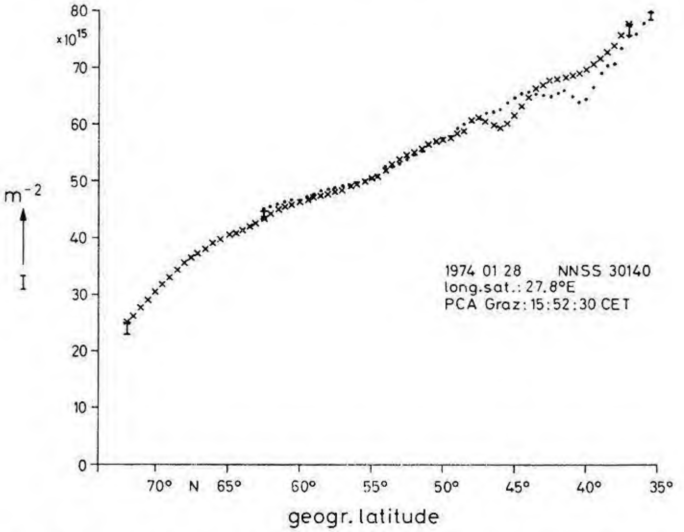

Fig. 5. Results of the combined evaluation for Graz (47.08° N, 15.49° E) and Lindau (51.62° N, 10.09° E). The characteristics of the pass of the NNSS-satellite used, are given in the drawing. (Date in year, month, day; number of the satellite; longitude of the satellite for 50° N; time, when the point of closest approach for Graz was reached in hours, minutes, seconds [UT + 1 hour]). Electron content vs. subionospheric latitude. Values from the combined evaluation: crosses for Lindau, dots for Graz. The range of values from single station evaluations, by means of three different methods, are indicated by vertical bars at the ends of each trace

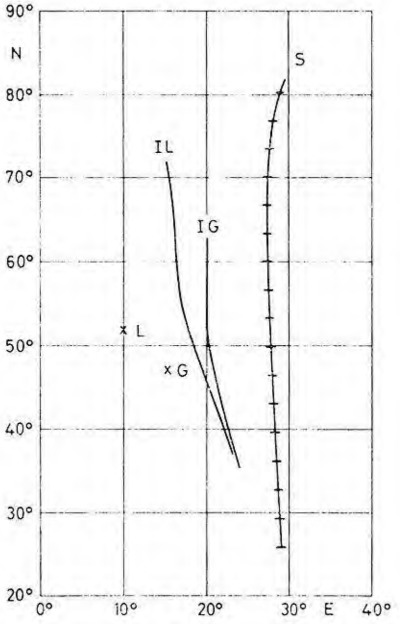

Fig. 6. Map showing the projection of the satellite orbit (*S*), the stations Lindau and Graz (L, G), the traces of the subionospheric points for Lindau (IL) and Graz (IG). Time marks on the satellite orbit every minute, beginning at 15:46 CET (south-north pass)

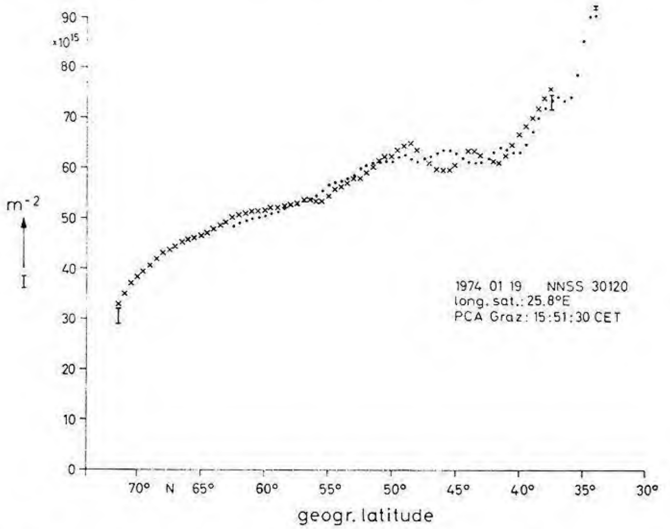

Fig. 7. Results of combined evaluation Graz — Lindau. Symbols as in Fig. 5

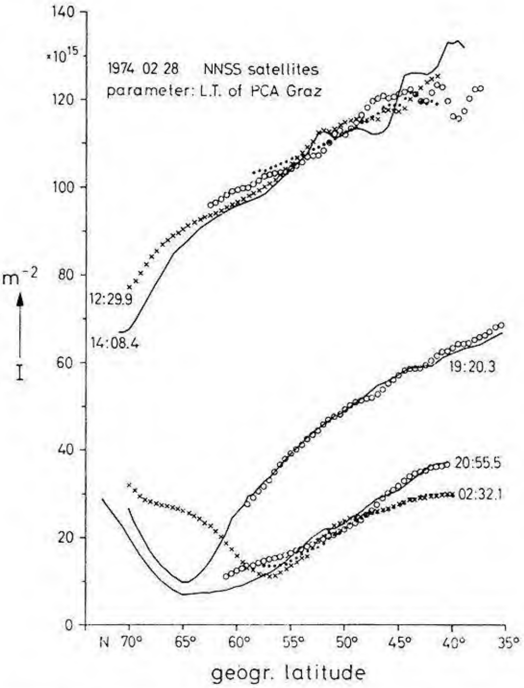

Fig. 8. Results obtained from 5 passes of NNSS satellites on the same day. Combined evaluation Graz — Lindau. Electron content vs. subionospheric latitude. Crosses or solid lines for Lindau, dots or circle for Graz. The local time (L. T.) (hours and minutes) is given for a subionospheric latitude of 50° N (station Graz)

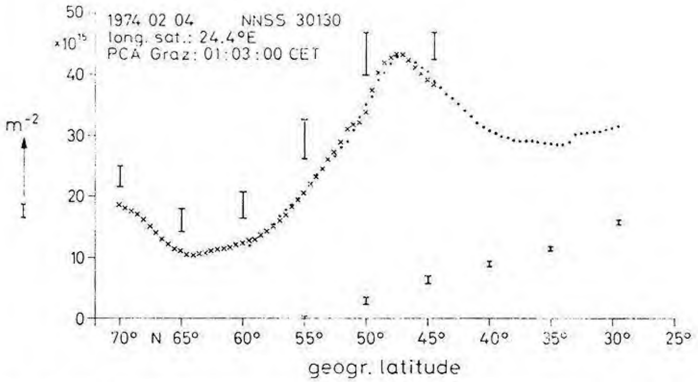

Fig. 9. Results of combined evaluation Graz – Lindau. Symbols as in Fig. 5. Bars above the curves for single station results from Lindau. Bars below the curves for single station results from Graz

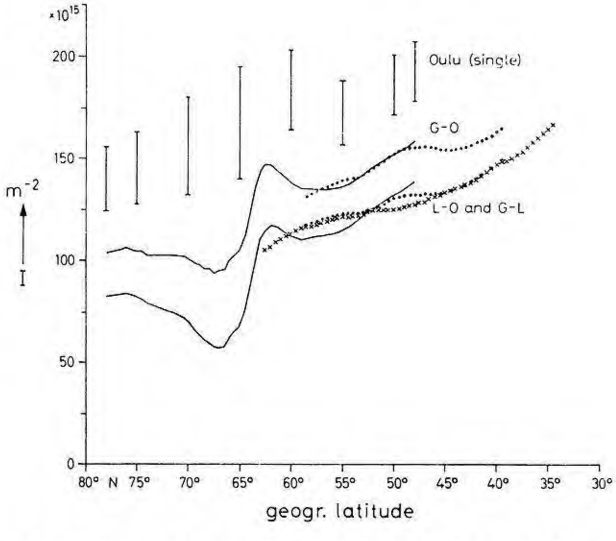

Fig. 10. Results of the combined evaluation for Lindau – Oulu (L–O, crosses for Lindau, solid line for Oulu), for Graz – Lindau (G–L, dots for Graz; the results for Lindau are not shown, as they were only slightly above the crosses of L–O), for Graz – Oulu (G–O, dots for Graz, solid line for Oulu). Electron content vs. subionospheric latitude. Bars above the curves correspond to single station results from Oulu. Characteristics of the satellite: NNSS 30190 on April 11, 1974. PCA Graz: 16:03:30, PCA Lindau: 16:02:20, PCA Oulu: 15:58:00 (all in hours:minutes:seconds CET). The geometry is shown in Fig. 11

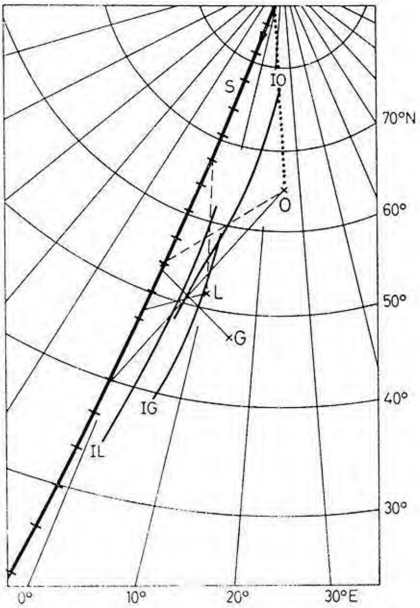

Fig. 11. Map showing the projection of the satellite orbit (*S*), the stations Oulu (O), Lindau (L) and Graz (G), and the traces of the subionospheric points for Oulu (IO), for Lindau (IL) and for Graz (IG). The characteristics of the satellite are given in the caption of Fig. 10. Timemarks on the orbit of the satellite are shown for each minute, beginning at 15:53 CET. Projections of the straight line from a receiving station to the satellite are shown for a subionospheric latitude of 50° N (all three stations, solid lines) and for a subionospheric latitude of 60° N (stations Oulu and Lindau, dashed lines). The projection for the map is gnomonic, centered on Oulu

Acknowledgement. We wish to thank Mag. rer. nat. E. Putz for the reading of the registrations made in Graz and in Oulu and for his help in preparing computer programs.

The support of the Fonds zur Förderung der wissenschaftlichen Forschung in Österreich who financed the construction of the measuring equipment used in Graz is gratefully acknowledged.

All calculations have been made on the UNIVAC 494 of the Rechenzentrum Graz.

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