Tidal Triggering of Earthquakes in the Swabian Jura?

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Abstract. Several statistical tests were used to investigate the possibility that the earthquakes in the Swabian Jura are triggered by the tidal stress in the earth. The results provide weak evidence that the earthquakes tend to occur when the tidal shear stress on the fault plane in the direction supporting the tectonic stress is greatest. A comparison of some of the statistical methods available for investigating tidal triggering effects is made.

Key words: Tidal triggering – Earthquake statistics - Swabian Jura.

1. Introduction

There have been many studies of the hypothesis that the times of occurrence of earthquakes are influenced by the periodic tidal stresses in the Earth. These have generally indicated the existence of such an effect only when they have separated the catalogue of earthquakes into small geographical regions, over which the earthquake mechanism may be reasonably expected to be fairly constant, for example Klein (1976), or when the orientation of the fault plane and slip vector of each earthquake has been taken into account in calculating the effective tidal stress, as in Heaton (1975). This is to be expected, since in an analysis which involves earthquakes of various mechanisms without allowing for these variations, a tidal effect is liable to be masked by the differences in the time dependences of the tidal forces acting in different directions.

An earthquake catalogue of 259 earthquakes from the Swabian Jura, in southern Germany, was felt to be particularly suitable for a study of this type because the fault plane mechanisms are very well known and show remarkable consistency. The opportunity was taken to apply to this one data set several of the statistical tests used previously in various studies, so that their results might be compared.

2. The Catalogue

The catalogue contained earthquakes in the region between latitudes 47°57' N and 48°32' N and between longitudes 8°26' E and 9°28' E which occurred between 1900 and 1976 inclusive. They ranged in local magnitude from 2.0 to 6.3 and in depth from 1 km to 15 km. Their origin times are given in the appendix.

All the fault plane solutions which have been carried out on these earthquakes (Schneider, 1968, 1971) show a strike slip mechanism with nearly vertical nodal planes lying roughly north-south and east-west, with the compressive axis in the NW-SE direction. The variance of the nodal plane directions is very small, being only about 5°. There is good evidence that the north-south plane is in fact the fault plane: this is indicated by observations of slickensides at the surface, the extension of isoseismic contours in the north-south direction, aftershock distribution, and by detailed studies of the focal mechanism (Schick, 1968). For this study the north-south striking plane was assumed to be the fault plane, and in this case the sense of the strike-slip motion is left lateral.

3. A **Model for Tidal Triggering**

In order to apply statistical tests to the data a model for the process under consideration must be formulated. A simple but reasonable physical model is that fracture occurs on the fault plane when the shear stress applied to it exceeds the cohesive strength of the fault, which may depend upon the normal stress on the fault plane. The applied stress will consist of the unknown accumulating tectonic stress upon which is superimposed the stress due to the solid earth tides. Whilst the stresses due to the tides are much smaller than the tectonic stresses normally expected, they vary rapidly, the major constituents having diurnal and semidiurnal periods, and so on this simple model may be expected to trigger the occurrence of earthquakes, provided that the rate of increase of tectonic stress immediately prior to an earthquake is relatively small. This model suggests that aftershocks are less likely to be tidally triggered than main shocks occurring after a period of quiescence, since at times when the crust is in a disturbed state the rates of change of parameters other than the tidal strains will be greatest. Different authors have adopted different approaches: Heaton (1975) used only main shocks, whereas Shlien (1972) specifically analysed an aftershock sequence. In view of the small size of the sample and the lack of a suitable criterion for identifying aftershocks, we have chosen to include all the recorded events.

On the basis of these ideas, two quantities were taken as being of interest: the tidal shear strain on a vertical plane of strike $N 10^{\circ}$ E (close to the mean of the fault plane solution orientations), and the tidal linear strain (extension) normal to this plane. The first of these quantities is proportional to the tidal shear stress on the fault plane; the second quantity is closely related to the tidal normal stress on the plane and might be expected to influence the strength of the fault.

For this fault, the tidal normal extension as a function of time has a predominantly 24 h period and is approximately in phase with the diurnal part of the local gravitational potential, whereas the shear strain is a mixture of components varying both diurnally and semidiurnally in quadrature with the components of the local gravitational potential. In both cases intermodulation of the lunar and solar components of the tide produces variations in amplitude over periods of about two weeks.

The theoretical solid earth strain tide was calculated using a program by Berger (Bilham et al., 1972). This program calculates the linear horizontal tidal extension at a given position and orientation. The shear strain on the fault is merely half the difference between the extensions in two directions at 45° on either side of the fault. Ocean load tides were not taken into account, but Beavan (1976) has calculated ocean load tidal strains at Schiltach, latitude $48^{\circ}17'$ N, longitude $8^{\circ}21'$ E, some 50 km from the main earthquake region, and shown that their contribution to the total theoretical tidal strains is small. The ocean loading correction makes a difference of less than 10° to the phase of the theoretical M2 tide at Schiltach, a result supported by measurements, so it seems reasonable to neglect ocean load tides for our purpose.

The theoretical earth tide is calculated for a laterally homogeneous, isotropic earth model. However, local inhomogeneities near the fault will modify the effective strain in its vicinity so that the real strain at a point will differ from the calculated strain (Bilham et al., 1974). Since there is no means of allowing for this effect, the theoretical strain must be used; but if the inhomogeneities present are on a scale small compared with the size of the fault plane, the theoretical strain field will be representative of the average strain over the fault.

4. Tests of the Triggering Hypothesis

The suggestion that these earthquakes may be tidally triggered is here tested using standard methods, the comparison of which we believe to be important. The techniques of Sect. 4.1 have been chosen because they deal directly with the information available on the amplitudes of the tidal strains at the times of the earthquakes, and being "non-parametric" tests do not involve assumptions about the distributions of these amplitudes, nor do they require the grouping of the observations into bins. Those of Sect. 4.2 have been more commonly applied in the past and are simpler in application, but deal less directly with the data in requiring the definition of a "phase" of the tide for each earthquake. In Sect. 5 we give the results of a study of the cross-correlation of the earthquake origin times with the tidal strain functions in order to obtain a qualitative comparison of this method with the others.

4.1. Tests on the Distribution of Strain Magnitudes at Origin Times

Following Shudde and Barr (1977) we tested the null hypothesis that the distribution of the tidal strains at the origin times of the earthquakes may have arisen by random sampling of the tidal strain curve. To do this, it is necessary to obtain a reference distribution, which may be formed by sampling the theoretical tidal strain curve at a large number of random times. The number of random samples must be sufficiently large that they serve to define the underlying distribution. If the number of random samples is not made considerably larger than the number of earthquakes, a difference between the two distributions could be due to random fluctuations in the artificially generated distribution resulting from the sampling process rather than to an effect associated with the distribution corresponding to the real earthquakes. We used 10,000 samples for these tests.

Pseudo-random numbers were generated using the linear congruential generator

 $v_{i+1} \equiv 41475557 v_i \pmod{2^{28}}$

where y_i is the *i* th number to be generated. Dieter (1971) indicates that this generator is of high quality. Random times uniformly distributed through the period of the catalogue were generated by a linear transformation from the numbers y_i and the tidal strains calculated for the random times exactly as for the real earthquakes. The histogram of the strains thus obtained was taken as the reference distribution against which the histogram of the strains at the times of the real earthquakes could be compared.

Figure 1 shows the histograms and cumulative distribution functions formed by these procedures, and it may be seen that the distribution of normal tidal strain calculated for the real earthquake origin times appears to differ little from the corresponding distribution for randomly generated events, but that in the case of the shear strain the difference is greater.

The significance of this difference was assessed using two tests: the Kolmogorov-Smirnov test and the Wilcoxon-Mann-Whitney test (Siegel, 1956), which make no assumptions concerning the form of the distributions in question. The Kolmogorov-Smirnov test uses as its statistic the largest difference between the two cumulative distribution functions; this is the largest vertical gap between the curve for the real earthquakes and the curve for randomly generated events in Fig. 1c or d and may be written as

 Max $|F(x) - G(x)|$

where $F(x)$ is the fraction of real earthquakes which occurred when the strain under consideration (normal or shear) was less than or equal to *x*, $G(x)$ is the fraction of randomly generated times at which the strain was less than or equal to *x,* and the maximum is taken over all values of *x.* The Wilcoxon-Mann-Whitney test uses the sum of the ranks of the observations from one of the distributions when the observations making up both distributions are all ranked together; this is related to the sum of the number of observations from one distribution which are less than each of the observations in the other distribution. Both tests are simple to apply, and the sampling distributions of the statistics, which give the probability associated with a particular value of the statistic calculated, are well known. (See, for example, Massey, 1951

Fig. l a and b. Histograms of shear and normal strains on the fault. *Stepped line:* for the earthquake origin times. *Continuous curve:* for random events clustered round the origin times. *Broken curve :* for random events uniformly distributed in time. c and d Cumulative distribution functions of shear and normal strains. *Arrowed curve:* for earthquake origin times. *Unmarked continuous curve :* for clustered random events. *Broken curve :* for uniformly distributed random events

and Mann and Whitney, 1947; Siegel, 1956 gives other references.) One-tailed tests are appropriate since the alternative hypotheses are directional: that is, if there is triggering the origin times will be displaced towards times when the left lateral shear strain or the normal extension is greatest, rather than towards either of the possible extremes of these quantities. Application of these tests leads to the significance levels shown in Table I ; these are the probabilities under the null hypothesis of the statistic calculated taking on a more extreme value than that actually found for the data.

Test used	Strain component	
	Shear	Normal
Wilcoxon-Mann-Whitney	0.015	> 0.1
Kolmogorov-Smirnov	0.025	> 0.1
Schuster	0.14	0.62

Table 1. Significance levels

It is necessary to consider in addition the effect of the existence of aftershock sequences on the results of the above tests, since it is possible that clustering of the earthquakes in time will result in a different distribution of the tidal strain amplitudes from that generated by a uniform distribution of earthquakes in time, even when there is no influence of the tidal strain in either case. It is not necessary to omit all aftershocks since an earthquake occurring only a few hours after the previous one occurs at a very different value of the strain, but the possibility of an effect must be investigated. Sets of 10,000 random times, clustered in a way approximating the clustering of the origin times, were therefore generated and the distributions of strain amplitudes calculated. These sets of times were drawn from a set of overlapping Gaussian distributions, each of which was centred on an origin time, so that the probability of sampling the tidal strain curve was greater close to the times when the real earthquakes had occurred. By choosing the standard deviation of the Gaussian distributions appropriately, the basic clustering in time of the real earthquakes is preserved in the reference set of times, but the latter will still sample the tidal strain curve randomly with respect to the short term variations to form the reference distribution of strains. Taking, on this basis, 25 h as a suitable value for the standard deviation leads to the distributions displayed in Fig. 1, from which it may be seen that they differ little from the distribution of strains obtained from uniformly distributed random times. This indicates that the departure of the distribution of shear strains at the origin times from that of shear strains at uniformly distributed random times cannot be attributed merely to clustering of the origin times. Indeed, if the distribution of shear strains at the clustered random times is taken as the reference distribution in the Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov tests applied as above, significance levels of 0.022 and 0.031 respectively are found, differing little from the significance levels obtained using the uniformly distributed random times. Whilst the reference distribution is now no longer strictly independent of the data, these significance levels are nevertheless still the probabilities associated with the null hypothesis that the distribution of shear strains at origin times arose from random sampling of the distribution obtained by the procedure described above. It was noted that the reference distributions of strains seemed insensitive to the method used to produce them: those shown in Fig. **1,** those formed by clustering the times with a 50 h Gaussian, and those formed by sampling at regular intervals of time (provided the interval was not a tidal period) all fall very close together.

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4.2. Tests on the Distribution of Tidal Phases

Knopoff (1964), Shlien (1971), Heaton (1975), and Klein (1976) all use tests based on the relationship of the time of occurrence of an earthquake to the times of the preceding and succeeding extrema in the curve of the chosen tidal function. Thus a tidal " phase " for the time of an event may be defined, for example, by taking the length of time from the maximum of the tidal strain curve immediately preceding the event to the maximum immediately following it as the local "period" of the tide, and dividing the length of time from the preceding maximum to the event by this period. The result is usually multiplied by 360° or 2π for convenience. This method makes no use of the value of the tidal strain (or other function) at the time of the event, using the tidal calculations only to determine the times of adjacent extrema, and has the disadvantage that as the amplitude of a tidal function will often differ greatly between adjacent maxima, the phase may be determined by a comparatively insignificant peak in the tidal curve. The advantage of the method is that under the null hypothesis the distribution of such phases is uniform across the interval chosen to represent a full period, and so is immediately available for comparison with the observed distribution. This is in contrast to the situation with regard to the strain amplitudes, when the distribution under the null hypothesis had to be ascertained by the random sampling methods of Sect. 4.1.

Figure 2 shows the histograms, plotted as rose diagrams, of the tidal phases defined by reference to the maxima nearest to the earthquake origin time in the shear and normal strain curves. A tendency for the earthquakes to occur around the times of greatest left lateral shear strain on the fault is discernable, but no such effect can be seen for the normal strain. The statistical test customarily used to examine the significance of the departure of such a distribution from uniformity was first proposed by Schuster (1897) and is clearly described by Heaton (1975) and Shlien (1971). Essentially, the phase for each earthquake is taken as the angle of a unit vector in two dimensions with respect to an

Fig. 2a and b. Histograms of phases at earthquake origin times of a left lateral shear strain; b normal linear strain. Phase increases clockwise from the top axis, which corresponds to maximum strain. The lengths of the axes are equal to the radius of a sector containing 40 events

axis; the statistic calculated for the distribution is the magnitude of the vector sum of all such vectors. Under the null hypothesis the unit vectors will perform a random walk in two dimensions, and the probability that the resultant vector will then have a magnitude greater than or equal to *R* is $\exp(-R^2/N)$, when there are N earthquakes.

The probabilities obtained for the distributions of phases with respect to the normal and shear strains are given in Table 1.

4.3. Interpretation of Significance Levels

The various significance levels we have calculated cannot easily be combined into a single significance level for the rejection of the null hypothesis. It is reasonable to interpret the results from the Wilcoxon-Mann-Whitney tests and the Kolmogorov-Smirnov tests as simply corroborating one another, in that they looked at differences between the same pairs of distributions, and to omit from consideration the results of tests on the distribution of phases since this test appears to be rather weak. However, we performed tests of the null hypothesis using two distinct alternative hypotheses, one involving the shear strain and the other the normal extension at the fault. Thus the probability under the null hypothesis that one of the results should reach a particular significance level is increased: clearly repeated trials of even a true null hypothesis will yield a "significant" result sooner or later by chance. If the two trials were independent then the significance levels could be combined by a standard technique such as that of Fisher (1970, pp. 99-100); but since the shear strain and normal extension are related to one another through the tidal potential this is not possible. This difficulty becomes acute when even more tidal functions are used, as is apparent in the study by Shudde and Barr (1977) where significance levels for fourteen tidal functions are calculated. Although several of these are very small, and would be deemed highly significant if taken alone, Shudde and Barr attribute them to random fluctuations under the null hypothesis, and conclude that they provide no evidence for tidal triggering.

We do not therefore attempt to give an overall significance level for a trial of the null hypothesis, but present the separate significance levels as the best indication of the ability of this data to reveal a tidal triggering effect.

5. Cross Correlation of Earthquake Times and Tides

Knopoff (1964) calculated the cross correlation function of the time sequence of southern Californian earthquakes with the tidal acceleration, treating the earthquakes as a series of delta functions in time. This cross correlation is given by

$$
C(\tau) = \int_{n=1}^{N} \sum_{n=1}^{N} (t - t_n) f(t - \tau) dt = \sum_{n=1}^{N} f(t_n - \tau)
$$

Fig. 3a and b. The cross correlation function for a left lateral shear strain; **b** normal linear strain. *Continuous curves:* earthquake origin times. *Broken curves.* times obtained by adding random numbers to the origin times

where $f(t)$ is the tidal function of interest and the N earthquakes occurred at the times t_n . This calculation was repeated for the Swabian Jura earthquakes, using for f the tidal normal and shear strains. The results are shown in Fig. 3, together with the cross correlations calculated for sets of 259 random times for comparison. The random times were generated in this case by the formula

$$
r_n = t_n + 48a_n - 24
$$

where r_n is the *n* th random time in hours and a_n is a pseudo-random number uniformly distributed between 0.0 and 1.0 produced as described in Sect. **4.1** above.

In neither the case of the shear strain nor that of the normal strain are the peaks in the cross correlation function larger for the times of the real earthquakes than for the sets of random times. Such a result provides no evidence for a tidal triggering effect; however, it is of interest that the peak of the correlation for the -shear strain does occur at exactly zero lag. The statistical

significance of such correlations is difficult to assess quantitatively. The cross correlation technique would be useful for demonstrating phase lags between a variable associated with triggering and the origin times, but this comparison shows that a much stronger effect or a much larger data set is necessary before it can be applied with confidence. We think that the methods of Sect. 4.1 are a more sensitive way to test the null hypothesis.

6. Discussion

Assuming that adequate allowance has been made for the clustering in time of the earthquakes, statistical tests based on the distributions of the strains at the times of earthquakes provide weak evidence that the origin times of the earthquakes in the Swabian Jura are not independent of the tidal shear strain. There is no evidence of any influence of the tidal normal extension. The implication for the simple model we have used is that the effect of change, due to the tidal normal extension, in the cohesive force preventing relative movement across the fault is small compared to the effect of tidal enhancement of the tectonic shear stress on the fault. Heaton's (1975) results also indicate that the shear stress plays the predominant role in any tidal effect, although the correlation he observes is not present in the shallow strike-slip earthquakes amongst his sample, and Klein (1976) explains many of his results by tidally enhanced shear on fault planes.

Statistical tests carried out on the distribution of the strains at the origin times of the earthquakes appear to be more sensitive in detecting a tidal effect than tests on the distribution of tidal phases. This is presumably because the strains at the actual times of the earthquakes are taken into account in the former kind of test. However, if an effect is accepted, analysis in the time rather than in the strain domain gives more information about the relationship of the earthquake times to features of the tidal strain curve, and both the histogram of phases and the cross correlation function may be useful in this respect.

On the basis of the significance levels obtained we cannot conclude that tidal triggering has been demonstrated; merely that the results are indicative that an effect may be present. Indeed, the tidal effect for earthquakes of this type appears to be too weak to be clearly demonstrable in a catalogue of this size and it would be most desirable to extend the catalogue in time rather than in space. However, when a catalogue from a larger geographical region is used, it is important that attention be paid to the details of the fault plane solutions. It is also essential that statistical tests be carried out only on the basis of a definite physical model in order to avoid the effects of multiple alternative hypotheses.

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Appendix

Times of occurrence of Swabian Jura earthquakes Origin times given as year (minus 1900), month, day, hour (GMT) and minute.

References

Beavan, R.: Earth strain: ocean loading and analysis. PhD dissertation, University of Cambridge 1976

- Bilham, R., Evans, R., King, G., Lawson, A., McKenzie, D.: Earth strain tides observed in Yorkshire, England, with a simple wire strainmeter. Geophys. J. **29,** 473-485, 1972
- Bilham, R., King, G., McKenzie, D.: Inhomogeneous tidal strains in Queensbury tunnel, Yorkshire. Geophys. J. 37, 217-226, 1974
- Dieter, U.: Pseudo-random numbers: the exact distribution of pairs. Math. Comp. 25, 855-883, 1971
- Fisher, R.: Statistical methods for research workers, 14th ed. Edinburgh: Oliver & Boyd 1970 Heaton, T.: Tidal triggering of earthquakes. Geophys. J. 43, 307-326, 1975
- Klein, F.: Earthquake swarms and the semidiurnal solid earth tide. Geophys. J. **45,** 245-295, 1976
- Knopoff, L.: Earth tides as a triggering mechanism for earthquakes. Bull. Seism. Soc. Am. **54,** 1865-1870, 1964
- Mann, H., Whitney, D.: A test of whether one of two random variables is stochastically larger than the other. Ann. Math. Stat. **18,** 50-59, 1947
- Massey, F., Jr.: The Kolmogorov-Smirnov test for goodness of fit. J. Amer. Statist. Ass. 46, 68-78, 1951
- Schick, R.: Untersuchungen über die Bruchausdehnung und Bruchgeschwindigkeit bei Erdbeben mit kleinen Magnituden (M < 4). Z. Geophys. **34,** 267-286, 1968
- Schneider, G.: Erdbeben und Tektonik in Siidwest-Deutschland. Tectonophysics. 5, 459-511, 1968
- Schneider, G.: Seismizität und Seismotektonik der Schwäbischen Alb. Stuttgart: Ferdinand Enke Verlag 1971
- Schuster, A.: On lunar and solar periodicities of earthquakes. Proc. Roy. Soc. **61,** 455-465, 1897
- Shlien, S.: Earthquake tide correlation. Geophys. J. 28, 27-34, 1972
- Shudde, R., Barr, D.: An analysis of earthquake frequency data. Bull. Seism. Soc. Am. 67, 1379-1386, 1977
- Siegel, S.: Nonparametric statistics for the behavioural sciences, international student edition. Tokyo: McGraw-Hill Kogakusha 1956