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Q of Mode $_0S_0$*

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Abstract. Estimates of Q of mode $_0S_0$ as measured after the Chilean (1960) and Alaskan (1964) earthquakes showed large scatter. The Indonesian quake of August 19, 1977, has provided a new opportunity to determine the attenuation factor for $_0S_0$. Time lapse spectra of *a priori* selected high quality data were analyzed using a maximum-likelihood method and Ricean statistics. Data from South Pole and Los Angeles gave Q values of $6324(1 \pm 21\%)$ and $6859(1 \pm 17\%)$ respectively. Taken together the result is $6687(1 \pm 13\%)$.

Key words: Q-$_0S_0$-Time lapse spectra – Maximum Likelihood Method – Ricean statistics.

Introduction

The value of Q for the purely radial mode of free oscillation $_0S_0$ should be one of the most reliable of physical quantities for the earth. It is uncontaminated by the effects either of the earth's rotation or of asymmetries in the earth's internal constitution. Furthermore its value appears to be high, so its signal persists for an extremely long time. Despite these attractive features, it has only been observed infrequently and the values of Q reported for this mode have a wide variation.

Until recently, analysis of mode $_0S_0$ has focussed on recordings made from only two earthquakes, the great Chilean (1960) and Alaskan (1964) earthquakes.

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Table 1. Values of Q for ${}_0S_0$ for Alaska and Chilean Earthquakes

Q	Author	Earthquake – Station
>7,500	Ness et al. (1961)	Chile (1960) – UCLA
900	Smith (1961)	Chile (1960) – Isabella
>25,000	Slichter et al. (1966)	Alaska (1964) – UCLA
13,000 ± 2,000	Slichter (1967)	Alaska (1964) – UCLA
4,229 ± 5%	Sailor and Dziewonski (1978)	Alaska (1964) – $UCLA_4$
3,996 ± 11%	Sailor and Dziewonski (1978)	Alaska (1964) – $UCLA_7$

Values of Q for this mode have been reported as listed in Table 1. Most of these analyses have been made from recordings made on the UCLA gravimeters/ultra-long period seismometers. For both earthquakes, analyses of these records made by measuring the spectral amplitude in each of several time-lapsed intervals, give values on the order of thousands. The much lower value by Smith was obtained from the spectral line shape, coincidentally from the only record in the list not taken at UCLA; we have performed a subsequent analysis which shows that the line will be broadened to Smith's value, if Q is of the order of thousands and account is taken of the $\sin x/x$ transform of the record window. The variation among the high values may be attributable to significant power input from even small amounts of aftershocks and from major glitches due to instrumental defects. Further, Sailor and Dziewonski (1978) have selectively rejected a number of estimates for some of the lagged points on *a posteriori* grounds; among the points rejected are those with the largest spectral amplitudes.

A new illustration of the variability of determinations of Q for this mode is given by Buland et al. (1979). Indicated Q values for five records of the Sumbawa earthquake of August 19, 1977, range from 2950 to 8090. The result of stacking all five records yielded a Q of 4100. The possibility continues to exist, in view of the large variation in values of Q, that all are compatible with each other within rather large error bounds. If this is the case, the problem of Q for ${}_0S_0$ is therefore reducible to one in which the identifiable errors are minimized; this, in turn, implies that we be able to compute the size of the residual error and test whether the values reported earlier are consistent.

II. Data and Analysis

As with Buland et al. (1979), the Indonesian earthquake of August 19, 1977, provided a rare opportunity to detect the radial mode of lowest order ${}_0S_0$ in a relatively undisturbed condition. Our data were collected with the ultra-long period seismometers (Nakanishi et al., 1976) in operation at the South Pole (SPA) and at UCLA (LMS). This earthquake, occurred at coordinates 11.1°S, 118.5°E., $h=20$ km and origin time 06:08:55.2 UT and had a surface wave magnitude of 7.9, all data from PDE.

The output of the seismometer, as described in Nakanishi et al., (1979) is analog filtered in the pass band 270 s to 7200 s at the 3db points to suppress the diurnal and semi-diurnal tides at the long period end and to suppress large amplitude surface waves at periods around 20 to 30 s. The filtered output

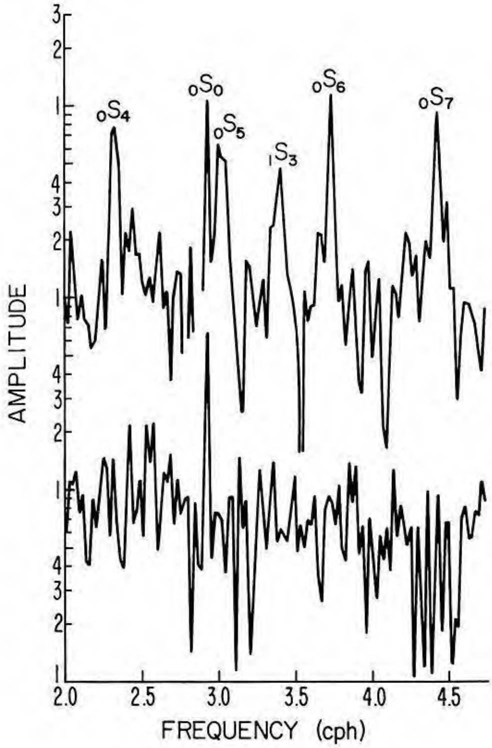

Fig. 1. Portion of spectrum at UCLA showing $_0S_0$ and other nearby modes. The upper curve is for the section starting at 40h. The lower is for the section starting at 386 2/3h. The modes other than $_0S_0$ are imbedded in the noise in the *lower curve*. The lower curve should be multiplied by 10

is sampled at 10 s intervals and recorded digitally. The records contain several large aftershocks and other large earthquakes as well as some unexplained glitches which are of instrumental origin. Only stretches of data without identifiable glitches or other interruptions were used in the data analysis. *No deglitching was applied to the data.* After this *a priori* selection of the raw seismographic data, no further selection has been applied. The preceding three sentences describe the basic differences between the present work and the preceding.

We have chosen a record length of 16,200 sample points of 45 h for each of the record segments to be analyzed. This number is appropriate for use with Singleton's Fast Fourier Transform since $16200 = 2^3 \times 3^4 \times 5^2$. Further, the effect of the periodicity of $_0S_0$ incommensurate with the record length is minimized. The nominal period of $_0S_0$ is 2.93246 cph or a period of 1227.64 s = 20.4606 min. The error in amplitudes due to the fact that 162000 s is 131.961 cycles of $_0S_0$ rather than an integral number of cycles is

$$1 - \frac{\sin 2\pi(0.039)}{2\pi(0.039)} = 1\,\%. \tag{1}$$

The record length is sufficiently long to resolve $_0S_0$ from $_0S_5$. The contribution of $_0S_5$ to amplitudes of $_0S_0$ from a similar $\sin x/x$ calculation is at most 2%. The rapid decay of $_0S_5$ compared with $_0S_0$ helps make this error small after two days (Fig. 1).

Table 2. Time lapse spectral estimates for ${}_0S_0$. (Note that start time is measured from 21 h 20 min on August 19, 1977, i.e., more than 15 h after the quake; each record length = 45 h)

Start time (hours)	${}_0S_0$ Amplitude (mV/cph)	Noise Level (mV/cph)	Start Time (hours)	${}_0S_0$ Amplitude (mV/cph)	Noise Level (mV/cph)
South Pole			UCLA		
0	9.1368	1.5	40.00	10.304	1.2
80.67	6.3314	1.0	122.67	9.6377	1.1
184.00	5.9886	0.9	194.67	8.1930	1.5
244.67	4.9114	1.0	314.67	6.6027	1.1
352.27	5.1269	1.2	386.67	6.5052	0.9
458.67	4.2927	0.9	462.67	5.6644	1.4
506.67	3.5833	1.2	507.75	5.5252	1.3
554.67	3.3947	1.1	578.67	5.9279	1.7
			626.67	4.4317	1.6

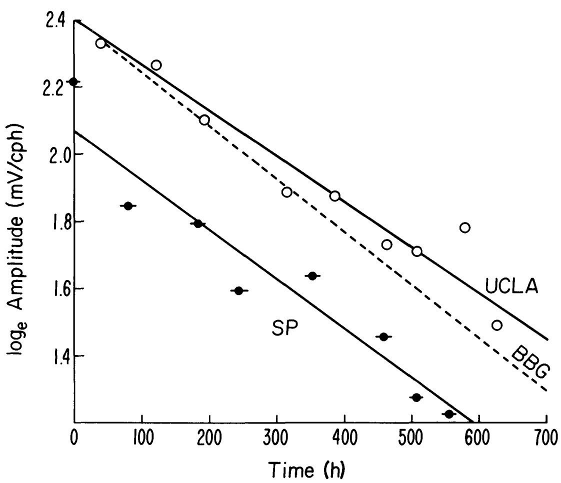

Fig. 2. Logarithmic spectral amplitudes for records at UCLA and South Pole. A straight line (*BBG*) with slope appropriate to $Q = 4100$ is indicated (Buland et al., 1979)

Spectra were taken of record segments in the frequency neighborhood of ${}_0S_0$ for the recordings at both SPA and LMS. As noted, the record segments are 45 h long and we exclude extraneous seismic events and other glitches. No overlap of record segments was allowed in either of the two station analyses. Eight time lags were taken for the South Pole record and nine for the UCLA record, a total of 275,400 points in all. For later use, the noise level in the neighborhood of ${}_0S_0$, is listed along with the results in Table 2. Time is measured from 21h 20m August 19, 1977. The amplitudes are given, in millivolts per cph at the output of the meters.

III. Maximum Likelihood Analysis

A linear least-squares fit through the points displayed in Fig. 2 is inappropriate because of the bias introduced by the logarithmic operator. Maximum likelihood methods are more appropriate since they take into account the signal-to-noise ratio. However, the model for the noise must be considered carefully. We report in Table 2 and Fig. 2 spectral amplitudes without regard to phase. Because spectral amplitude is a scalar positive variable, its noise spectrum cannot be taken to be Gaussian; if it were Gaussian, this would imply that the tabulated values, which are the result of adding noise to the signal, could under some circumstances be negative.

The Ricean distribution function is appropriate for absolute spectral amplitudes, if the noise on the seismogram is Gaussian. Rice (1944, 1945, 1948; see also Middleton, 1968, Chapter 9) appears to have been the first to have shown that the probability of finding peak spectral amplitude estimates between A_i and and $A_i + dA_i$ is

$$P(A_i)\,dA_i = \frac{A_i}{N_i^2} \exp\{-\tfrac{1}{2}(A_i^2 + S_i^2)/N_i^2\}\, I_0\left(\frac{A_i S_i}{N_i^2}\right) dA_i \tag{2}$$

where S_i is the signal strength and N_i is the root mean square noise level of the system in the absence of the signal. We take the signal strength for the i^{th} time lapse t_i to be

$$S_i = A_0 \exp(-\pi t_i/QT) \tag{3}$$

where T is the period of mode ${}_0S_0$.

For large S_i/N_i we find, from the first term of the asymptotic expansion of the modified Bessel function I_0, that

$$P(A_i)\,dA_i \approx \left(\frac{A_i}{2\pi S_i}\right)^{\frac{1}{2}} \frac{1}{N_i} \exp\{-\tfrac{1}{2}(A_i - S_i)^2/N_i^2\}. \tag{4}$$

This distribution is approximately Gaussian. If it were Gaussian we would have

$$P_i(A_i)\,dA_i = \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \frac{1}{N_i} \exp\{-\tfrac{1}{2}(A_i - S_i)^2/N_i^2\}. \tag{5}$$

From Table 2, the ratios of A_i/N_i vary between 3.4 and 8.8. Let us take as a most unfavorable case, the values $S_i/N_i = 2.0$ and $A_i/S_i = 0.7$. We find that the functions in (2) and (4) differ by $5\frac{1}{2}\%$. Since this upper bound is small, we use (4) in our maximum likelihood estimation. The differences between (2) and (5) for these same conditions are about 13% and cannot be considered to be small. We have performed the maximum likelihood analysis on the values of Table 2 using the expression

$$\operatorname{Min} \Phi = -\operatorname{Max} \sum_i^{8 \text{ or } 9} \log P(A_i) \tag{6}$$

using the approximation (4) to Ricean statistics. The minimization is performed with regard to the variables Q and A_0 in (3). We get $Q_0 = 6324$ for ${}_0S_0$ measured at the South Pole and 6859 at UCLA.

With regard to estimates of error in the values of Q, we can write

$$\Phi \simeq \Phi_o + {}^1/_2\,(\delta Q)^2/\sigma_Q{}^2 \tag{7}$$

since the function $\exp\Phi$ is normal in the vicinity of the extremum by the central limit theorem. In the above expression Φ_o is the value of Φ at the extremum, $\sigma_Q{}^2$ is the variance we are seeking and δQ is a variation in Q from the value which yields the extremum. We obtain σ_Q by finding the value of δQ such that

$$\Phi - \Phi_o = {}^1/_2 \tag{8}$$

This method was used in Knopoff (1961). We find the results

$$\begin{array}{ll} \text{South Pole} & Q=6324\ (1\pm 21\%), \\ \text{UCLA} & Q=6859\ (1\pm 17\%). \end{array} \tag{9}$$

The error is listed as a multiplicative percentage rather than as an additive percentage because of the exponential character of the operation involving the factor Q.

Since the amplitude of mode ${}_0S_0$ is uniform over the entire surface of the earth, we can combine the data of both stations after allowance for the different sensitivities of the two instruments. The instrument at South Pole is 1.38 times less sensitive than that at UCLA. The 17 record segments now overlap in time but still contain independent estimates. The analysis yields

$$Q=6687\ (1\pm 13\%)$$

Our results are not in significant disagreement with that of Buland et al. (1979) for their longest record segment.

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