J. Geophys. 48, 158–160, 1980

Journal of Geophysics

High Precision Measurement of the Frequency of Mode ${}_0S_0$ *

W. Zürn[1], L. Knopoff[2], and P.A. Rydelek[2]

1 Geowissenschaftliches Gemeinschaftsobservatorium, Universitäten Karlsruhe/Stuttgart, Heubach 206, D-7620 Wolfach, Federal Republic of Germany

2 Institute of Geophysics and Planetary Physics, University of California at Los Angeles, Los Angeles, California 90024, USA

Abstract. Mode ${}_0S_0$ was excited by the Indonesian earthquake of 19 August 1977 and recorded at the South Pole and at UCLA. Phase estimates from 16 independent time lapse spectra were used to determine the frequency of this mode to high precision. The result is $f({}_0S_0) = 2.932851$ cph with a standard deviation of 11 ppm.

Key words: Frequency – ${}_0S_0$ – Time lapse spectra – Sumbawa earthquake.

Introduction

The frequencies of the earth's free vibrations form an important set of data and are used to infer the global average internal structure of the earth. The precision with which these eigenfrequencies can be measured depends in part on the length of seismic records available for spectral analysis; these depend in turn on the initial amplitude of a mode after an earthquake, its specific attenuation factor Q and the noise level of the instrument and/or station at the time. Recorded noise may be due to instrumental disturbances, aftershocks or earthquakes at other locations and to meteorological sources. The influence of changes in atmospheric mass on recordings by a gravimeter, for example, has been demonstrated by Warburton and Goodkind (1977) and Slichter et al. (1979) for tidal periods; these processes must certainly be important in the free mode band of periods as well.

The measurement of the frequency of mode ${}_0S_0$ is favored by its high Q; on the other hand it is excited only by very large earthquakes with a usable signal-to-noise ratio. Derr (1969) reports four observations of this mode; the present-day standard of measurement of the frequency of ${}_0S_0$ is (Slichter 1967)

$$f({}_0S_0) = 2.9324 \pm 0.0003 \text{ cph}$$

The Sumbawa earthquake of 19 August 1977 has provided an additional opportunity to measure the periods of this and other low order modes for the earth (Buland et al. 1979; Knopoff et al. 1978; Knopoff et al. 1979; Linton et al. 1979, Riedesel et al. 1979).

* Contribution No. 229, Geophysikalisches Institut, Universität Karlsruhe, Hertzstr. 16, D-7500 Karlsruhe, Federal Republic of Germany and Publication No. 2071, Institute of Geophysics and Planetary Physics, University of California, Los Angeles

Measurement of the Frequency of ${}_0S_0$

Knopoff et al. (1979) determined Q of ${}_0S_0$ from recordings of the Sumbawa earthquake using ultralong period seismographs at South Pole (SPA) and Los Angeles (LMS). For the determination of Q for this mode, we used amplitude information from time lapse spectra while phase information was ignored. Details about the data and the data reduction can be found in the above paper. The phases for the different portions of the records are listed in Table 1.

A fast Fourier transform of the longest clean record available from the two stations (9,900 min starting at minute 332,480.0 in 1977 from SPA) gave the following spectral amplitudes in the vicinity of ${}_0S_0$:

$$f_{483} = 2.927273 \text{ cph} \quad A_{483} = 1.2540$$
$$f_{484} = 2.933333 \text{ cph} \quad A_{484} = 7.3647$$
$$f_{485} = 2.939393 \text{ cph} \quad A_{485} = 1.7700$$

Table 1. Phase estimates for $f = 2.933333$ cph as a function of lapse time of the start of each record (in minutes from 0000 hours U.T., 1 January 1977). The origin time of the earthquake was 331568.92 min. Each record was 45.0 h long. Corrections t_{ci} were measured with stop-watch against time signals. The sample denoted with a question mark was deleted in the final analysis. Reference time $t_0 = t_1$

t_i (min)	$t_i - t_1$ (h)	Station	t_{ci} (s)	Phase (°)	Phase (cycles)
332480	0.00	SPA	0.4	−164.567	−0.4571
334880	40.00	LMS	− 1.5	73.670	0.2046
337320	80.67	SPA	0.3	− 13.654	−0.0379
339840	122.67	LMS	−12.5	− 84.945	−0.2360
343520	184.00	SPA	0.2	− 49.177	−0.1366
344160	194.67	LMS	−13.5	−141.703	−0.3936
347160	244.67	SPA	0.1	− 16.274	−0.0452
351360	314.67	LMS	−16.0	−122.898	−0.3414
353616?	352.27	SPA	0.0	−147.189	−0.4089
355680	386.67	LMS	−17.0	−170.921	−0.4748
360000	458.67	SPA	− 0.1	122.919	0.3414
360240	462.67	LMS	−18.5	−131.794	−0.3661
362880	506.67	SPA	− 0.1	177.294	0.4925
362945	507.75	LMS	−19.0	130.097	0.3614
365760	554.67	SPA	− 0.2	− 64.893	−0.1803
367200	578.67	LMS	−20.5	146.084	0.4058
370080	626.67	LMS	− 0.5	−125.273	−0.3480

0340-062X/80/0158/$01.00

where the subscripts are the estimate number in the Fourier spectrum of the record and the amplitudes, A, are in relative units. From the near symmetry of these amplitudes we conclude that the frequency of ${}_0S_0$ must be very close to f_{484} and is bounded by

$$2.936363 \text{ cph} > f({}_0S_0) > 2.930303 \text{ cph}$$

where the two limiting values are the midfrequencies between the three spectral estimates. If the phase of the mode is not affected by earthquakes subsequent to the start, t_1, of the first lapsed record, then the phase information from the time lapse spectra can be used to measure $f({}_0S_0)$ more accurately.

The phases in the spectra of the records are shifted with respect to those in the actual ground motion, due to instrumental phase shifts that include the effects of the free mode filters (Nakanishi et al. 1976). At both stations the frequency responses are sufficiently close to identical that we assume these phase shifts to be the same for all spectra; no corrections for instrumental effects have been applied.

The quartz clocks that control the digital sampling of the data drift at both stations. This drift was checked regularly against time signals and appropriate corrections (Table 1, column 4) have been applied.

A zero order approximation to the phases of ${}_0S_0$ in the lapsed records is given by the phases for the frequency $f_0 = 2.933333$ cph. The phases for f_0 differ from those of ${}_0S_0$ by an amount which depends on the difference between f_0 and $f({}_0S_0)$, the Q of the mode, and the length of the time series. Since all these factors are the same for all spectra, no corrections needed to be applied for these effects.

The phase p_i (in cycles) of the mode is defined to decrease linearly with starting time, t_i, for each segment of the seismogram, namely

$$p_i = -f({}_0S_0)\cdot(t_i - t_0) + p_0 \tag{1}$$

where (t_0, p_0) are a reference time and phase. Because phase is uncertain by an integer number of cycles, the results are only describable as p_i (modulo 1.0). The straight line of Eq. (1) is therefore mapped onto a sawtooth function with unknown slope (Fig. 1). There are many cycles of ${}_0S_0$ between the different starting times t_i of the record segments. Without *a priori* bounds on $f({}_0S_0)$ and in the presence of noise, it would be difficult to find the sawtooth-function that fits the data in a maximum likelihood sense. There is an obvious analogy to the aliasing problem of spectral analysis.

The *a priori* knowledge gleaned from the spectral analysis of the 9,900 minute record described above permits us to "demodulate" the sawtooth function. If we choose the "carrier-frequency" to be close enough to $f({}_0S_0)$, then the demodulated phases p_i' will span a phase interval of less than one cycle over all the record segments, $i=1,2,\ldots$ We choose the estimate $f_0 = 2.933333$ cph from the above analysis as the demodulating frequency and let $\delta f = f({}_0S_0) - f_0$. Equation (1), as modified, is,

$$p_i' = -\delta f\cdot(t_i - t_0) + p_0 \tag{2}$$

where $p_i' = [p_i + f_0\cdot(t_i - t_0)]$ (modulo 1.0). In other words, the value of f_0 allows us to estimate the number of full cycles in the interval $(t_i - t_0)$. If this estimate is accurate, then p_i' will satisfy the straight line equation within one cycle, except for noise. The demodulated phases are plotted in Fig. 2 and we note that f_0 is sufficiently

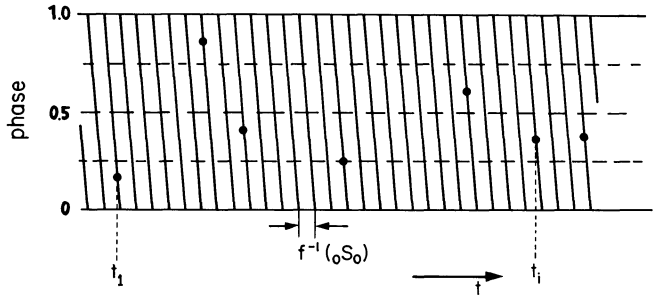

Fig. 1. Sketch of phase as a function of lapse time before demodulation. *Solid circles* denote phase estimates up to an arbitrary integer number of cycles

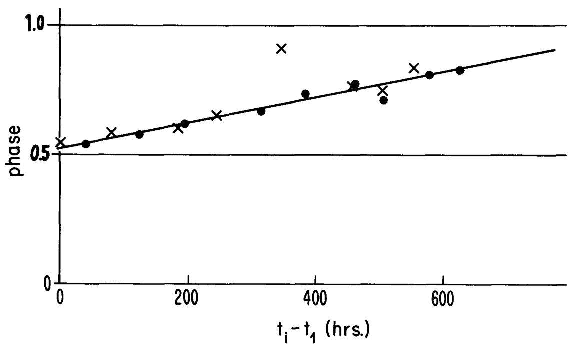

Fig. 2. Phase estimates (in cycles) as function of lapse time after demodulation with $f = 2.9333333$ cph. *Crosses* and *solid circles* are data from SPA and LMS respectively. *Straight line* is the least-squares fit to the estimates

close to $f({}_0S_0)$ that the p_i' span less than one cycle, for all values of i.

The statistics of phase estimation for sinusoidal signals in the presence of Gaussian noise are close enough to normal for the noise levels in our spectra (Knopoff et al. 1979) that a least squares solution to Eq. (2) is justified (Middleton 1968, Fig. 9.4). This is in contrast to the Ricean statistics we were obliged to use to study spectral amplitudes. Because noise-free ${}_0S_0$ is in phase for both stations and because instrumental and data processing phase shifts are identical, we can analyze the phase data for both stations jointly, after the clock corrections have been applied.

As can be seen in Fig. 2, the phase estimate labelled with a question mark in Table 1 deviates conspicuously from the general trend of the other estimates. Inspection of the relevant time series revealed that a small earthquake was recorded during this segment; this may have produced an erratic phase estimate. This particular time series did not produce any more scatter in the spectral amplitudes than the other data and was not rejected in the amplitude analysis performed in Knopoff et al. (1979). In the present analysis this point has been deleted and we obtain, from the least squares solution for the remaining 16 time lapse spectra, the value

$$f({}_0S_0) = (2.932851 \pm 0.000031) \text{ cph}$$
$$= 2.932851 \text{ cph} \pm 11 \text{ ppm}.$$

Using seismograms from the IDA-network (and a different method of analysis) Riedesel et al. (1979), obtained

$$f({}_0S_0) = (2.932794 \pm 0.000015) \text{ cph}$$
$$= 2.932794 \text{ cph} \pm 5 \text{ ppm}.$$

These two results are in disagreement at the 1 s.d. level but agree at the 2 s.d. level. The effects of mode conversion, especially from nearby mode ${}_0S_5$, are probably small.

Acknowledgements. The superlative skills of Dr. Tadashi Yogi in operating the ultralong period seismometer at the South Pole during the year November 1976 to November 1977 made this research possible. Many discussions with Dr. Erhard Wielandt are appreciated. Drs. Dieter Emter, Karl Fuchs, and Gerhard Müller read the manuscript critically. We are grateful to Professor Karl Fuchs, Geophysical Institute, University of Karlsruhe, Germany for supporting the participation of Walter Zürn in this program. This work was supported by the National Science Foundation under grant numbers DPP 76-17234 A01 and EAR 78-23841. The Office of Polar Programs provided invaluable support for the program at the South Pole.

References

Buland, R., Berger, J., Gilbert, F.: Observations from the IDA network of attenuation and splitting during a recent earthquake. Nature **277**, 358–362, 1979

Derr, J.. Free oscillation observations through 1968. Bull. Seismol. Soc. Am. **59**, 2079–2099, 1969

Knopoff, L., Countryman, R.L., Rydelek, P.A., Smythe, W.D., Yogi, T., Zürn, W.: Free oscillations of the earth and tilt at the South Pole. Antarct. J.U.S., **XIII**, 4, 41–43, 1978

Knopoff, L., Zürn, W., Rydelek, P.A., Yogi, T.: Q of mode ${}_0S_0$. J. Geophys. **46**, 89–95, 1979

Linton, J.A., Smylie, D.E., Jensen, O.G.: Gravimeter observations of free modes excited by the August 19, 1977 Indonesia earthquake. Bull. Seismol. Soc. Am. **69**, 1445–1454, 1979

Middleton, D.: An introduction to statistical communication theory. New York: McGraw-Hill Book Co. 1968

Nakanishi, K.K., Knopoff, L., Slichter, L.B.: Observation of Rayleigh wave dispersion at very long periods. J. Geophys. Res. **81**, 4417–4421, 1976

Riedesel, M., Berger, J., Gilbert, F.: Frequency and attenuation of ${}_0S_0$ (Abstract). E.O.S., Trans. Am. Geophys. Union **60**, 379, 1979

Slichter, L.B. Spherical oscillations of the earth. Geophys. J. R. Astron. Soc. **14**, 171–177, 1967

Slichter, L.B., Zürn, W., Syrstad, E., Knopoff, L., Smythe, W.D., Uffelman, H.: Long period gravity tides at the South Pole. J. Geophys. Res. **84**, 6207–6212, 1979.

Warburton, R.J., Goodkind, J.M.: The influence of barometric pressure variations on gravity. Geophys. J. R. Astron. Soc. **48**, 281–292, 1977

Received March 6, 1980; Accepted May 28, 1980