

The Shape of the Geomagnetic Field Through the Last 8,500 Years Over Part of the Northern Hemisphere

K.M. Creer and P. Tucholka

Department of Geophysics, University of Edinburgh, King's Buildings, Edinburgh EH9 3JZ, UK

Abstract. Regional type curves depicting secular variations of declination and inclination through the last 10,000 calendar years constructed for north-western Europe (356° E, 55° N) and east-central North America (270° E, 46° N) by stacking palaeomagnetic data derived from lake sediment cores are described and analysed. The spectral content and phase relationships of the two pairs of curves show that they have a complex origin with both drifting and standing geomagnetic sources contributing to them. The strongest evidence of drifting sources is provided by the inclination type-curves which exhibit maximum correlation for a phase shift of ~650 years suggestive of westward drift at a rate of about 0.13 degrees a year. At the same time, comparison of the declination type-curves strongly suggests that waxing and waning standing sources were dominant. We show that the difference in relative importance of drifting as compared to standing geomagnetic sources implied by the patterns of correlation deduced respectively for declination and inclination can, at least in principle, be attributed to observation point/geomagnetic source geometry by modelling the secular variation that would be produced by standing but oscillating equatorial dipoles and radial dipoles located deep within the outer core, by a pair of drifting deep-seated radial dipoles of constant intensity and by drifting sheets of radial dipoles (taken to represent current-loops) located at shallow depth within the outer core. Each of these model sources produces secular variation curves with distinctive shapes and phase relationships. Hence, an attempt is made to identify qualitatively the types and locations of the sources which dominated the secular variations as recorded by our type-curves. One of our most important conclusions is that there appears to be a 'turning-point' at ~4,750 years before present when the relative amplitudes of the active 'standing' sources changed but the characteristics of the drifting sources appear to have remained relatively unchanged.

Key words: Palaeomagnetism – Lake sediments – Secular variation – Geomagnetic field

Introduction

The properties of lake sediments as recorders of geomagnetic secular variations render them particularly suitable for the investigation of the part of the geomagnetic spectrum with characteristic times ca. 10^3 years. There is a cut-

off at the shorter period end of the spectrum at ca. 100 years imposed by the smoothing of the recorded signal associated with the magnetization recording process (Tucker, 1979; Creer, 1982).

Type-curves illustrating geomagnetic secular variations (SV) in declination and inclination for N.W. Europe and east-central N. America are shown in Figs. 1 and 2 respectively. These SV curves which run back to 10,000 calendar years before present (present = 1950 AD) were produced by stacking records from individual cores of lake sediment. The plotted points were interpolated from the measured data points at uniform increments of 40 years and the horizontal bars indicate the standard deviations of the mean value at each time horizon. The pair of SV curves for 356° E, 55° N were constructed (Creer and Tucholka, 1982a) from records obtained from Loch Lomond and Lake Windermere, U.K. (Turner and Thompson, 1981) and the sequences of labelled features are also to be recognized (though not exactly reproduced) along individual core records from Switzerland (Creer et al., 1980), Poland (Creer et al., 1979) and Greece (Creer et al., 1981). The pair of SV curves for 270° E, 46° N were constructed using results from Lakes St. Croix and Kylen, Minnesota (Banerjee et al., 1979) and from Lakes Superior and Huron (Mothersill, 1979; 1981) as described by Creer and Tucholka (1982b). The time-scales were derived from numerous radiocarbon age determinations, carried out in different laboratories in U.K. and U.S.A. All these radiocarbon ages were calculated using the Libby half-life (5,568 years) and 95% of the isotopically corrected activity of NBS oxalic acid as the reference standard (Broecker and Olson, 1961). In this paper they have been converted to calendar years (Clark, 1975). We have labelled the principal features characterizing the European SV curves with letters from the Greek alphabet and those characterizing the N. American SV curves with letters from the Roman alphabet. Capital letters are used for declination and lower case letters for inclination.

Previous attempts to interpret the relationship between Holocene SV records obtained from N. American and U.K. lake sediments led to different, though not necessarily conflicting conclusions. The first such attempt was made by Creer, Gross and Lineback (1976) and by Creer (1977) who deduced that waxing and waning or oscillating standing geomagnetic sources had dominated the SV because of different periodicities (~2,200 years in Lake Michigan and ~2,700 years in Lake Windermere) apparent in the decli-

Offprint requests to. K.M. Creer

nation records. Later, when improved inclination records had been obtained, it became clear that, at least visually, there was a good correlation across the N. Atlantic with a phase shift of some 500 years which was interpreted as evidence of westward drift (Creer, 1981a). The paradox of standing sources appearing to dominate the SV as read from declination records and, at the same time, of drifting sources appearing to dominate the SV as read from inclination records persisted even when stacked SV type-curves for N. America and U.K. were compared (Creer and Turcholka, 1982a).

The coexistence of evidence of standing and drifting geomagnetic sources through Holocene time is not surprising since, as is well known, both types of source must be invoked to explain observatory geomagnetic records covering the last few centuries (Yukutake and Tachinaka, 1969). The problem is to explain why a comparison of declination SV type-curves and a similar comparison of inclination SV type-curves lead to different conclusions – surely if drifting and standing sources have coexisted, evidence of both should be forthcoming from all elements of the geomagnetic field, including declination and inclination.

In this paper, we describe further analyses of the relationships between the N. American and U.K. SV type-curves.

Normalization and Straightening of the Type-Curves

The first step in our comparison of the respective pairs of declination and inclination type-curves was to subtract the long wavelength trends. We did this by fitting cubic splines with different numbers of knots and finally chose to remove the trends defined by the curves obtained using one knot only. Second, we normalized the respective pairs of curves by equalizing the root-mean-square deviations from the mean of the straightened curves. This involved increasing the amplitudes of the U.K. curves by factors of 3.1 for declination and 1.1 for inclination (refer to the scale divisions marked on Figs. 1 and 2). This use of different normalization factors for declination and inclination cannot be attributed to a systematic difference in the efficiency of the magnetization process as it proceeded in the U.K. and Great Lakes sediments since this would attenuate declination and inclination SV amplitudes equally: it must be related somehow to the geomagnetic source – observation point geometry and this is further discussed below.

It is risky at this stage to attempt any interpretation of periodicities of the order of the length of the records available i.e. ca. 5,000 years or more, both because of technical difficulties associated with taking sediment cores and because of the limitations of available time-series analysis methods.

Within-Region Correlations

Objectives

We identify declination type-curves by the letter *D* and inclination type-curves by the letter *I*. N. American data are identified by the number 1 and U.K. data by the number 2. Thus *D1* refers to the N. American declination type-curve.

With the object of determining the basic spatial and temporal properties of the dominant geomagnetic sources

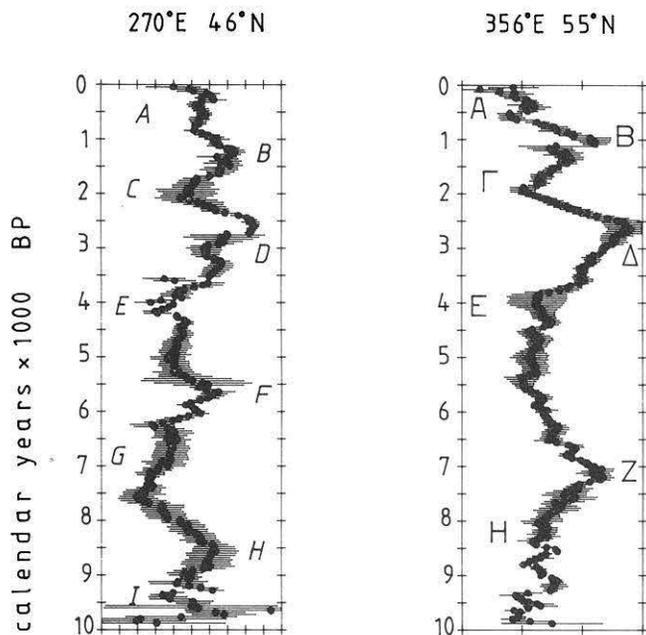


Fig. 1. Type-curves depicting secular variations in declination obtained by stacking palaeomagnetic data from individual cores of lake sediment. On left: curve for east-central N. America – Lakes Superior and Huron and Minnesota lakes (mean position 270° E 46° N). On right: curve for N.W. Europe – Lake Windermere and Loch Lomond, U.K. (mean position 356° E, 55° N). Data points interpolated at 40 years intervals. Standard error bars shown. Each scale division of declination represents 10°. Arbitrary zero.

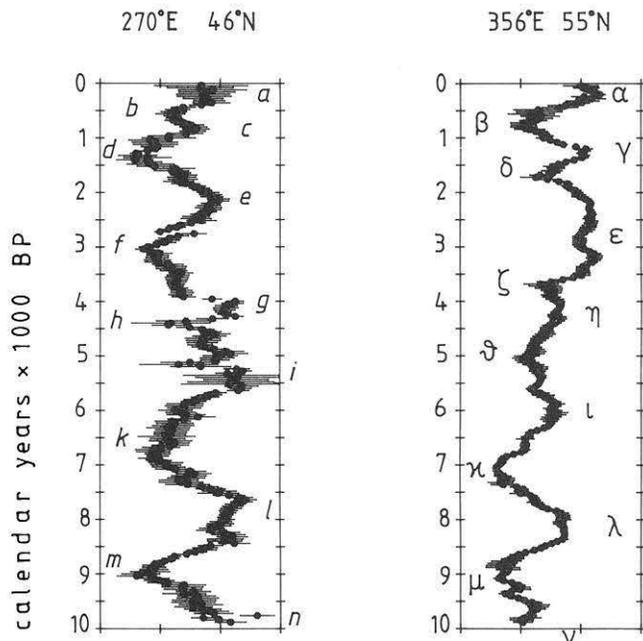


Fig. 2. Type-curves depicting secular variations in inclination obtained by stacking palaeomagnetic data from individual cores of lake sediment. On left: curve for east-central N. America – Lakes Superior and Huron and Minnesota lakes (mean position 270° E 46° N). On right: curve for N.W. Europe – Lake Windermere and Loch Lomond, U.K. (mean position 356° E 55° N). Data points interpolated at 40 years intervals. Standard error bars shown. Each scale division of inclination represents 10°. Inclination values are centred around zero.

Table 1. Correlative features of type curves 0–4,760 B.P.

Pair	$\Delta\rho$	T	$\Delta\phi$	$\Delta\phi/T$	S or D ?
$D1/D1$	–	1,100	–	–	–
$I2/I2$	–	1,050	–	–	–
$D1/I2$	0.98	1,050	130	+0.13	$S+D$
$I1/I2$	~0.3	~1,000	~250	~0.25	(very weak)
$D1/D1$	–	2,100	–	–	–
$I1/I1$	–	2,000	–	–	–
$D2/D2$	–	1,950	–	–	–
$I2/I2$	–	1,970	–	–	–
$D1/I1$	1.04	2,100	+600	~+0.28	$D+S$
$D1/D2$	1.18	~2,000 ^a	~0	~0	S
$I1/I2$	0.88	2,200 ^a	+720	+0.33	$D+S$
$D1/I2$	0.98	2,030	+130	+0.13	
$D2/I1$	1.28	2,100	–750	–0.36	
$D2/I2$	0.62	~3,000?	~0	0	$S+D$

^a for 1,000–4,760 B.P.

$D1, D2$ respectively N. American and U.K. declination curves
 $I1, I2$ respectively N. American and U.K. inclination curves
 $\Delta\rho$ = difference between peak positive and negative correlation coefficients
 T = inferred period
 $\Delta\phi$ = phase shift for positive peak in ρ
 S, D refer to standing or drifting source

which caused the SV patterns recorded by our type-curves, we have computed correlation coefficients, ρ , as a function of phase-shift ϕ of the timescales for the following combinations: (i) autocorrelations $D1/D1$, $D2/D2$, $I1/I1$ and $I2/I2$ and (ii) declination-inclination pairs $D1/I1$ and $D2/I2$. ϕ was changed between –2,000 years and +2,000 years in 40 year increments. To calculate the correlation coefficient we used the standard formula:

$$\rho = \frac{\sum_{i=1}^N (\Delta X_i \Delta Y_i)}{\sqrt{\sum_{i=1}^N (\Delta X_i)^2 \sum_{i=1}^N (\Delta Y_i)^2}}$$

where $\Delta X_i = X_i - \bar{X}$ and $\Delta Y_i = Y_i - \bar{Y}$.

ρ can vary from –1 for an exact negative relationship through 0 for no correlation at all to +1 for an exact positive relationship. The significance of the calculated value of ρ can be tested against Student's t -distribution for $N-2$ degrees of freedom. As the number of points used always exceeded 51 we can read off upper limits to the values of ρ from a table listing critical values of ρ for probability levels $P=0.05, 0.01, 0.001$ for 50 degrees of freedom. Using Table 7.1 of Rao et al. 1966 we find that the probabilities that the tested relationship is accidental is less than 5% for $\rho=0.273$, less than 1% for $\rho=0.354$ and less than 0.1% for $\rho=0.443$.

When we started this investigation (Creer and Tucholka, 1982a), it soon became clear that the correlation curves showed a distinct change in character just after ca. 5,000 B.P. which, for the purposes of computation we have placed at 4,760 B.P. Therefore we performed separate analyses of the data for the intervals 0–4,760 B.P. and 4,760–8,520 B.P. All the results are summarized in Tables 1 and 2.

Autocorrelation

Autocorrelation provides a simple yet effective method of spectral analysis because dominant periodicities show up

Table 2. Correlative features of type curves 4,760–8,520 B.P.

Pair	$\Delta\rho$	T	$\Delta\phi$	$\Delta\phi/T$	S or D ?
$D1/D1$	–	2,000	–	–	–
$I2/I2$	–	2,100	–	–	–
$D1/I1$	1.40	2,250	625	+0.28	$D+S$
$D2/I2$	1.04	1,965?	1,113	+0.57	$S+D$
$I1/I2$	1.34	2,260	460	–	assumed D
$D1/I2$	1.44	2,100	–165	–0.08	?
$D1/D1$	–	3,670	–	–	–
$D2/D2$	–	3,460	–	–	–
$D1/D2$	1.48	3,735	–425	–0.11	$S+D$
$I1/I1$	–	2,850	–	–	–
$D2/I1$	–	2,670?	885	0.33	$S+D$

Key to symbols given under Table 1

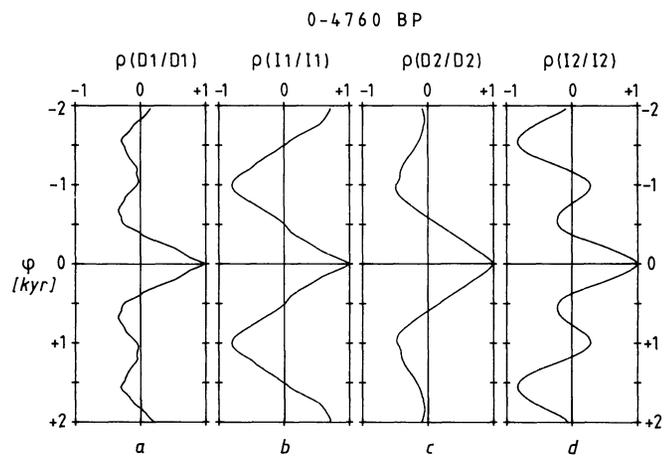


Fig. 3a–d. Autocorrelation curves for 0–4,760 B.P. – correlation coefficient ρ plotted against phase shift ϕ . ρ varies from –1 for an exact negative relationship through 0 for no relationship to +1 for an exact positive relationship. ϕ is plotted in Kyr along the vertical scale. **a** N. American straightened declination, $D1$; **b** N. American straightened inclination, $I1$; **c** U.K. straightened declination, $D2$; **d** U.K. straightened inclination, $I2$. Long wavelength trends were subtracted from type-curves before computation of ρ vs. ϕ .

as a succession of maxima and minima along the autocorrelation curve of ρ vs. ϕ . The phase difference $\Delta\phi$ between successive maxima or minima is a measure of the main periodicities in the SV curves. The method only works for periodicities shorter than the length of the record under investigation but within this limitation, we found it to be more reliable than sophisticated methods of time series analysis, the results of which will be discussed below.

0–4,760 B.P. There is a marked similarity between the autocorrelation curves $D1/D1$ (Fig. 3a) and $I2/I2$ (Fig. 3d): the spectral content of the N. American declination type-curve is similar to that of the U.K. inclination type-curve. The form of these curves which are characterized by alternating larger and smaller amplitude peaks may be simulated by autocorrelating a synthetic curve composed of two periodicities, one of them about half the other. Therefore, we interpret these curves as containing the following approximate periodicities: $D1/D1$, 1,100 years ($\Delta\phi$ between the first

4760–8520 B.P.

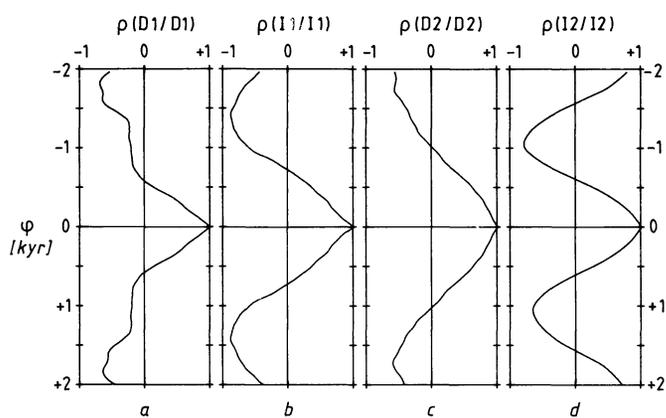


Fig. 4a–d. Autocorrelation curves for 4,760–8,520 B.P. – correlation coefficient ρ plotted against phase shift ϕ . ρ varies from -1 for an exact negative relationship through 0 for no relationship to $+1$ for an exact positive relationship. ϕ is plotted in Kyr along the vertical scale. **a** N. American straightened declination, $D1$; **b** N. American straightened inclination, $I1$; **c** U.K. straightened declination, $D2$; **d** U.K. straightened inclination, $I2$. Long wavelength trends were subtracted from type-curves before computation of ρ vs

pair of minima gives T and $\Delta\phi$ between the second pair of minima gives $3T$ and 2,100 years (given by $\Delta\phi$ between the pair of subsidiary maxima). Similarly for $I2/I2$ we identify $T \sim 1,050$ years and 1,970 years. The $D2/D2$ and $I1/I1$ curves show evidence of a single periodicity only, given by $\Delta\phi$ between the pair of subsidiary minima. For both curves, we thus estimate $T \sim 1,950$ years. The accuracies of these estimates of periodicity depend on the precision with which the maxima and minima of the autocorrelation curves can be identified. Typically this is about ± 50 years.

4,760–8,520 B.P. All four autocorrelation curves show only a single pair of minima in the range of phase shifts applied to the type-curves ($-2,000$ to $+2,000$ years). These correspond to the following approximate periodicities: 2,100 years for $I2/I2$ (Fig. 4d), 2,240 years for $I1/I1$, (Fig. 4b), 2,540 years for $D1/D1$ (Fig. 4a) and 3,410 years for $D2/D2$ (Fig. 4c). While the two inclination periods may possibly be associated with the 1,950 years period identified for the younger part of the records, the two longer periods are significantly different.

Correlation of Declination-Inclination Pairs

The phase relationship between declination and inclination SV curves provides an important clue as to the properties of the time-varying geomagnetic sources which produced them. This will be discussed fully below. Meanwhile, we proceed to describe the observed correlations.

0–4,760 B.P. The correlation curves for N. America and U.K. have quite different shapes. The $D1/I1$ correlation (Fig. 5a) is best ($\rho = +0.52$) for a phase shift ($\Delta\phi$) of 720 years. The common periodicity (T) may be estimated from the phase difference between successive maxima or minima which gives $\sim 2,100$ years. Thus $\Delta\phi/T \sim 0.33$. The $D2/I2$ correlation (Fig. 5b) is rather poor ρ varying from $+0.27$ to -0.29 . The phase shift between the two minima

0–4760 B.P.

4760–8520 B.P.

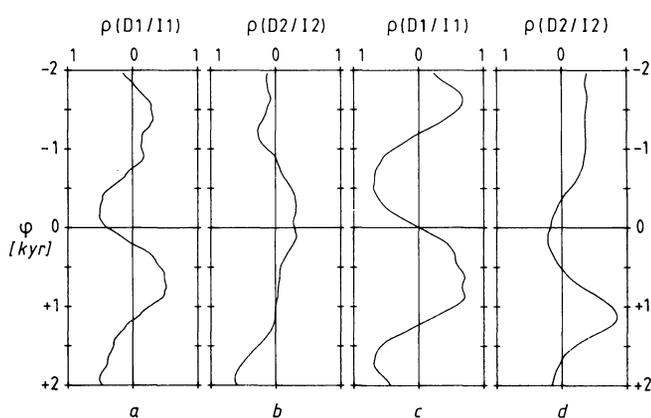


Fig. 5a–d. Correlation curves, ρ vs. ϕ for **a** N. American ($D1/I1$) and **b** U.K. ($D2/I2$) declination and inclination pairs 0–4,760 B.P.: **c** and **d** respectively the same for 4,760–8,520 B.P. Long wavelength trends removed

gives $T \sim 3,000$ years. $\Delta\phi/T \sim -0.1$, i.e. declination and inclination variations are almost in phase.

4,760–8,520 B.P. These correlation curves (Fig. 5c, d) are similar in form and moreover the former is similar to the corresponding ($D1/I1$) curve for the younger (Fig. 5a) time interval. The phase differences for maximum correlation are: for N. America, $\Delta\phi \sim 625$ years ($\rho = +0.72$) and for U.K., $\Delta\phi \sim 1,110$ years ($\rho = +0.84$). The N. American period is $\sim 2,250$ years and the U.K. period is only weakly defined being greater than $\sim 2,000$ years. For N. America, $\Delta\phi/T \sim 0.25$ and for U.K., $\Delta\phi/T \sim 0.55$.

Between-Region Correlations

Previous Work

The inclination SV type-curves for N. America and western Europe show marked similarities which can be identified on visual inspection (Creer, 1981a), particularly for those parts of the records older than features h and θ . The declination SV type-curves are also very similar in form, particularly post $\sim 5,000$ B.P. Before $\sim 5,000$ B.P. an inverse correlation has already been inferred and the suggestion made that an adjustment should be made to one of the time-scales such as to obtain the best anticorrelation for $\Delta\phi = 0$ (Creer and Tucholka, 1982a). We now re-examine the $D1/D2$ and $I1/I2$ correlations and we also examine other correlations $D1/I2$ and $D2/I1$.

0–4,760 B.P. Figure 6a illustrates a strong positive correlation which peaks at $\rho \sim +0.68$ for zero phase shift. A similar result is obtained for window 1,000–4,000 B.P. (Fig. 6b) with $\rho \sim +0.78$.

4,760–8,520 B.P. The correlation curve (Fig. 6c) exhibits two peaks separated by a phase shift of ca. 1,850 years corresponding to a quasi-period of ca. 3,700 years. The pair of declination curves are correlated almost inversely, the negative peak ($\rho \sim -0.80$) occurring for $\Delta\phi \sim 400$ years such that correlated features appear younger along the

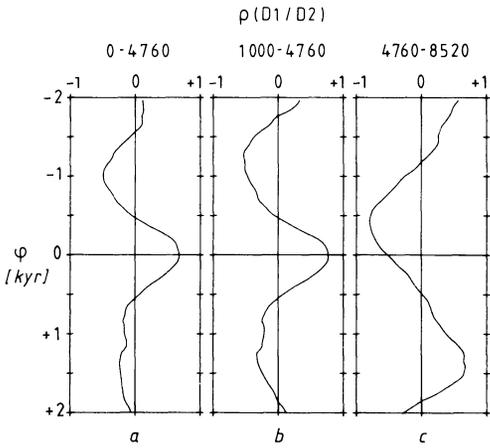


Fig. 6a-c. Correlation of N. American and U.K. straightened declination type-curves, $D1$ and $D2$. **a** 0-4,760 B.P.; **b** 1,000-4,760 B.P.; **c** 4,760-8,520 B.P.

U.K. curve. The maximum, $\rho \sim +0.68$ occurs for $\Delta\phi \sim +1,475$ years.

Inclinations $I1/I2$

0-4,760 B.P. Figure 7a shows that the $I1/I2$ correlation is poor for all phase shifts but when the most recent thousand years of record are removed, i.e. for window 1,000-4,760 B.P., ρ attains a maximum of +0.32 for a phase shift of +700 years and the phase difference $\Delta\phi \sim 2,300$ years between the two maxima in ρ is suggestive of a common periodicity in both N. American and U.K. inclination records.

4,760-8,520 B.P. The coefficient, ρ , peaks at +0.70 for a phase shift of +460 years (Fig. 7c) and the phases of the maxima and minima indicate the presence of a common periodicity of 2,200-2,350 years.

Declination-Inclination Pairs $D1/I2$ and $D2/I1$

The observed similarity between the autocorrelation $D1/D1$ and $I2/I2$ curves (see above) leads us to consider the cross relationships $D1/I2$ and $D2/I1$. The results should throw some light on the geomagnetic source/recording point geometry.

0-4,760 B.P. Figure 8a shows that the $D1/I2$ correlation curve has much the same form as each of the respective autocorrelation curves. $D1$ lags behind $I2$ by ~ 130 years. The phase difference between the two inner minima gives $T \sim 1,110$ years and between the outer minima they give $3T \sim 3,100$ years, i.e. $T \sim 1,030$ years so that $\Delta\phi/T \sim 0.12$. The unequal heights of the maxima and minima suggest that a second period of $T \sim 2,000$ years is also present. Figure 8b shows the $D2/I1$ correlation curve which has a quite different shape: $D2$ leads $I1$ by ~ 750 years. The common periodicity is estimated at $\sim 2,100$ years and hence $\Delta\phi/T \sim 0.36$.

4,760-8,520 B.P. Figure 8c shows the $D1/I2$ curve: $D1$ leads $I2$ by only ~ 160 years, so that they are very nearly in phase ($\Delta\phi/T \sim 0.08$) and the common periodicity is estimated at $\sim 2,100$ years. The form of the $D2/I1$ curve (Fig. 8d) sug-

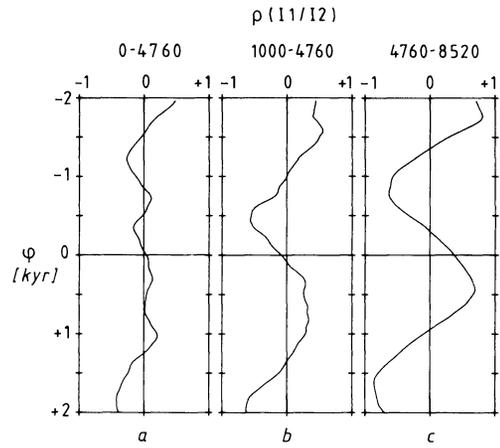


Fig. 7a-c. Correlation of N. American and U.K. straightened inclination type-curves, $I1$ and $I2$. **a** 0-4,760 B.P.; **b** 1,000-4,760 B.P.; **c** 4,760-8,520 B.P.

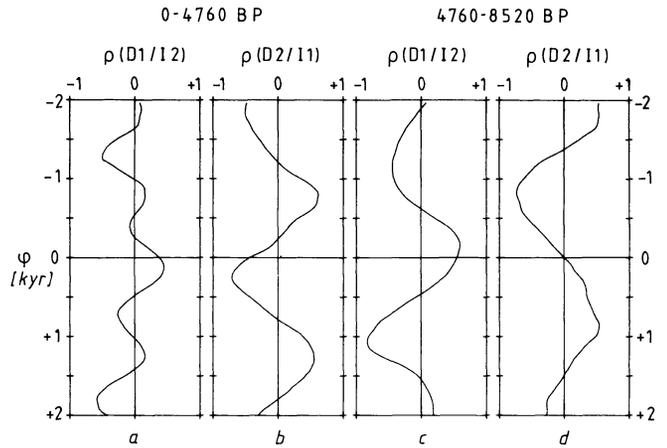


Fig. 8a-d. Cross correlation of declination and inclination between N. America and Europe. **a** $D1/I2$; **b** $D2/I1$ for 0-4,760 B.P.; **c** and **d** same respectively for 4,760-8,520 B.P.

gests a longer period of about 2,660 years, $D2$ lagging behind $I1$ by about $T/4$.

Time-Series Analysis

The autocorrelation method described above is not practical for the determination of periodicities which approach the length of the available record. Therefore we carried out time series analyses on each declination and inclination type curve separately using the maximum entropy method (Burg, 1967; 1968) and we also analysed the declination-inclination pairs simultaneously by representing them as series of complex numbers (Denham, 1975) using programs developed by Barton (in press 1982) who discussed the application of established methods of time-series analysis to palaeomagnetic data sets.

Analyses were carried out for six different prediction error filters (PEF), of 400, 800, 1,200, 1,600, 2,000 and 2,400 years. These correspond to 10-60 time increments of 40 years, i.e. approximately 1/12-1/2 of the length of the data sets being analysed. If the PEF is chosen too low,

Table 3. Periods inferred from spectral analyses

Time interval	Curve	Periodicities		
0–4,760 B.P.	<i>D1</i>	~1,000	2,800	
	<i>D2</i>		~ 2,700	
	<i>I1</i>		~ 2,000	
	<i>I2</i>	1,050	3,350	
	<i>D1/I1</i>	~1,000	2,200	3,300
	<i>D2/I2</i>	~1,000	2,500	
4,760–8,520 B.P.	<i>D1</i>		3,050	
	<i>D2</i>		4,000–4,400	
	<i>I1</i>			2,200
	<i>I2</i>			2,000
	<i>D1/I1</i>	2,500	3,200	
	<i>D2/I2</i>	1,700	2,000	

Prediction error filter: 1,200 and 1,600 years

the spectrum obtained is over smoothed and if it is chosen too high frequency shifting and spontaneous splitting of the spectral peaks occur, so the correct choice of PEF is important and it is not always obvious when this choice has been made. We quote our results obtained for PEF values of 1,200 and 1,600 years for which the positions of the spectral peaks showed a tendency to stabilize before splitting at higher PEF values. We shall discuss the spectra for younger and older parts of the record separately. The results which are summarized in Table 3 confirm the conclusions already drawn above.

0–4,760 B.P.

Both U.K. inclination (*I2*) and N. American declination (*D1*) type curves show a spectral peak at $T \sim 1,000$ years which is stable for PEFs from 800 to 2,400 years. Other peaks at about $2T$, $3T$ and $4T$ occur in the spectra for different PEFs but not all in any given spectrum. These results are consistent with those obtained by the simpler approach described above.

The U.K. declination spectrum (*D2*) shows a single peak at $T \sim 2,700$ years and the N. American inclination (*I1*) spectrum shows a single peak at $T \sim 2,000$ years. We obtained $T \sim 2,000$ years for both *D2* and *I1* autocorrelation curves (see above).

The complex number method resolves a peak at $T \sim 1,000$ years for all tested values of PEF for both *D1/I1* and *D2/I2* pairs but none of the other peaks are stable over an appreciable range of PEF (compare with results above).

4,760–8,520 B.P. Both inclination spectra show a single peak at $T \sim 2,100$ years. This is in agreement with the results given by the autocorrelation method. The declination spectrum for *D1* shows a single peak at $T \sim 3,000$ years and *D2* shows a single peak at $T \sim 4,000$ – $4,500$ years (U.K.). The autocorrelation method gave rather shorter periods: $T \sim 2,550$ years for *D1* and $T \sim 3,500$ years for *D2*.

The complex number method shows peaks at $T \sim 3,200$ years, 2,500 years and 1,100 years for (2,250 years) *D1/I1* and at $T \sim 2,000$ years and 1,700 years ($\sim 2,000$ years) for *D2/I2*, the values in brackets being those obtained by autocorrelation.

Source Models

Basic Types of Source

In order to interpret the results described above, we shall now consider the properties of some basic types of geomagnetic source. The simplest kind of source one can imagine is a dipole or a current loop and the latter may conveniently be represented by a sheet of dipoles. In principle, secular variations can be produced by variations in the strength (M) and orientation of the main geomagnetic dipole and/or by changes in topography and/or drift of the non-dipole field (NDF). The former may be simulated in part by steady drift or fluctuations in azimuth of a geocentric equatorial dipole (GED), i.e. by changes in ratio of the Gauss coefficients g_1^1 and h_1^1 and/or by fluctuations in the intensity of the GED relative to that of the axial component (Gauss coefficient g_0^0) of the main dipole. The shape of the NDF may be modelled by representing it as a set of radial dipoles (RD) located in the outer core. For example, Allredge and Hurwitz (1964) used a set of eight deep-seated RD at $r = 1,750$ km. Secular variations may then be simulated by drift and/or by fluctuations in intensity of these RD relative to one another and also to the main dipole (Creer, 1977, 1981b; Creer and Tucholka, 1982a) or alternatively by sheets of dipoles located near the top of the outer core if the NDF is to be represented by a set of current loops.

The shape and phase relationships of the declination and inclination waveforms produced by different models depend critically on the size and depth of the geomagnetic source and on whether it is drifting, oscillating or pulsating. We shall illustrate this point here by considering a few specific models, without attempting a comprehensive survey.

Drifting Radial Dipoles

Yukutake and Tachinaka (1968, 1969) deduced that the symmetry of the drifting part of the NDF through historic time has been essentially quadrupolar with one positive and one negative focus in each of the northern and southern hemispheres. Although theirs is not a unique interpretation of instrument-based geomagnetic data for the last four centuries, it is nevertheless pertinent to consider a model, based on their analysis, consisting (for the northern hemisphere) of two drifting RD, one with N pole pointing up and the other with S pole pointing up. The curves shown in Fig. 9a were computed for two such RD at 45° N latitude and separated by 180° of longitude at $r = 1,750$ km. Their strength, relative to the main dipole is given by $m/M = 0.17$ and the drift rate is such that they make two revolutions, in a retrograde sense, relative to the mantle in the time interval spanned by the curves (10,000 years). The observer is on latitude 45° N so that each RD passes directly underneath him once every 5,000 years. The characteristics of the resulting SV are (i) flat and rather square declination anomalies and (ii) cusp shaped inclination anomalies which are markedly asymmetrical about the axial dipole inclination for 45° N ($\sim 63^\circ$). It is relevant to note that the inclination variations observed along the Lake Superior (Mothersill, 1979) and Lake Huron (Mothersill, 1981) records and also along the Lac de Joux (Creer et al., 1980) record prior to about 5,000 B.P. are of very similar form, having sharp minima and rounded maxima, though these characteristics

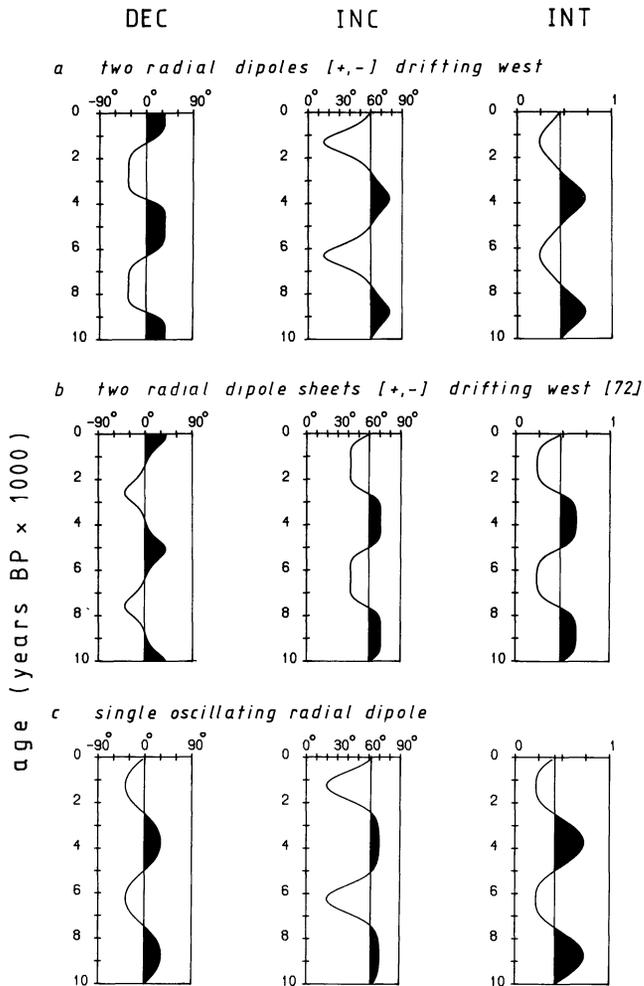


Fig. 9a–c. Synthetic SV curves produced at 45° N, 0° E **a** by two westward drifting radial dipoles (RD) which start at 45° N, 90° E (N pole up) and 45° N, 270° E (S pole up) at 10,000 B.P. Drift rate is $0.072^\circ/\text{year}$. Ratio of RD moment m to main dipole moment $M=0.20$, $r=1,750$ km. **b** by two westward drifting radial dipole sheets at $r=3,400$ km, moment of each sheet $=0.26 M$, same drift rate and initial conditions as **a**; **c** single RD at 45° N, 30° E with $m/M=0.20$ at $r=1,750$ km oscillating with period $T=5,000$ years

tend to disappear when individual records are stacked to produce a type-curve. These characteristic wave forms hold at least for the range of latitudes for which the data used in compiling our stacked SV type-curves were obtained.

The motion of the SV field vector through time can be represented by a D vs. I (Bauer) plot or by plotting virtual geomagnetic poles (VGP) corresponding to the D , I pairs on a stereogram. In either case, the path traced out forms a loop, the sense of motion being clockwise for westward drifting sources and anticlockwise for eastward drifting sources (Skiles, 1970). The converse of this rule does not generally hold however (Creer and Tucholka, 1982a).

Sheets of Radial Dipoles at Shallow Depth

Let us now consider the form of the SV curves which would be produced by widely dispersed geomagnetic sources (e.g. electric current loops) located near the top of the outer core. Two current loops of opposite polarity were each rep-

resented by a sheet of radial dipoles centred under 45° N latitude at $r=3,400$ km with N poles pointing up in one hemisphere and S poles pointing up in the other. The total moment of each sheet is $0.26 M$. The drift rate and initial conditions are given in the caption to Fig. 9. Figure 9b shows the resulting SV curves as would be observed on the Earth's surface at latitude 45° N. The SV waveforms for these broad shallow sources are distinctly different from the corresponding waveforms for deep-seated radial dipoles: the declination peaks are pointed while the inclination peaks are rather square.

It is important to note that both of the drifting-source models we have considered produce, for a particular observation point, declination and inclination curves with the same periodicity but which are out of phase by $T/4$. This phase relationship distinguishes such sources from oscillating or pulsating sources.

Single Radial Dipole Fluctuating in Intensity

We shall consider an oscillating RD only, because the general characteristics of the SV it produces will be the same for a RD which fluctuates in intensity either repeatedly or occasionally. The relative magnitude and shape of the declination and inclination waveforms depend on the source/observer geometry. For a RD located just to the east of the observer, declination and inclination are in phase (Fig. 9c). Here the RD is located at 45° N, $r=1,750$ km as above and $m/M=0.20$ where m and M are respectively the moments of the radial and central dipoles. The asymmetry of the inclination waveform becomes more marked for larger amplitude perturbations, i.e. for larger m/M .

Figure 10 illustrates the effects of source/observer geometry: declination, inclination and intensity curves are shown for observers at 15° intervals of longitude around latitudes 55° N and 45° N. The RD is at 45° N, 0° E, $r=1,750$ km. The curves illustrate two complete periods of oscillation. Figure 11 shows the corresponding family of curves for an oscillating geocentric equatorial dipole (GED) aligned along azimuth 0° E with $m/M=0.42$ (about twice the present value). Thus, the inclination of the geomagnetic axis to the geographic axis would fluctuate by up to 23° .

The effects of attenuation due to conductivity of the core and mantle would be to reduce further the amplitudes of the signals from the RD as a function of increasing distance from the observer but the signal from the GED, which is a representation of sources distributed through the whole outer core, need not suffer attenuation in the same way. It is not clear how the problem of attenuation should be dealt with: clearly it would be wrong to treat the outer core as a conducting solid for which the treatment would be simple.

Attention is drawn to the following points which are especially relevant for two observation points separated by 90° of longitude like the two geographical regions to which our SV type-curves apply. Note that these 'rules' relate to SV produced by oscillating (or pulsating) standing sources and not to drifting sources.

- (i) The declination/inclination (D/I) phase difference is always 0 or $T/2$.
- (ii) Declinations are in phase for observation points on the *same* side of the source (i.e. to the east or west) and

OSCILLATING RADIAL DIPOLE UNDER 45° N 0° E

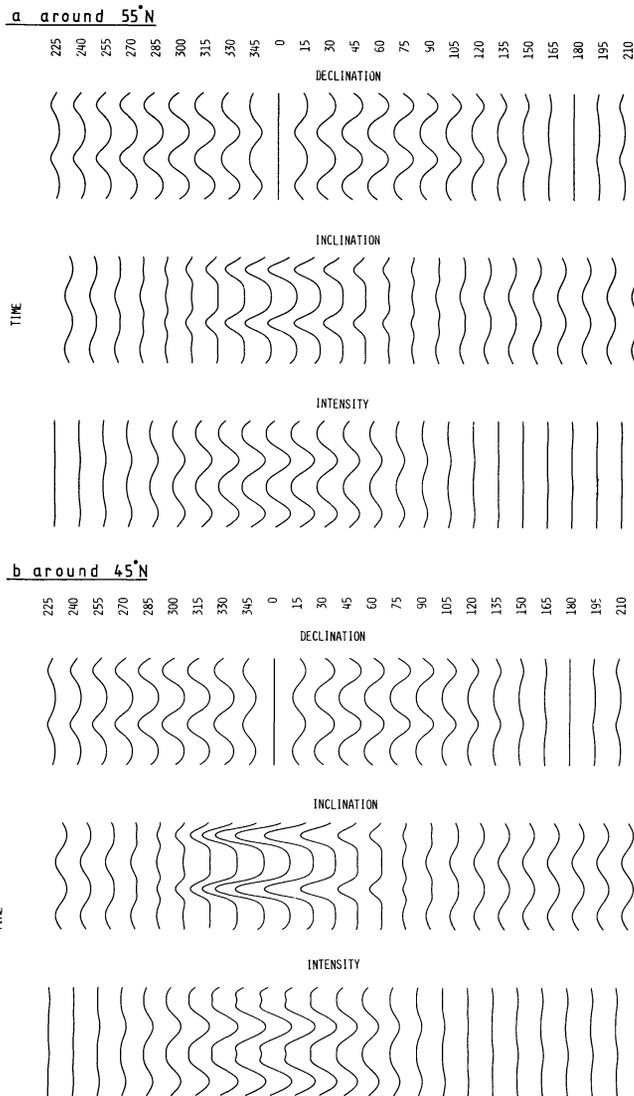


Fig. 10 a and b. SV in declination, inclination and intensity observed around **a** 55° N, **b** 45° N produced by an oscillating radial dipole at 45° N 0° E with $m/M=0.18$ (**a**) and 0.24 (**B**), $r=1750$ and $T=5,000$ years. Longitude of observer shown at head of each column of plots.

out of phase for points on *opposite* sides of the source (for longitude differences up to $\sim 180^\circ$).

(iii) Inclination variations are in phase and symmetrical in form on opposite sides of the source for longitude separations of up to $\sim 90^\circ$ for the GED and up to about $\sim 60^\circ$ for the RD, the latter being dependent on r .

(iv) The relative amplitudes of the declination variations ($D1:D2$) recorded at two observation points will, in general, be different from the relative amplitudes of the corresponding inclination variations ($I1:I2$).

(v) Considering declination and inclination curves for two observation points (1 and 2), observer/source geometries exist such that $D1/I2$ or $D2/I1$ correlations (or anticorrelations) are stronger than those between $D1/D2$ or $I1/I2$.

(vi) The magnetic field vector traces out a linear path as a function of time whether shown as a Bauer plot of D vs. I or transformed to a stereographic plot of the virtual

OSCILLATING EQUATORIAL DIPOLE ORIENTATED ALONG 0° MERIDIAN

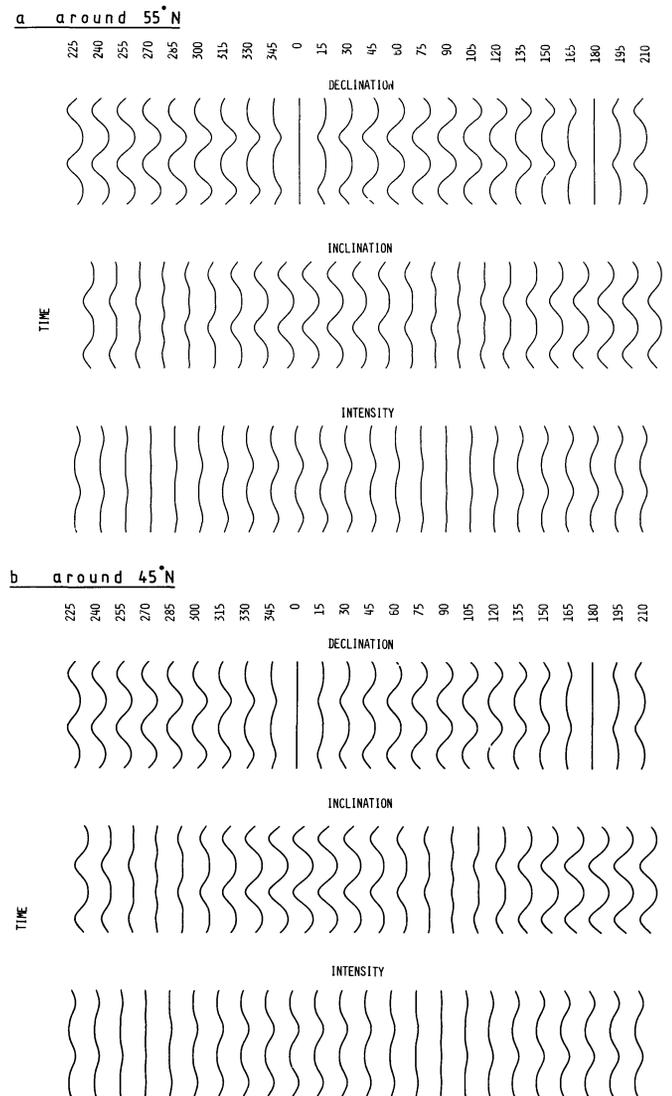


Fig. 11 a and b. SV in declination, inclination and intensity observed around **a** 55° N, **b** 45° N produced by a geocentric equatorial dipole aligned along 0°–180° with $m/M=0.42$ and $T=5,000$ years. Longitude of observer shown at head of each column of plots

geomagnetic pole (VGP) positions (Creer and Tucholka, 1982a).

Two or More Oscillating or Fluctuating Radial Dipoles

The phase relationships summarized above no longer hold. The magnetic vector or VGP traces out an elliptical path which may appear similar to the loops traced out by the secular variations produced by a single drifting RD, especially if the palaeomagnetic record covers only a portion of the ellipse or loop (Creer and Tucholka, 1982a).

Effects of Smoothing

The higher frequencies initially present in the geomagnetic signal are filtered out of the SV record carried by sediments because the remanent magnetization requires a finite time, Δt , to become stabilized (Tucker, 1980; Creer, 1982).

The high frequency cut-off corresponds to $\Delta t \sim 100$ years for lake sediments and $\sim 1,000$ years or more for marine sediments.

The effects of filtering are twofold: (i) the shapes of the maxima and minima become more rounded so that the differences in shape distinguishing deep-seated (Fig. 9a) from shallow-broad (Fig. 9b) sources become blurred and (ii) a phase change equal to $\Delta t/2$ is introduced.

Discussion of Geomagnetic Sources

The essential characteristics of the correlations observed between the two pairs of type-curves are summarized in Table 1 for the interval 0–4,670 B.P. and in Table 2 for the interval 4,760–8,520 B.P. The main results of spectral analysis are shown in Table 3.

We have shown above that the SV waveforms originating from drifting and standing-oscillating sources exhibit important differences in shape and phase. Also, broad-shallow drifting sources produce different waveforms from deep-seated drifting sources, but the D/I phase difference at a given observation point is $T/4$ for both types of source. The D/I phase difference for a single standing-oscillating source is however either 0 or $T/2$. Thus we should be able to identify the types of source which produced the observed SV at our two observation points, provided that only a few sources made strong contributions. Judging from the pattern of periods identified (Tables 1, 2 and 3) it would appear that we need consider the combined effects of only about three different geomagnetic sources.

0–4,760 B.P.

The type-curves appear to be dominated by two periodicities (Table 1). The shortest, $T \sim 1,000$ years is observed in the $D1$ and $I2$ autocorrelations and in the $D1/I2$ correlation and very weakly in the $I1/I2$ correlation. These results could have been produced by either an oscillating GED or an oscillating RD. Referring to Fig. 11 it can be seen that if the GED axis were aligned approximately along a 10° – 190° azimuth the amplitudes of the U.K. declination variations ($D2$) and of the N. American inclination variations ($I1$) would be much smaller than those of the $D1$ and $I2$ variations. A GED type of source is favoured over an RD source for which (see Fig. 10) the amplitudes of the $D1$ and $D2$ curves would be very similar and the $I2$ amplitude would be excessively large. The weak registration of the 1,000 years period in the $I1/I2$ correlation is also explained in this way.

A periodicity of about 2,000 years is indicated by all four autocorrelations and by five cross-correlations (Table 1). The phase difference for $D1/I1$ is $0.28 T$ which suggests that drifting sources dominated the N. American SV, though standing sources must play some part because $\phi = 0.25 T$ for pure drift. We note that the N. American VGP path (see Fig. 3 of Creer and Tucholka, 1982a) traces out clockwise loops for the whole of this time interval except for the millenium between about 1,600 and 600 B.P. The existence of standing sources is indicated by the in-phase relationship for $D1/D2$. However $I1/I2$ shows a phase difference of $0.33 T$ suggestive of both drifting and standing sources. Translated into a drift rate the phase difference of 720 years indicates westward drift at $0.12^\circ/\text{year}$ whereas if the 2,000 years period were pro-

duced by a drifting source exclusively, the drift rate would be $0.18^\circ/\text{year}$. Thus, we are prompted to propose the existence of another standing source with period $\sim 2,000$ years. Since the strong $D2/I1$ correlation indicates this periodicity, we suggest that the component of the GED along the 100° – 280° azimuth was oscillating with a period of about 2,000 years.

The interpretation becomes more complicated when we attempt to explain the longer period of $\sim 3,000$ years indicated by the $D2/I2$ correlation (note that spectral analyses indicate 2,500 years – Table 3). This would suggest that the SV observed in U.K. is affected by yet another source, possibly a radial dipole since it appears not to have affected the N. American SV record. This could be similar to one of the radial dipoles with which Allredge-Hurwitz (1964) modelled the geomagnetic field for epochs 1945 and 1955 AD: the one at 35°N , 41°E would appear to be in about the right place, producing declination and inclination curves which are in phase (Fig. 10) as observed (Table 1).

Thus we propose four separate geomagnetic sources (Table 4) to account for the relationship observed between the SV type-curves.

4,760–8,520 B.P.

The autocorrelations for $D1$ and for $I2$ show evidence of the presence of $T \sim 2,000$ years. It also shows up in the $D1/I1$ correlation with a phase shift of $0.28 T$ suggestive of a drifting source mechanism and in the $I1/I2$ correlation for which the phase shift of $+460$ years allows us to roughly estimate westward drift at $\sim 0.19^\circ/\text{years}$. This is unlikely to be the true drift rate because a standing source must make some contribution to the SV records since the $D1/I1$ phase shift differs from $0.25 T$. Nevertheless, the N. American VGP path for $\sim 8,500$ – $6,000$ B.P. executes an open clockwise loop consistent with westward drifting sources (see Fig. 3 of Creer and Tucholka, 1982a).

Both declination type-curves, $D1$ and $D2$ are dominated by a longer period $\sim 3,600$ years which shows up in both $D1$ and $D2$ autocorrelations and in the $D1/D2$ correlation for which the phase shift is $\sim -0.11 T$ indicating a contribution from both standing and drifting sources with the former dominant. The negative sign shows that $D1$ and $D2$ are almost opposite in phase, so the standing source must be located at some longitude between the two sites. If it is placed at a high latitude, the amplitudes of SV in inclination will be much smaller than those in declination (see Fig. 12) which would explain why this periodicity does not show up in the inclination analyses. We note that one of the RD computed by Allredge and Hurwitz (1964) for the geomagnetic field for epochs 1945 and 1955 A.D. is located in a suitable position, 76°N , 342°E .

The $D1/D2$ phase shift is -425 years as compared with the $I1/I2$ phase shift of $+460$ years. This difference in sign could be explained if $D1$ and $D2$ were each the sum of an out-of-phase standing component and a drifting component of comparable magnitude. On the other hand, $I1$ and $I2$ mainly consist of a component from the drifting source because the amplitudes produced by the standing source at 76°N latitude would be very small.

Finally a third component with $T \sim 2,700$ years appears in the $I1$ autocorrelation record and in the $D2/I1$ correlation. We think that possibly this may result from our failure to separate the 2,000 years and 3,600 years components in

Table 4. Suggested source mechanisms 0–4760 B.P.

Type of source	Location	Period (yr)	Standing (<i>S</i>) or drifting (<i>D</i>)	Elements affected
GED	10°–190°	~1,000	<i>S</i>	<i>D1, I2</i>
GED	100°–280°	~2,000	<i>S</i>	<i>D2, I1</i>
2RD ^a	around 45° N	~2,000	<i>D</i>	<i>D1, I1, I2</i>
RD	(35° N 41° E) ^b	≥2,500	<i>S</i>	<i>D2, I2</i>

^a Approximately as in Yukutake-Tachinaka (1968) model

^b Or in this general position which is that of one of the eight RD with which Alldredge Hurwitz (1964) modelled the 1,945 A.D. field

Table 5. Suggested source mechanisms 4,760–8,520 B.P.

Type of source	Location	Period (yr)	Standing (<i>S</i>) or drifting (<i>D</i>)	Elements affected
GED	10°–190° E	~2,000	<i>S</i>	<i>D1, I2</i>
2RD ^a	around 45° N	~2,000	<i>D</i>	<i>I1, I2</i>
RD	(76° N 342° E) ^b	3,600	<i>S</i>	<i>D1, D2</i>

^a Approximately as in the Yukutake-Tachinaka (1968) model

^b Or in this general position which is that of one of the eight RD with which Alldredge-Hurwitz (1964) modelled the 1,945 A.D. field

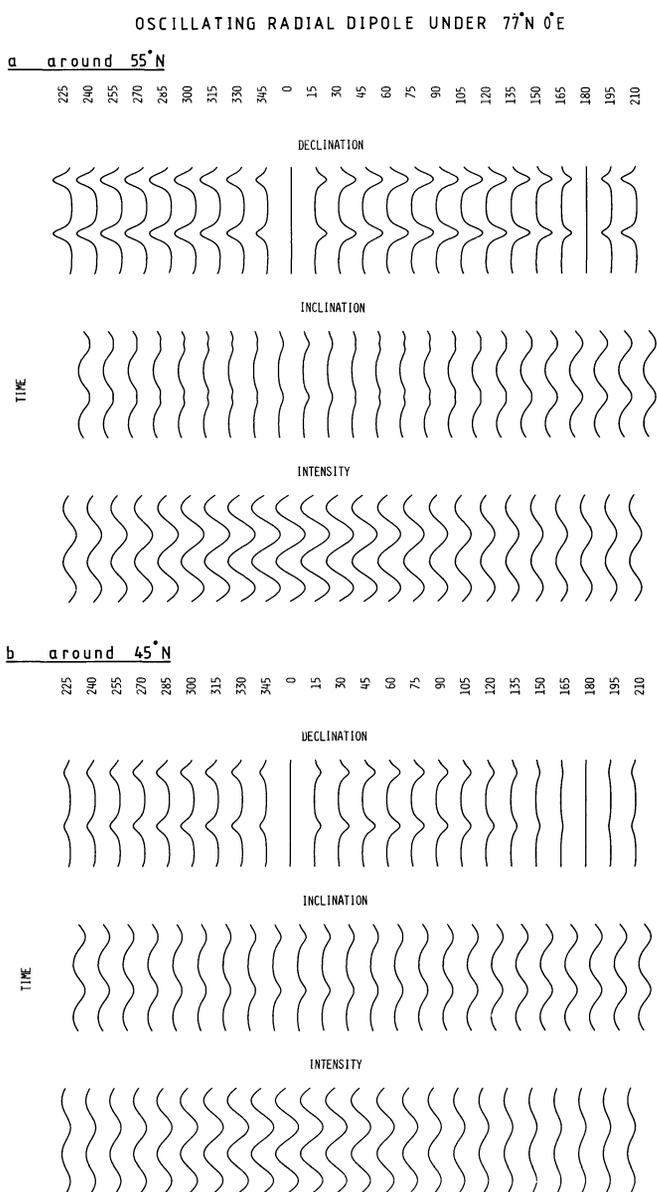


Fig. 12a and b. SV in declination, inclination and intensity observed around **a** 55° N, **b** 45° N produced by an oscillating radial dipole at 70° N, 0° E with $m/M=0.20$, $r=1,750$ km and $T=5,000$ years. Longitude of observer shown at head of each column of plots

these records. The 2,700 years period does not show up in the maximum entropy time-series analyses of declination or inclination for any PEF but this method does suggest a significant difference between the periodicities associated with the *D1* (3,100 years) and *D2* (4,400 years) records. Our autocorrelation method gives ~3,600 years for both *D1* and *D2* and the discrepancy is very likely due to the fact that our records span only one period of signal which is contaminated by the signal from the drifting source.

Acknowledgements. This work and PT's research post have been funded by a research grant from the U.K. Natural Environment Research Council (GR3/2238).

References

- Allredge, L.R., Hurwitz, L.. Radial dipoles as the sources of the Earth's main magnetic field. *J. Geophys. Res.* **69**, 2631–2640, 1964
- Banerjee, S.K., Lund, S.P., Levi, S.. Geomagnetic record in Minnesota lake sediments – absence of the Gothenburg and Eriean excursions. *Geology* **7**, 588–591, 1979
- Barton, C.E.: Analysis of palaeomagnetic time series – techniques and applications. Geophysical Surveys, in press 1982
- Broecker, W.S., Olson, E.A. Lamont radiocarbon measurements. *Radiocarbon* **3**, 176–204, 1961
- Burg, J.P.. Maximum entropy spectral analysis. 37th Ann. Int. Meeting Soc. Explor. Geophys., Oklahoma City, 1967
- Burg, J.P.. A new analysis for time series data. Adv. Study Inst. on Signal Processing, NATO, Enshede, 1968
- Clark, R.M.. A calibration curve for radiocarbon dates VIII. *Antiquity XLIX*, 251–266, 1975
- Creer, K.M. Geomagnetic secular variations during the last 25000 years: an interpretation of data obtained from rapidly deposited sediments. *Geophys. J.R. Astron. Soc.* **48**, 91–109, 1977
- Creer, K.M.. Long period geomagnetic secular variations since 12000 yr B.P. *Nature* **292**, 208–212, 1981 a
- Creer, K.M.: Palaeomagnetism of the Quaternary Period. In: Magnetostratigraphy and geodynamics – results of researches on the international geophysical projects, A.N. Khramov, ed. Moscow pp. 16–25. 1981 b (in Russian).
- Creer, K.M.: Lake sediments as recorders of geomagnetic field variations – applications to dating post-glacial sediments. *Hydrobiologia*, **92**, 587–596, 1982
- Creer, K.M., Gross, D.L., Lineback, J.. Origin of regional magnetic variations recorded by Wisconsin and Holocene sediments from Lake Michigan, USA and Lake Windermere, England. *Bull. Geol. Soc. Am.* **87**, 531–540, 1976
- Creer, K.M., Hogg, T.E., Malkowski, Z., Mojski, J.E., Niedziolka-

- Kroll, E., Readman, P.W., Tucholka: Palaeomagnetism of Holocene lake sediments from North Poland. *Geophys. J.R. Abstract Soc.* **59**, 287–314, 1979
- Creer, K.M., Hogg, T.E., Readman, P.W., Reynaud, C.: Palaeomagnetic secular variation curves extending back to 13400 years B.P. recorded by sediments deposited in Lac de Joux, Switzerland. *J. Geophys.* **48**, 139–147, 1980
- Creer, K.M., Readman, P.W., Papamarinopoulos, S.: Geomagnetic secular variations in Greece through the last 6000 years obtained from lake sediment studies. *Geophys. J.R. Astron. Soc.* **66**, 193–219, 1981
- Creer, K.M., Tucholka, P.: Secular variation as recorded in lake sediments: a discussion of North American and European results. *Phil. Trans. R. Soc., Lond.* **A 590**, 87–102, 1982a
- Creer, K.M., Tucholka, P.: Construction of type-curves of geomagnetic secular variation for dating lake sediments from east-central N. America. *Can. J. Earth Sci.* **19**, 1106–1115, 1982b
- Denham, C.R.: Spectral analysis of palaeomagnetic time series – techniques and applications. *J. Geophys. Res.* **80**, 1897–1901, 1975
- Mothersill, J.S.: The palaeomagnetic record of the Late Quaternary sediments of Thunder Bay. *Can. J. Earth Sci.* **16**, 1016–1023, 1979
- Mothersill, J.S.: Late Quaternary palaeomagnetic record of the Goderich Basin, Lake Huron. *Can. J. Earth Sci.* **18**, 448–456, 1981
- Rao, C.R., Mitra, J.K., Matthai, A.: Formulae and tables for statistical work. Calcutta: Statistical Publishing Society 1966
- Skiles, D.D.: A method of inferring the direction of drift of the geomagnetic field from palaeomagnetic data. *J. Geomagn. Geoelectr.* **22**, 441–462, 1970
- Tucker, P.: Selective post-depositional in a synthetic sediment. *Phys. Earth. Planet. Int.* **20**, 11–14, 1979
- Tucker, P.: A grain mobility model of post-depositional realignment. *Geophys. J.R. Astron. Soc.* **63**, 149–163, 1980
- Turner, G.M., Thompson, R.: Lake sediment record of the geomagnetic secular variations in Britain during Holocene time. *Geophys. J.R. Astron. Soc.* **65**, 703–725, 1981
- Yukutake, T., Tachinaka, H.: The westward drift of the geomagnetic secular variation. *Bull. Earthquake Res. Inst. Tokyo* **46**, 1027, 1968
- Yukutake, T., Tachinaka, H.: Separation of the Earth's magnetic field into the drifting and the standing parts. *Bull. Earthquake Res. Inst. Tokyo* **47**, 65, 1969

Received July 1, 1982; Revised September 6, 1982;

Accepted September 6, 1982