J. Geophys. 42, 237 – 256, 1976

Journal of Geophysics

Geothermal Models of the Crust and Uppermost Mantle of the Fennoscandian Shield in South Norway and the Danish Embayment

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Abstract. A narrow heat flow transition zone between the Fennoscandian Shield and the North Sea Basin has been investigated along a profile from the Precambrian of South Norway to the Danish Embayment in North Jylland. Along this profile the surface heat flow varies from about 42 mW m^{-2} (measured) in South Norway to 60–70 mW m^{-2} (estimated) in Denmark. Geothermal, seismic, gravity and other geophysical and geological data have formed the basis for construction of heat production and thermal conductivity models in the depth interval 0–50 km. The related steady-state temperatures and heat flow distributions are calculated by a numerical solution of the heat conduction equation in two dimensions. Three models are presented, a preferred model and two others which yield temperatures assumed to be close to the lowest and highest possible values. The preferred model gives temperatures of about 350 °C at the crust-mantle boundary in the Shield and approximately 700 °C beneath the Danish Embayment. These differences are associated with considerable variations in the heat flow from the mantle. In the main model variations from 16–17 mW m^{-2} in the shield region to about 40 mW m^{-2} in the Danish Embayment have been found. Some geophysical and petrological implications are discussed. In the sedimentary basin partial melting in the lower crust and at shallow depth in the uppermost mantle seems to be likely.

Key words: Geothermal models – Heat flow transition zone – Crust uppermost mantle – Fennoscandian Shield and the Danish Embayment.

1. Introduction

South Scandinavia consists of three main structural-geological units: The southern part of the Paleozoic Caledonian orogenic belt, the southeastern part of the Precambrian Fennoscandian (Baltic) Shield, and the northeastern part of the North Sea Basin (Fig. 1). The shield area is subdivided into two parts by the Permian Oslo Graben. Our present knowledge of geological and crustal physical conditions indicates that the transition zone between the Fennoscandian Shield,

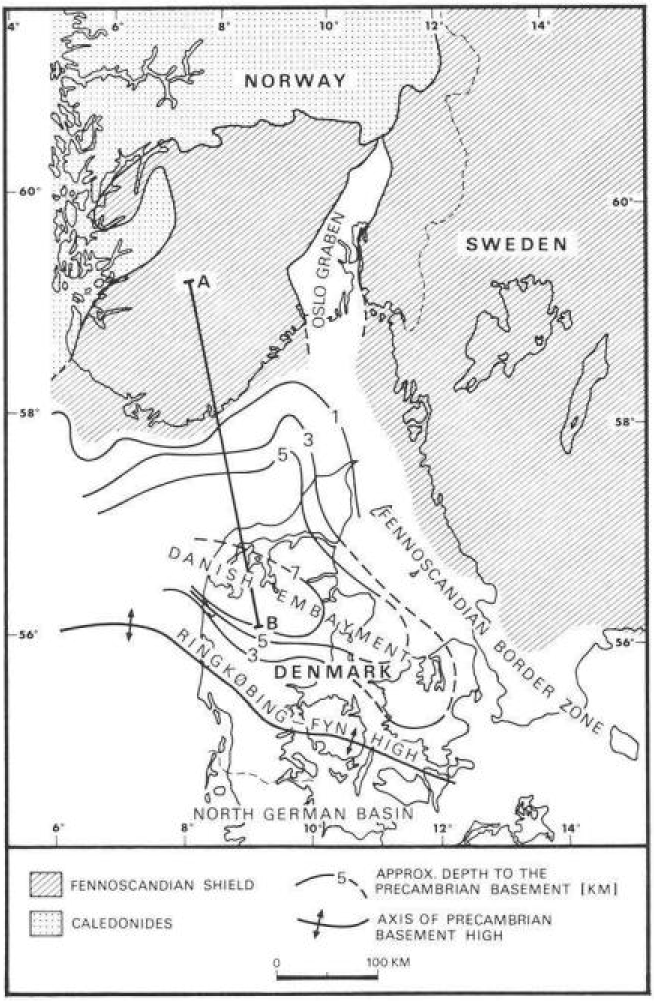

Fig. 1. Main structural-geological units of South Scandinavia and the position of the geothermal profile (AB)

which is a province of low heat flow, and the North Sea Basin where the heat flow in general seems to be considerably higher, also represents a region with great lateral temperature variations in the crust and upper mantle and with great variations in the heat flow from the mantle. To examine this anomaly, general geothermal models are constructed along a profile from the shield region in South Norway to the Danish Embayment (profile AB, Fig. 1). This is the area for which we have the most geophysical and geological information.

Based on heat flow values, thermal parameters for the near-surface rocks, seismic and gravity results, models of the radioactive heat production and of the thermal conductivity distribution for the depth interval 0–50 km are made. The related temperature models are calculated from a numerical solution of the heat conduction equation

$$A+\nabla\cdot(K\nabla T)=0 \tag{1}$$

where A is the heat production per unit volume per unit time, K is the thermal conductivity, and T is the temperature. This equation is valid for steady-state conditions in a non-homogeneous but isotropic medium without heat transfer by moving materials.

Heat production and conductivity are functions of the position, while conductivity is also a function of the temperature and the pressure. The variations perpendicular to profile AB are assumed to be so small that the problem can be treated as two-dimensional.

Equation (1) is to be solved with respect to $T=T(x, z)$ using models containing known or assumed thermal conductivity and heat production distribution together with some necessary boundary conditions. Three geothermal models are constructed. What are considered the most probable values of the geothermal parameters are used in model A. In model B and C the parameters are combined in such a way as to yield what is assumed to be close to the lowest and highest possible temperature, respectively. To construct geothermal models, knowledge of the geology, the crustal and uppermost-mantle physics including the measured geothermal parameters of the area, is essential and shall be treated in some details.

2. Geology, Crustal and Uppermost-Mantle Physics

South Norway constitutes a typical Precambrian migmatite area that has undergone a complicated metamorphic, magmatic and structural development (Barth and Dons, 1960; Smithson, 1965; Falkum, 1966, 1972). The area is dominated by high-metamorphic (medium and high grade) granitic gneiss, augen gneiss, banded gneiss and intrusive granites and charnockites. The banded gneiss, which is composed of alternating layers of amphibolite, pyribolite, acidic and intermediate gneiss, garnet-cordierite-sillimanite gneiss, quartzite and pyribolite-anorthosite, is of special interest as an indicator of the composition of the deeper parts of the crust. The area is considered to be poly-orogenic with the Sveconorwegian orogeny (approx. 1,000 m.y.) as the last one. The main part of the present surface rocks seems to be generated by anatexis and related processes to a large extent. The mean specific density of the main rock units varies from $2.6 \cdot 10^3$ kg m^{-3} for some of the intrusive granites to 2.8 for some banded gneiss formations. The mean density of 658 samples from the above mentioned rock types collected by Professor S. Saxov and the author in connection with a gravity study of the area is $2.68 \cdot 10^3$ kg m^{-3}.

The general geological conditions of the sedimentary basin are rather well-known from the many exploration drillings covering the area (Sorgenfrei and Buch, 1964; G. Larsen, 1966; Sorgenfrei, 1969; Rasmussen, 1972, 1974) except for the Skagerrak area and the deepest part of the Danish Embayment. To find the thickness of the sedimentary formations the seismic and magnetic investigations of Sellevoll and Aalstad (1971) and Casten and Hirschleber (1971) together with

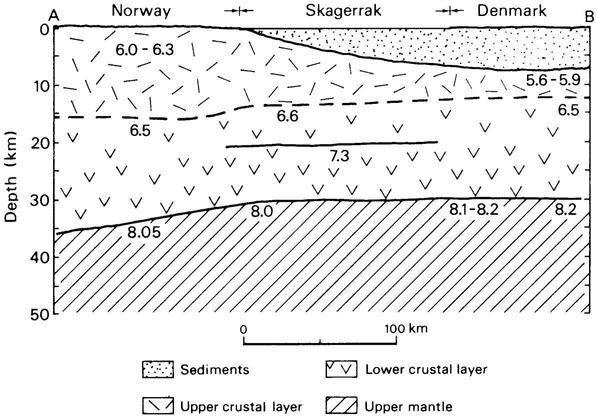

Fig. 2. Crustal structure along profile AB (Fig. 1) from seismic investigations. Results from lines within a distance of ± 50 km are projected on to the profile. The figures indicate *P*-wave velocities (km s^{-1})

results from drill holes and other geophysical data have been compiled and interpreted (Fig. 1). The total thickness of the Paleozoic, Mesozoic, and Cenozoic sedimentary sequence in the central part of the Danish Embayment is supposed to be 7–9 km. Radiometric dating of some basement cores from Jylland and Fyn indicates a basement age similar to that of the Sveconorwegian orogeny of the shield region (O. Larsen, 1971, 1972).

The general crustal structure of Scandinavia is known from several deep seismic sounding projects (Sellevoll, 1973). Results from the area of profile AB (Hirschleber et al., 1966; Aric, 1968; Weigel et al., 1970; Casten and Hirschleber, 1971; Sellevoll and Warrick, 1971) compiled in Figure 2 show typical continental velocity-depth values. For simplicity the terms 'upper crustal layer' and 'lower crustal layer' are used, although variations within these units are likely both as to physical and petrological conditions. In all investigations the crust-mantle boundary (the Moho) has been found by well-defined depths and *P*-wave velocities. In the models of South Norway, the crustal thickness is about 32 km in the coast region increasing to about 36 km at the northern part of profile AB. To the south the crustal thickness decreases to 29–32 km below the Skagerrak and North Jylland. The crust seems to be sub-divided by the 6.5–6.6 km s^{-1} refractor – the Conrad 'discontinuity' – which, according to the cited authors, has been observed in the whole area with a well defined velocity but with some uncertainties concerning the depth. For South Norway the depth values in the interpretational models are 14–17 km, and in the Skagerrak and North Jylland 12–14 km. The extremely low values of 8–9 km in the models of Hirschleber et al. (1966) are not realistic, due in part to their use of too low a velocity for the sediments. In the models, the Precambrian basement of North Jylland has a lower velocity than the related rocks at similar pressure close to the surface in South Norway. There are reasons to believe that the basement rocks to the south are more granitic. For both areas there are clear indications of a Pg-velocity increase with depth, which cannot be explained by the effect of pressure alone.

Below the Skagerrak an intermediate velocity of 7.3 km s^{-1} has been observed at a depth of about 20 km. Velocities of about 7 km s^{-1} at that level are also reported at other places in South Scandinavia (Kanestrøm and Haugland, 1971; Gregersen, 1971). Although such intermediate velocities have not been introduced in the models to the north and south along profile AB, it must be assumed that the *P*-wave velocity of the lowest part of the crust is higher than 6.5–6.6 km s^{-1}. This assumption is based on an analysis of the regional gravity field of South Scandinavia (Balling, in preparation). From these investigations an average density of the upper crustal layer of $2.7–2.8 \cdot 10^3$ kg m^{-3} increasing to $2.8–2.9 \cdot 10^3$ kg m^{-3} in the upper part of the lower crust is found. At the base of the crust the density seems to be 2.9–3.1 followed by $3.3–3.4 \cdot 10^3$ kg m^{-3} in the uppermost mantle.

The upper crustal layer is supposed to be dominated by high-metamorphic migmatites similar to those observed at the surface in the Precambrian of South Norway but with an acidity decreasing with depth. Gravity investigation of some of the intrusive granites in South Norway shows a vertical range of the density contrast of 2–5 km, which together with the seismic results support the assumption of this decreasing acidity.

The petrological nature of the lower crust in stable continental areas has been a matter of discussion. The detailed experimental investigations of Green and Ringwood a.o. (see e.g. Green and Ringwood, 1967, 1972) show that it is unrealistic to label the lower crust 'basaltic' or 'gabbroic' as is usual. It is to be assumed in general that gabbro is not stable in the lower crust. Here large amounts of high-density and high-metamorphic elements of the banded-gneiss formations observed in South Norway are presumed to be present.

The geothermal models are constructed on the assumption that the uppermost mantle consists of olivine-peridotite (see e.g. Ringwood, 1969).

Near-surface geothermal conditions of the Precambrian in South Norway have been investigated by Swanberg et al. (1974) and Haenel et al. (1974). Heat flow values are determined at 19 localities, the average being 38.6 mW m^{-2}. Very low values (20–25 mW m^{-2}) are found near the centre of the 1,000 km^2 Egersund anorthosite complex with associated low heat production. A large number of heat production values from mainly granites are available (Killeen and Heier, 1974, 1975a, b). The linear relation between heat flow and heat generation based on only a few localities in the area (Swanberg et al., 1974) is most likely of limited importance. As the vertical extent of the surface formations, with a few exceptions, is less than a few kilometres, and the heat generation varies very much between the different rock units, no linear relation is to be expected, except perhaps, if average values of large geological units are used. The very low heat flow values of 20–25 mW m^{-2} are of great importance, as they determine the maximum possible heat flow from the mantle.

In the above-mentioned heat flow values the climatic corrections are supposed to be somewhat under-estimated. A surface value of 42 mW m^{-2} shall be used in the models.

The average geothermal gradient of central parts of the Danish Embayment within a depth of about 5 km is around 27–30 °C km^{-1} (Madsen, 1975). Thermal conductivities of rocks from the southeastern part of the North Sea Sedimentary

Basin have been published by e.g. Majorowicz (1973), Wesierska (1973), and Kappelmeyer and Haenel (1974, p. 55). Taking the temperature dependence into consideration the average conductivity of the sedimentary formations in question is estimated to be 2.2–2.3 W m^{-1} °C^{-1}. Consequently, the heat flow is estimated to be about 60–70 mW m^{-2}.

3. Thermal Conductivity Models

The thermal conductivity depends on the material, the temperature, and the pressure. For the majority of materials to be dealt with in the present case in the range of temperatures involved, Clark (1957, 1969) and others have given the total conductivity as

$$K = K_l + K_r \tag{2}$$

where K_l is the lattice (or phonon) component and K_r is the radiative component. K_l is approximately related to the temperature by

$$K_l = (a + bT)^{-1} \tag{3}$$

where a and b are constants.

The investigations of Fukao et al. (1968) and Schatz and Simmons (1972) on olivines show that the contribution of K_r is about 50% at 900–1,300 °C. This is important in estimating the conductivity of the upper mantle. As to crustal rocks at temperatures below 800–900 °C, Equation (3) seems to be an acceptable approximation to the total conductivity, (e.g. Birch and Clark, 1940; Kawada, 1964 and 1966). Here it is appropriate to use

$$K(T) = \frac{K_0}{1 + cT} \tag{4}$$

where $c = a/b$ and K_0 is the conductivity at 0 °C.

The effect of pressure on thermal conductivity has been little studied theoretically and experimentally. Dry metamorphic and magmatic rocks of low porosity exposed to an increase of pressure from 0 to about 1 kb seem to have a conductivity increase of about 10% on an average due to reduction of pore volume (Walsh and Decker, 1966; Hurting and Brugger, 1970). According to Walsh and Decker the effect of water saturation is of the same magnitude, and the saturated samples showed no significant effect due to pressure. At higher pressures the elastic properties of the medium are of importance. A further conductivity increase to the depth interval of 50 km seems to be only a few percent (Bridgman, 1924; Fujisawa et al., 1968) and will be ignored.

The c-value in Equation (4) for the upper crustal rocks of interest is about 0.6–1.3 · 10^{-3} °C^{-1} with an average value of about 1.0 · 10^{-3} °C^{-1}. For the lower crustal rocks the temperature dependence seems to be somewhat less, with a c-value of about 0.7 · 10^{-3}. K_0-values of 2.8–3.2 W m^{-1} °C^{-1} are used for the upper crustal layer, and 2.3–2.6 W m^{-1} °C^{-1} for the lower crustal layer. These K_0-values represent the values of water-saturated samples and therefore, no pressure correction function is needed. The estimate of these values is based on the geophysical and geological models, conductivity data of rocks in South Norway, data from other areas with rocks of similar composition (e.g. Wenk and Wenk, 1969; Čermák and Jessop, 1971; Smithson and Decker, 1974) together with the values of rock-forming minerals (Horai and Simmons, 1969). In order to approximate the olivine-peridotite model of the upper mantle the conductivity has been calculated for a theoretical composition of 80% olivine ($Fo_{90}\,Fa_{10}$) and 20% enstatite using the experimental data of Schatz and Simmons (1972) on single crystals of olivine and enstatite. The conductivity of this mantle composition has been calculated in the interval between 250 and 1,250 °C. A conductivity-temperature function has been found by fitting a fourth degree least square function to these data:

$$K(T)=a_0+a_1T^1+a_2T^2+a_3T^3+a_4T^4$$

with $a_0=1.017\cdot10^1$ (model A), $a_1=-2.694\cdot10^{-2}$, $a_2=4.679\cdot10^{-5}$, $a_3=-3.734\cdot10^{-8}$, and $a_4=1.115\cdot10^{-11}$.

Due to some uncertainties with respect to the composition of the upper mantle and the effect of variable grain size on the conductivity (Fukao, 1969), an interval of variation of ±0.5 W m^{-1} °C^{-1} in relation to the model A function has been assumed. The thermal conductivity functions are shown in Figure 3.

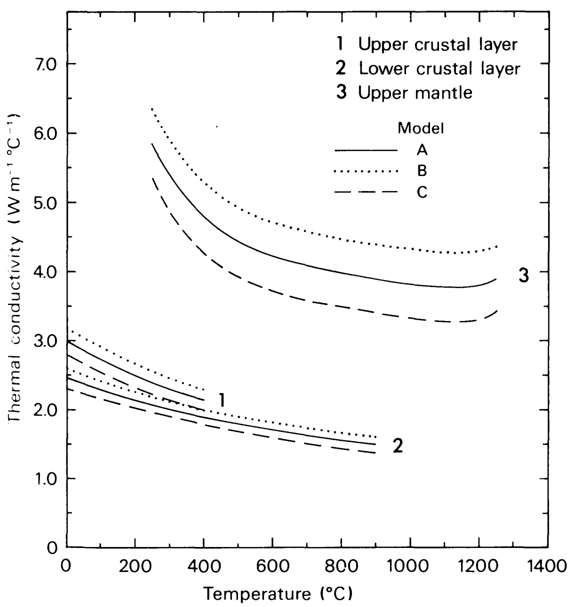

Fig. 3. Assumed thermal conductivity-temperature functions of the main crustal units and the upper mantle. The K_0-values for the upper crustal layer are 3.0, 3.2, and 2.8, and for the lower crustal layer 2.45, 2.6, and 2.3 W m^{-1} °C^{-1} in model A, B, and C, respectively. For the upper crustal layer the c-value is 10^{-3} and for the lower crustal layer $0.7\cdot10^{-3}$ °C^{-1}. The upper mantle functions are described in the text

4. Heat Production Models

The heat production of the surface rocks in the shield area varies strongly from place to place. Very low values of about $0.1 \cdot 10^{-6}$ W m^{-3} have been found for anorthosite and amphibolite, and 2.9–$3.3 \cdot 10^{-6}$ W m^{-3} for granitic gneiss complexes. Extremely high heat production of up to $6.6 \cdot 10^{-6}$ W m^{-3} has been revealed in some of the South Norwegian granites (Killeen and Heier, 1975a, b). Only a few data from the most common migmatites are available, and the average surface values between 1.7–$2.1 \cdot 10^{-6}$ W m^{-3} used in the models are thus only preliminary. But no doubt, there is a considerable decrease in heat production with depth speaking of average values of large units. A heat production decrease with increasing metamorphic degree has been found by Heier and Adams (1965) and Lambert and Heier (1967). Theoretical considerations of Lachenbruch (1970) and field observations of Swanberg (1972) also support the idea of a strong concentration of heat producing elements in the upper part of the continental crust. This is in agreement with the assumed geological models.

Besides the values due to authors already cited, a large number of heat generation data published by Čermák (1975), and compilations of a.o. Shaw (1967), Kappelmeyer and Haenel (1974), and Smithson and Decker (1974) have been used as basis for the model constructions. For the sediments average values between 1.1 and $1.5 \cdot 10^{-6}$ W m^{-3} are assumed. Below the sediments, values of 2.1–$2.9 \cdot 10^{-6}$ W m^{-3} are used for the presumed granitic rocks. Along the whole profile the heat production is assumed to be reduced to 0.5–$0.8 \cdot 10^{-6}$ W m^{-3} at the level of the 6.5–6.6 km s^{-1} refractor and with a further reduction through the lower crust to 0.15–$0.25 \cdot 10^{-6}$ W m^{-3} at the base of the crust. At the upper mantle $0.01 \cdot 10^{-6}$ W m^{-3} has been used. Model B contains the highest and model C the lowest heat production values. In model A the mean values have been used. Between these fixed values at the mentioned levels, an exponential decrease with the depth z of the form

$$A(z) = A_0 e^{-\mu z} \tag{5}$$

has been assumed (Figs. 4 and 5).

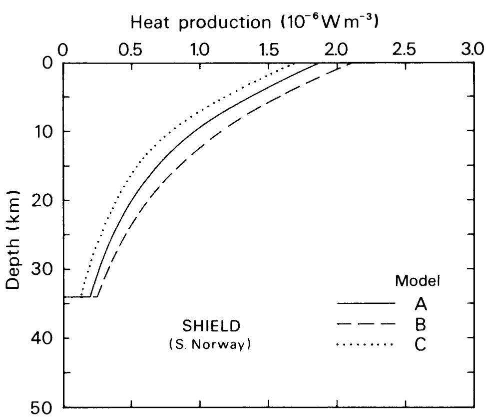

Fig. 4. Assumed heat production models of the Shield (central part)

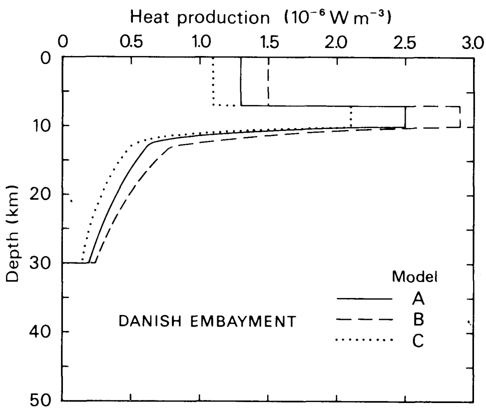

Fig. 5. Assumed heat production models of the Danish Embayment (central part)

5. Solution of the Heat Conduction Equation

We may write Equation (1) as

$$2A+\nabla^2(KT)+K\nabla^2T-T\nabla^2K=0. \tag{6}$$

This is a non-linear differential equation which is to be solved subject to well defined boundary conditions. A modified Gauss-Seidel method in which the temperature dependence on the conductivity is included will be applied to the set of algebraic equations which are obtained from a finite difference approximation to the differential equation. The region for which the differential equation is valid will be divided into a $N\times M$ rectangular mesh. The nodal points of the mesh (i, k) are separated by distances Δx and Δz in the x (horizontal) and z (vertical) directions, respectively. The central difference approximation to Equation (6) using a five-point formula is given by

$$\begin{aligned}&\frac{2A_{i,k}\Delta z^2}{K_{i,k}}+\frac{K_{i-1,k}+K_{i,k}}{r^2K_{i,k}}T_{i-1,k}+\frac{K_{i+1,k}+K_{i,k}}{r^2K_{i,k}}T_{i+1,k}\\&+\frac{K_{i,k-1}+K_{i,k}}{K_{i,k}}T_{i,k-1}+\frac{K_{i,k+1}+K_{i,k}}{K_{i,k}}T_{i,k+1}\\&-\left[2+\frac{2}{r^2}+\frac{K_{i-1,k}+K_{i+1,k}}{r^2K_{i,k}}+\frac{K_{i,k-1}+K_{i,k+1}}{K_{i,k}}\right]T_{i,k}=0\end{aligned} \tag{7}$$

where $A_{i,k}$ are predetermined heat production parameters and $r=\Delta x/\Delta z$. This equation is valid for non-boundary points (i, k). Starting at point $i=1$ and $k=1$, a mixed set of known and estimated values in Equation (7) results in a residual non-zero value on the right hand side of Equation (7), i.e.

$$\begin{aligned}R^0_{i,k}=&\frac{2A_{i,k}\Delta z^2}{K^*_{i,k}}+\alpha^*_{i-1,k}T^0_{i-1,k}+\alpha^*_{i+1,k}T^0_{i+1,k}\\&+\alpha^*_{i,k-1}T^0_{i,k-1}+\alpha^*_{i,k+1}T_{i,k+1}-\alpha^*_{i,k}T^0_{i,k}\end{aligned} \tag{8}$$

where the α's are the appropriate coefficients of Equation (7). From the foregoing a first order estimate in terms of the original zero order estimates $T^0_{i,k}$ and $\alpha^*_{i,k}$ is written:

$$T^1_{i,k} = T^0_{i,k} + R^0_{i,k}/\alpha^*_{i,k} \tag{9}$$

for all internal points of the mesh. This equation defines an iteration procedure. The K- or α-values are adjusted after say 50 iterations of the $T^j_{i,k}$ to take account of their temperature dependence. The general expression for $T^n_{i,k}$ is:

$$T^n_{i,k} = \frac{1}{\alpha^m_{i,k}} \left[\frac{2A_{i,k}\Delta z^2}{K^m_{i,k}} + \alpha^m_{i-1,k} T^n_{i-1,k} + \alpha^m_{i+1,k} T^{n-1}_{i+1,k} + \alpha^m_{i,k-1} T^n_{i,k-1} + \alpha^m_{i,k+1} T^{n-1}_{i,k+1} \right] \tag{10}$$

where the conductivities have been adjusted m times.

At the boundary a combination of prescribed temperature and prescribed heat flow shall be used.

The starting estimates of the $T_{i,k}$- and $K_{i,k}$-values at all nodal points are determined by solving the one-dimensional heat conduction equation:

$$A(z) + \frac{d}{dz}\left(K(T)\frac{dT}{dz}\right) = 0 \tag{11}$$

with different parameters.

Case (a)

$$A(z) = A_0, \qquad K(T) = K_0$$

$$T(z) = T_0 + \frac{q_0}{K_0} z - \frac{1}{2}\frac{A}{K_0} z^2 \tag{12}$$

where $T_0 = T(0)$ and $q_0 = K_0 \left(\frac{dT}{dz}\right)_{z=0}$.

Case (b)

$$A(z) = A_0; \qquad K(T) = \frac{K_0}{1+cT}$$

$$T(z) = \frac{1}{c}\left[(1+cT_0)\exp\left\{\frac{c}{K_0}(q_0 z - \tfrac{1}{2}A_0 z^2)\right\} - 1\right]. \tag{13}$$

Case (c)

$$A(z) = A_0 e^{-\mu z}; \qquad K(T) = \frac{K_0}{1+cT}$$

$$T(z) = \frac{1}{c}\left[(1+cT_0)\exp\left\{\frac{c}{K_0}\left(\frac{A_0}{\mu^2}(1-\exp(-\mu z)) - \frac{A_0}{\mu} z + q_0 z\right)\right\} - 1\right]. \tag{14}$$

In order to investigate the accuracy of the described numerical method, with special reference to the influence of variable Δz values, a number of test models have been examined. Results from one of them, a model with variable heat production and variable conductivity of the type c described above will be shown.

The parameters are similar to those of the crustal unit of profile AB:

$T_0 = 10\,°C$,

$q_0 = 60\,mW\,m^{-2}$,

$A_0 = 2 \cdot 10^{-6}\,W\,m^{-3}$,

$K_0 = 3\,W\,m^{-1}\,°C^{-1}$,

$c = 10^{-3}\,°C^{-1}$,

$\mu = 7 \cdot 10^{-5}\,m^{-1}$.

Solutions are found for the depth interval 0–40 km. At the upper boundary the temperature is 10 °C. At the side boundaries $K(T)\frac{\partial T}{\partial x}$ is equal to zero, and at the lower boundary $K(T)\frac{\partial T}{\partial z}$ is equal to 33.17 mW m^{-2}, which defines the boundary conditions. Numerical solutions have been found with the Δz-values of 1, 2.5, 5 and 10 km. With Δz equal to 1 km the deviations between the exact solution given by Equation (14) and the numerical solution are less than 0.1 °C, and even with Δz equal to 10 km the differences are relatively small (Table 1). If poor starting solutions are used (say 10 °C km^{-1}) the computing time increases markedly.

In the solutions for the profile AB of Figures 1 and 2 the mesh size used was $\Delta x = 15$ km and $\Delta z = 2.5$ km. With these values the seismic boundaries, which are assumed to be also thermal parameter boundaries, are approximated within the accuracy of their determination. The results compiled in Table 1 show that no problems arise due to variations between these boundaries.

The boundary conditions are as follows: The mean annual surface temperature is used at $z = 0$ km. In the Skagerrak and South Norway corrections for the effect of water and topography are applied. The profile length is 345 km, which is enough to make the horizontal heat flow below point A and B negligible $\left(\left(\frac{\partial T}{\partial x}\right)_{1,k} = \left(\frac{\partial T}{\partial x}\right)_{24,k} = 0\right)$. The vertical heat flow across the lower boundary

Table 1. The analytical solution and numerical solutions for the test model

Depth	Analytical solution	Numerical solutions (°C)			
		Δz			
(km)	(°C)	1 km	2.5 km	5 km	10 km
0	10	10	10	10	10
5	108.0	107.9	107.8	107.3	
10	201.1	201.0	200.7	199.8	196.2
15	291.2	291.1	290.7	289.4	
20	379.9	379.8	379.3	377.6	371.2
25	468.5	468.5	467.9	465.9	
30	558.4	558.3	557.6	555.3	546.3
35	650.3	650.2	649.4	646.8	
40	745.0	745.0	744.1	741.1	729.6

($z = 50$ km) has been found by "trial and error" starting with the values found by a one-dimensional consideration, with the horizontal heat transfer neglected. This heat flow is determined to produce agreement between the model results and the surface heat flow values observed. For the shield area this is 42 mW m^{-2}, which has been used in all models. For the Danish Embayment 65 mW m^{-2} has been used in model A and 60 and 70 mW m^{-2} in model B and C, respectively. Between the coast regions linear interpolation has been used.

The starting estimates of the temperatures and the conductivities have been found by using Case c (Eq. 14) for the main crustal units and Case a and b (Eqs. 12 and 13) for the upper mantle and the sedimentary region.

After 1,000–1,500 iterations the calculated temperatures at the region of temperature maximum are within 1–2 °C of the asymptotic values.

The computations were carried out at the CDC 6400 at the Aarhus University, Computing Centre. The computation time was about 10 min for each model with the stated accuracy.

6. Computation Results

The models calculated take into account temperature distribution, conductivity distribution, and heat flow distribution. From the model data the average temperatures, vertical geothermal gradients and conductivities for the Shield and the Embayment are calculated (Tables 2 and 3). The Shield values represent average values of the northernmost 90 km of the profile, and the Embayment values are average values of the southernmost 60 km. In these areas the lateral variations

Table 2. Computed temperature distribution and heat flow from the mantle

Depth	Average temperature (°C)					
	Shield			Embayment		
(km)	Model A	Model B	Model C	Model A	Model B	Model C
0	11	11	11	8	8	8
5	78	70	84	148	137	155
10	137	120	152	264	232	292
15	189	162	214	373	313	433
20	242	203	280	489	397	587
25	292	241	346	607	479	749
30	339	275	410	712	549	898
35	376	299	463	765	584	977
40	394	311	492	815	616	1,052
45	412	321	519	866	649	1,129
50	429	332	547	918	682	1,206
Heat flow from the mantle (mW m^{-2})	16–17	11–13	21–22	41	31	51

Table 3. Computed vertical geothermal gradients and thermal conductivity

Unit	$\Delta T/\Delta z$ (°C km^{-1})			$k(T)$ (W m^{-1} °C^{-1})		
	Model A	Model B	Model C	Model A	Model B	Model C
Shield						
Upper crustal layer	10–14	9–13	12–16	2.5–3.0	2.8–3.2	2.3–2.8
Lower crustal layer	9–11	6–10	12–14	1.9–2.1	2.1–2.3	1.8–2.0
Upper mantle	3–4	2–3	5–6	4.7–4.9	5.7–5.9	3.8–4.0
Embayment						
Sedimentary layer[a]	28	26	29	2.25	2.2	2.3
Upper crustal layer	22	17–18	26–27	2.2–2.5	2.4–2.7	2.0–2.3
Lower crustal layer	23–24	16–17	29–34	1.7–1.9	1.9–2.1	1.5–1.8
Upper mantle	10–11	6–7	14–15	3.9–4.1	4.6–4.8	3.3–3.4

[a] Average values

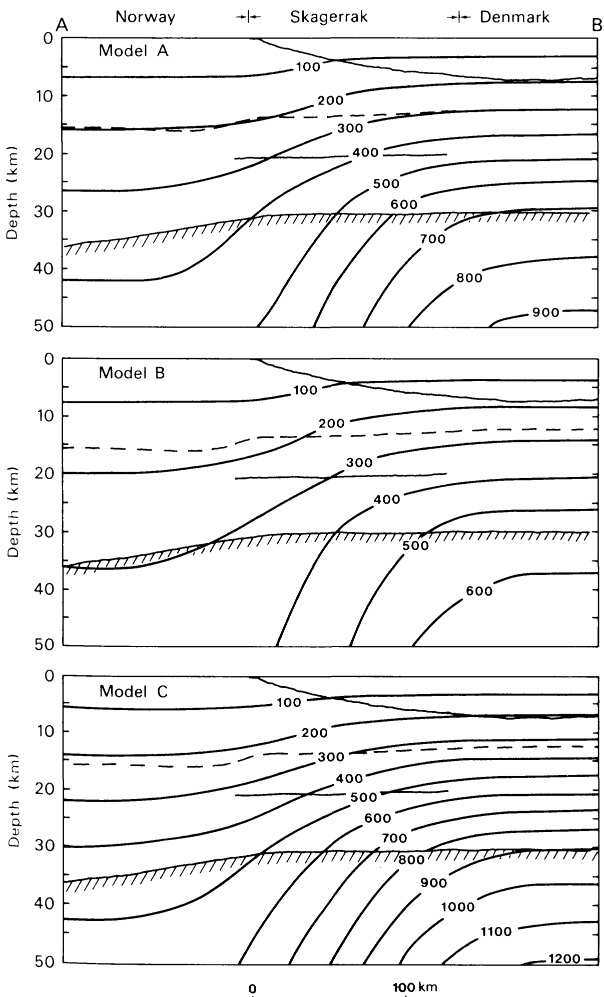

Fig. 6. Isothermal lines (°C) of the computed temperature models A, B, and C. The preferred parameters have been used in model A. Models B and C have been constructed to give what is assumed to be close to the lowest and highest possible temperatures, respectively

are very small (Fig. 6). The temperatures in the models of South Norway have been determined within rather narrow limits, whereas the values of North Jylland show a rather large variation as shown in models B and C. It appears that even over a small horizontal distance considerable temperature differences may be discovered. At a depth of 50 km the differences between the temperatures in the models of the Precambrian in South Norway and the Danish Embayment are 350–660 °C (model B and C), with the preferred value approximately 500 °C (model A).

The main reason for the temperature differences in the two areas and the spread between the models is the rather large differences in heat flow, not just at the surface, but through the whole depth interval considered. As the total crustal heat production in the models is almost the same in both areas, the heat flow from the mantle differs considerably with 11–22 mW m^{-2} in the shield models, and 31–51 mW m^{-2} in the models of the Danish Embayment. The models imply a considerable lateral heat flow in the uppermost mantle from North Jylland into the Skagerrak, the maximum values being between 12 and 15 mW m^{-2}. Due to this lateral heat transfer the heat flow from the mantle in the southern area is 5–10% higher than the one-dimensional values. In the coast region of Norway the models show an increase of the surface heat flow of around 10% due to this phenomenon. None of the models have isothermal lines that follow the seismic boundaries (Fig. 6). At the Moho the model temperatures are 280–470 °C in the shield region and 550–900 °C in the Embayment. The preferred values are approximately 350 °C and 700 °C in the Shield and the Embayment, respectively.

7. Discussion

The computed results show that small changes in the geothermal parameters may produce considerable temperature variations. This applies especially to areas with normal or high surface heat flow. In such areas it is essential that accurate heat flow determinations are available. Use of the model A value (65 mW m^{-2}) together with the other model parameters of the models B and C would reduce the differences between these two models by 200–260 °C at a depth of 50 km in the Danish Embayment. Due to the low heat flow in the shield area a small change in this parameter will only cause small variations in the temperature distribution. A change of the average surface value from 42 mW m^{-2}, used in all models, by ± 3 mW m^{-2} will only alter the temperature at a depth of 50 km by about ± 50 °C.

Since there is apparently a high degree of uniformity in seismic velocities and density distribution along the profile, no attempt was made to compute models with variations in the parameters. It should be emphasized that large variations in the heat production distribution in the crust will produce only insignificant temperature variations, provided that the integrated heat production within a given crustal unit is maintained within reasonable limits. As previously mentioned, too few heat production data are available from the migmatites in Southern Norway to construct a definitive model. However, moderate changes from the model parameters used would not change the temperature distribution significantly. Recently, Massé (1975) has stated that P^* (Pb) velocities lower than 7.2 km s^{-1} are misinterpretations of retrograde reflections in several refraction studies of the Scandinavian region. The use of possible alternative seismic models to those shown in Figure 2 need not alter the results.

It is obvious that steadily increasing temperature differences with depth between the two regions cannot continue, and at some depth the temperature curves must converge. This can arise due to great differences in heat production and/or large increase in heat transfer, e.g. heat transfer by moving material. This last phenomenon is closely connected to the problem of partial melting (Fig. 7). The lowest possible temperatures of the beginning of anatexis in gneisses are those that produce melting in the system Q—Ab—Or—H_2O. It is not possible to make accurate calculations on the effect of anatexis on the temperature depth curves, partly due to lack of experimental data giving information of the thermal conductivity variations in systems with anatexis, partly because this would demand rather exact knowledge of the mineralogical nature of the lower crust. But it is reasonable to assume that the high model C temperatures for the lower crust in the Danish Embayment have to be reduced due to partial melting. From a petrological point of view temperatures above 850 °C in the lower crust are not very likely in this area. With the model A temperatures and a water content of only 0.2% the pyrolite solidus is reached at a depth of about 70–80 km in the Danish Embayment. In an upper mantle zone of partial melting the temperature curve is supposed to follow the melting curve. When extrapolating in particular the model A and C curves for the Embayment to greater depths not only melting problems have to be considered. Above about 1,200 °C a large thermal conductivity increase due to radiation is expected. Such an increase will also make the temperature curves in the two regions converge more easily.

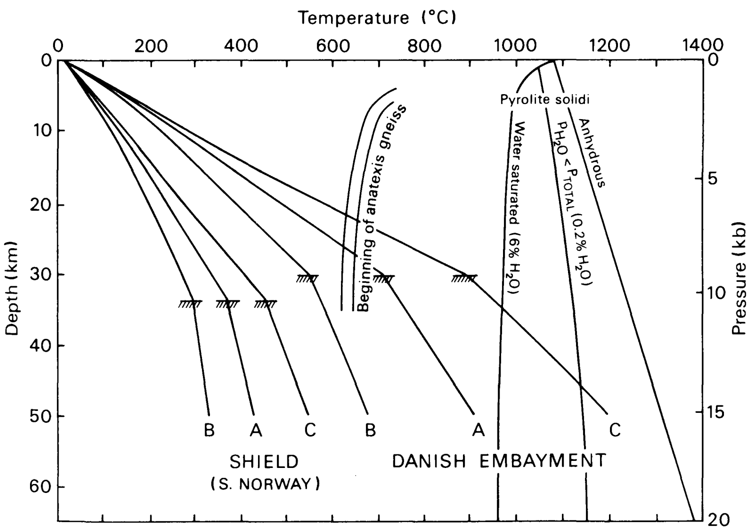

Fig. 7. The average temperature-depth functions of Table 2 in relation to the pyrolite solidi from Green (1973) and the zone of beginning of anatexis from Winkler (1974). The position of the Moho is indicated ($\overline{TTTT}$)

Using the division of metamorphic grade of Winker (1974), the temperature models indicate that the lower crust in the shield region is to be found in the *P*, *T* field of very low grade to low grade, while for the Danish Embayment medium to high grade is most likely. Here the preferred temperatures show high grade conditions at the base of the crust. As the lower crust in the shield area has previously had considerably higher temperatures than those to be expected at present, medium and high grade metamorphic rocks are also likely in this region.

In discussing the mineralogical and petrological nature of the crust the investigations on the stability of dry basaltic (gabbroic) rocks are of importance (e.g. Green and Ringwood, 1967, 1972; T.H. Green, 1967; Ito and Kennedy, 1970, 1971). There seems to be a close agreement between the authors as to the experimental data. The experiments which are carried out around 1,100–1,250 °C and with pressures up to 40 kb establish a general pattern of mineralogical variation with increasing pressure from low-pressure pyroxene + plagioclase ± olivine assemblages through pyroxene + plagioclase + garnet ± quartz to plagioclase-free assemblages dominated by garnet + clino-pyroxene. Gabbro transforms under dry conditions via a garnet granulite transition zone with varying width into eclogite. In the present case the main interest is concentrated on the extrapolation of these transitions to lower pressures and temperatures. Here there is some significant disagreement (Green and Ringwood, 1972; Kennedy and Ito, 1972), especially in the extrapolation of the plagioclase-out line. The temperature-depth curves of the lower crust in the Danish Embayment are most likely to be found in the garnet granulite stability field. If the statement of Green and Ringwood (1972, p. 277) "eclogite is the stable mineralogy for dry basaltic rocks along normal geothermal gradients in the continental crust (stable or shield region)" is true, this applies to South Norway. Kennedy and Ito (1972) find garnet granulite to be the stable mineralogy in such regions. This disagreement may be of minor importance in the present discussion. Due to their high density (about $3.5 \cdot 10^3$ kg m^{-3}) eclogites cannot be present in large amounts in the crust, and are probably not formed in any significant amount in the continental crust of normal geothermal gradients. This may be due to the presence of some water content. Therefore, it is assumed that rocks of basaltic composition are present as amphibolites also in the main parts of the lower crust in South Scandinavia. If only a very small water content should be present in the lower crust of the Danish Embayment region, garnet granulite is likely to be the stable composition of basaltic rocks. Only under such conditions, and at temperatures higher than those in the calculated models could gabbro have been formed. Before a more detailed discussion of these problems is considered, more information on seismic velocity and the density distribution is necessary. However, in general it is not correct to classify the lower crust "gabbroic" or "basaltic". Attention should be drawn to the high density amphibolites and/or pyroxene + garnet + plagioclase assemblages.

Most petrophysical properties are temperature dependent. This applies to density, seismic velocity, and especially electric conductivity. There may be conductivity differences of one or two orders of magnitude in the lower crust and upper mantle along the investigated profile, in which case magnetotelluric investigations could give essential information about the geothermal conditions.

A mapping of the depth to the Curie temperature surface, which is of great interest, could be obtained by a magnetic survey.

In great parts of the North Sea area and in particular in areas with thick sedimentary formations the surface heat flow is estimated to be around 60–70 mW m^{-2} or even somewhat higher. Decreasing values are found in the direction of the shield area and along basement highs as e.g. the Ringkøbing-Fyn high (Fig. 1) (Evans and Coleman, 1974; Madsen, 1975). The same trend has been observed in the southeastern part of the Danish-Polish Trough of which the Danish Embayment constitutes the northwestern part. Here the heat flow is decreasing from the basin area towards the Precambrian Russian Platform (Majorowicz, 1973; Wesierska, 1973).

Deviations from these trends have been found in North West Germany, where low values (47 and 52 mW m^{-2}) are measured at localities in an area with thick sedimentary rocks (Haenel, 1971).

Heat flow values close to those of South Norway have been found also in Finland (Puranen et al., 1968) and in Sweden (Parasnis, 1975). With minor modifications the geothermal models here presented may be valid for the whole transition region between the Fennoscandian Shield and the North Sea Basin.

Considerable temperature variations in the crust and upper mantle and variations in the heat flow from the mantle associated with narrow heat flow transition zones have been found in several cases. The Sierra Nevada-Basin and Range transition in the western United States (Roy et al., 1968, 1972) have been investigated in greater details. Recently Čermák (1975) has investigated in details transitions in Czechoslovakia and adjacent areas. The geothermal models of the Kapuskasing area in the Canadian Shield (Čermák and Jessop, 1971) with temperatures around 400 °C at the base of the crust and a heat flow from the mantle of about 20 mW m^{-2} are close to those of the shield region in South Norway. The Danish Embayment geothermal models show temperatures similar to those calculated for the Alps and the foreland (Buntebarth, 1973).

8. Conclusion

The geothermal model calculations show that the heat flow transition between the Fennoscandian Shield and the North Sea Basin is associated with considerable lateral variation in temperature and in heat flow from the mantle. At the base of the crust variations from 300–450 °C in the models of South Norway to 700 ± 150 °C in the Danish Embayment models have been found. The preferred model shows heat flow from the mantle of about 40 mW m^{-2} for the Danish Embayment and 16–17 mW m^{-2} in the shield area.

Considerable physical differences in the lower crust and upper mantle beneath the Precambrian shield region and the basin region produced by lateral temperature variations must be expected. Magnetotelluric investigations, more heat flow data from the basin area, more heat generation data from common shield rocks and magnetic investigations would increase the knowledge of the geothermal nature of the crust and upper mantle along the transition zone. Crustal and upper mantle physical and geological investigations in the area of South Scandinavia are of great general interest and would contribute to a better understanding of the composition and formation of the continental crust and upper mantle.

Acknowledgements. The present work was in part carried out at the Institut für Geophysik der Technischen Universität Clausthal. The author wishes to thank Professor O. Rosenbach, Dr. G. Buntebarth, Clausthal, and Professor S. Saxov, Århus, for their cooperation in arranging the stay at the German Institute, and for fruitful discussions.

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Received April 6, 1976; Revised Version July 22, 1976