J Geophys (1984) 55:76–84

Journal of Geophysics

Magnetospheric hydromagnetic waves: their eigenperiods, amplitudes and phase variations; a tutorial introduction

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Abstract. The detection of hydromagnetic waves on spacecraft and at the Earth's surface indicate disturbances of the geomagnetosphere. These disturbances are diagnostic of processes and boundaries occurring within the plasma of the Earth's space environment. In order to delineate the processes and boundaries a dense network of measuring points is desirable. Parameters that have proved useful are the frequency and the relative amplitudes and phases of the waves at different positions in space. This paper summarises some of the characteristics of hydromagnetic waves in the magnetosphere and the modification of the signal brought about by the boundary conditions imposed by the ionosphere-neutral atmosphere-Earth system.

Key words: Geomagnetic pulsations – Ground-satellite correlations – Magnetosphere – Eigenperiods – Hydromagnetic waves

Introduction

The Sixth Workshop on IMS observations in Northern Europe was a meeting of some 60 scientists who discussed two main topics during the week:

1) Reactions of the Ionosphere and Neutral Atmosphere to Magnetospheric Activity, and

2) Geomagnetic pulsations: correlated observations from satellites and the ground.

The aim of the material in this paper was to present useful background theory on hydromagnetic waves as a setting for the detailed ground-satellite studies in four particular intervals that followed. A tutorial approach is adopted where typical magnetospheric eigenperiods for different wave modes are given. Forced oscillations of the magnetosphere are considered, leading to the possibility of reasonances and phase variations of pulsations at different positions in space. The importance of the ionospheric conductivities is illustrated by discussing situations ranging from superconducting ionospheres to non-uniform ones. In this short paper the author is aware of the omission of many important papers which could only be included in a very much longer account. Recently, several good reviews on different aspects of geomagnetic pulsations have been published (e.g. Southwood, 1978; Nishida, 1978; Lanzerotti and Southwood, 1979; Kokubun, 1980; Green, 1981; Hughes, 1982; Green, 1982; Hughes 1983; Southwood and Hughes, 1983) and the interested reader is referred to these for a more complete picture.

The vast majority of geomagnetic pulsation studies to date have been concerned exclusively with either data recorded on the ground or with measurements from spacecraft. The aim in this short paper is to outline some of the features of hydromagnetic waves which may be useful in making ground-magnetosphere correlations. In particular, we consider characteristic periods, amplitudes and phase variations for waves with periods in the range 10–1,000 s, i.e. those with wavelengths comparable to the lengths of geomagnetic flux tubes.

Sources of hydromagnetic wave energy that may excite such long-period waves exist both inside and outside the magnetosphere. External sources directly related to the solar wind include, for example, waves generated at the magnetopause by the Kelvin-Helmholtz instability (Southwood, 1968; Pu and Kivelson, 1983); flux-transfer events, which occur at the magnetopause when the interplanetary magnetic field has a southward component (Russell and Elphic, 1979); and waves generated at the bow shock (Greenstadt et al., 1980). Energetic trapped charged particles within the magnetosphere may interact with, and amplify, hydromagnetic waves through the bounce resonance instability (e.g. Dungey, 1965; Southwood et al., 1969; Hughes et al., 1978) or the drift instability (e.g. Hasegawa, 1969; Lanzerotti and Hasegawa, 1975; Walker et al. 1982).

In reviewing hydromagnetic wave theory, Dungey (1967) pointed out that for a plasma permeated by a uniform ambient magnetic field, B_0, two modes are important. Hydromagnetic wave energy in the fast mode propagates approximately isotropically, whereas in the guided or Alfvén mode the energy is directed along B_0. Southwood (1974) and Chen and Hasegawa (1974a) considered fast-mode waves within the magnetosphere coupling to the guided mode and building up standing-wave resonances on geomagnetic field lines. The amplitude peak at resonance and the polarization reversal, predicted by the theory on a latitudinal traverse of the resonance, agreed with observations from high-latitude magnetometer arrays (e.g. Samson et al., 1971).

A standing-wave resonance at a particular location implies that an integral number of half wavelengths of the hydromagnetic wave fit into the magnetic flux tube linking the ionospheres in the northern and southern hemispheres. The eigenperiods of such standing waves depend on the ambient magnetic field strength, the plasma density along the flux tube and the mode of the excited wave. Thus, in general, the eigenperiods associated with guided standing waves vary with the position of the flux tube that is supporting the wave. The position of the flux tube in space can be conveniently defined by the latitude of the foot of the field line in the E region of the ionosphere or by the L value of the field line (McIlwain, 1961).

Eigenperiods

In this section some estimates of different eigenperiod calculations are reviewed and typical values for some important modes for different plasma regimes in the magnetosphere (plasmasphere, plasmatrough and detached plasma) are presented.

The simplest estimate that can be made of the period of a hydromagnetic standing oscillation on geomagnetic field lines is given by the time of flight approximation:

$$\text{Period} = 2\int \frac{dl}{V_A},$$

where the integration is performed along the field line, and V_A is the Alfvén velocity (Obayashi and Jacobs, 1958). In 1954, Dungey considered a non-uniform magnetic field (dipole) and plasma and derived the coupled differential equations, relating the wave electric field to the plasma velocity. These equations have not been solved, but with certain simplifying assumptions two particular guided modes have yielded numerical solutions (Radoski, 1967a, b; Cummings et al., 1969; Orr and Matthew, 1971). For a hydromagnetic wave varying as $e^{il\Phi}$ where Φ is the longitude, the two special cases are:

i) The axially symmetric case or toroidal mode where the azimuthal motion of the plasma, V_Φ, at a particular latitude and all longitudes ($l=0$), results in a twist of the geomagnetic field lines which are assumed to be anchored in the ionospheres. This torsional motion gives an east-west wave magnetic field b_Φ in the magnetosphere with an associated electric field E_ν directed along the principal normal to the geomagnetic field shells. In this mode the wave energy $\mathbf{E}_\nu \times \mathbf{b}_\Phi$ is guided along the geomagnetic field lines.

ii) The guided-poloidal mode may arise for a hydromagnetic disturbance which is localized in longitude (the wave is made up of modes with large l) where the magnetic disturbance b_ν is mainly in the meridian plane, and the wave electric field is E_Φ.

These three estimates (time of flight, toroidal and guided poloidal) of characteristic periods are, in general, different from each other and will vary with L. Table 1 summarises some calculated periods applicable to $L=6.6$ for a dipole magnetic field. The magnetospheric plasma is assumed to be composed of protons and electrons and the number density distribution follows a power law

$$n = n_0 \left(\frac{r_0}{r}\right)^m$$

where n_0 and n are the proton number densities at r_0 and r respectively; r_0 is the equatorial geocentric distance $= 6.6$ earth radii and n_0 is 1 cm^{-3}.

It is interesting to note that for the first harmonic the time of flight approximation gives values for the period which are some 21% and 17% lower than toroidal mode for $m=3$ and 4 respectively, while for $m=6$ the periods are identical (Radoski, 1966). For these three cases ($m=3$, 4, 6) the first-harmonic guided poloidal periods are about 37% higher than the symmetric toroidal values. The predicted values for the second-harmonic periods for the three different wave modes in Table 1 for a particular plasma distribution are in close agreement with each other.

Table 1. Eigenperiods in seconds for $L=6.6$ and $n_0 = 1/\text{cm}^3$

	First harmonic			Second harmonic		
Density index, m	3	4	6	3	4	6
Time of flight	45[a]	49[a]	65	23[a]	25[a]	33
Toroidal mode ($l=0$)	57	59	65	24	26	33
Guided poloidal mode	78	81	89	24	26	33

References [a] Warner & Orr (1979
All other values from Cummings et al. (1969)

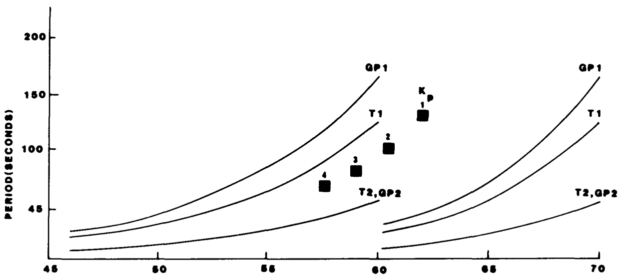

Fig. 1. Some typical eigenperiods for standing waves in the latitude range 45°–70° with the plasmapause located at 60° ($L=4$). *GP1* and *T1* are the fundamental guided poloidal and toroidal eigenperiods; *GP2* and *T2* are the second-harmonic guided poloidal and toroidal eigenperiods. The *squares* represent the estimated periods of surface waves at the plasmapause for four different positions of the plasmapause corresponding to different levels of magnetic activity

Figure 1 shows a schematic variation of the way eigenperiods for five different wave modes change with latitude. The plasmapause position is at $L=4$ and typical average plasma densities for the plasmasphere and plasmatrough have been assumed. The guided poloidal fundamental (or first-harmonic) eigenperiods, *GP1*, are seen to be longer than the corresponding first-harmonic toroidal period T_1. The second-harmonic periods, *GP2* and *T2*, of the guided poloidal and toroidal modes are almost identical. Figure 1 also shows estimates of the surface-wave period that may on occasions be excited at the plasmapause. The surface-wave periods are indicated by four squares which represent four different plasmapause positions corresponding to different levels of magnetic activity (K_P equals 1, 2, 3 and 4). The theory of surface waves generated at a plasma gradient by a broadband source has been formulated by Chen and Hasegawa (1974b); Orr (1983) evaluated the periods shown in Fig. 1 using average plasmaspheric charged-particle densities given by Park et al., (1978). The latitude-dependent dominant periods detected at seven European observatories in the range $2 < L < 4$ (Stuart and Usher, 1966; Voelker, 1965) agree well with the fundamental toroidal-mode calculations (Orr and Matthew, 1971) for the plsmaspheric plasma densities indicated by Angerami and Carpenter (1966), which has been used in the preparation of Fig. 1.

Figure 2 summarises some results for high-latitude geomagnetic field lines which differ significantly from a dipole geometry; it includes estimates of the eigenperiods from time of flight calculations (Warner and Orr, 1979) for differ-

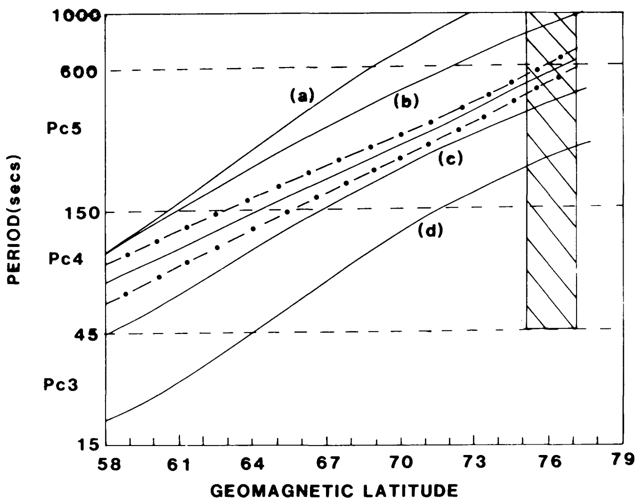

Fig. 2. The time of flight period calculations under different plasma conditions (Warner and Orr, 1979) compared with the observations of Samson and Rostoker (1972). The different plasma conditions are represented by (a) the extended plasmasphere, (b) detached plasma – upper limit, (c) detached plasma – lower limit, (d) plasmatrough. The *barred region* shows the experimentally observed continuum of periods seen above 75°. The latitude-dependent observations, within the *dash-dot lines*, fall within the limits for detached plasma

ent plasma density distributions appropriate to (i) extremely quiet magnetic conditions when the plasmasphere fills out to high L-shells, (ii) occasions when enhanced detached plasma density exists beyond the plasmapause and (iii) typical plasmatrough densities.

The latitudes at which Pc 4 and Pc 5 pulsations showed a maximum amplitude over the Alberta chain of magnetometers was noted for a large number of events (Samson and Rostoker, 1972). A linear regression to these data is shown in Fig. 2 with a pair of dash-dot lines indicating the 95% confidence limits. The observed periods are consistent with the time of flight calculations for enhanced plasma densities in the plasmatrough. These enhanced densities may arise either from regions of plasma detached from the plasmasphere (Chappell, 1974) where the ionic component will be mainly protons, or from the presence of heavier ions.

At sychronous orbit ATS-6 often detects several spectral peaks simultaneously in the Pc 3–4 frequency range (Takahashi and McPherron, 1982). On occasions these peaks are harmonically related and are most clearly seen in the east-west magnetic field component, b_Φ. The fundamental mode seems to be absent in the dynamic spectra (the anticipated amplitude is low for the near-equatorial ATS-6 position) but harmonics as high as the tenth have been measured in the dayside pulsations. The results are consistent with the standing Alfvén wave model

The possibility of magnetospheric flux tubes supporting standing Alfvén waves with a period approximately twice as long as the 'fundamental' mode was suggested by Allan and Knox (1979a). These quarter-wave modes may occur when the height-integrated Pedersen conductivity is greater than a certain critical value in one ionosphere and less than this critical value in the conjugate ionosphere. For boundary conditions equivalent to a superconducting ionosphere in one hemisphere (giving a node of electric field) and a non-conducting conjugate ionosphere (giving a node of magnetic field), Allan (1983) computed eigenvalues from the toroidal and guided poloidal wave equations for L in the range 2–8 and gives some illustrative quarter-wave periods for plasmaspheric resonances. Stuart and Lanzerotti (1982) observed a wave packet which lasted for over 1 h with a wave period of approximately 6 min at the conjugate stations St. Anthony and Halley ($L \sim 4.2$). The ionosphere in the northern hemisphere was under local night conditions, whereas the Halley ionosphere was unlit. A polarization reversal was identified on the Bell Laboratories network which, together with whistler data, points to a plasmaspheric resonance but with a wave period that is too long for the estimated plasma density if it is a half-wave resonance. This event is a good candidate for a quarter-wave resonance (Allan, 1983).

Forced oscillations of the magnetosphere

We consider now the response of the magnetosphere to some source which generates fast-mode waves with a magnetic field component $b_z \sin(\omega_f t)$. It is assumed that the magnetospheric plasma density distribution is such that the guided mode eigenperiods have similar features to those drawn in Fig. 1. Orr and Hanson (1981) and Gough and Orr (1984) have used such a model to investigate the variation in amplitude and phase of geomagnetic pulsation signals at ground observatories from mid- to high latitudes. They assumed that each individual magnetospheric flux tube responds independently to the driving force according to the differential equation describing forced damped simple harmonic motion:

$$\ddot{b}_\Phi + 2\gamma\, \dot{b}_\Phi + \omega_n^2\, b_\Phi = \omega_n^2\, b_z\, c \sin(\omega_f t) \qquad (1)$$

where b_Φ is the transverse-mode magnetic wave field. The fast-mode wave, b_z will propagate through the magnetosphere coupling to, and driving, adjacent flux tubes at the forcing frequency ω_f. The coupling constant c is assumed to be the same for each separate harmonic oscillator. The general solution of Eq. (1) consists of two parts; the complementary function or transient solution and the particular integral or steady state solution (Orr and Hanson, 1981). Thus the individual flux tubes will oscillate with a transverse mode, b_Φ, steady-rate response at the driving frequency ω_f; combined with a transient transverse-mode response at a frequency close to the natural eigenfrequency, ω_n, of the geomagnetic flux tube.

For a source field with forcing frequency ω_f as shown in Fig. 3a we see that there are three latitudes at which $\omega_f = \omega_n$ and resonant enhancement of the magnetic field line oscillations may occur; namely, latitudes A, B and C corresponding to resonances in the plasmasphere, at the plasmapause and in the plasmatrough respectively. The idealised 180° phase change predicted in the magnetospheric magnetic wave component, b_Φ, across each resonance is shown in Fig. 3b. The phase changes are further illustrated in Fig. 4: solutions of Eq. (1) are given where a 20 mHz sinusoidal forcing term is applied to plasma-loaded flux tubes whose natural frequencies are 30, 20 and 10 mHz, i.e. they are above, at and below the driving frequency. This damping factor γ/ω_n has been taken as 0.1 which is typical of day-time conditions (Gough and Orr, 1984). The response of the different flux tubes shows the following:

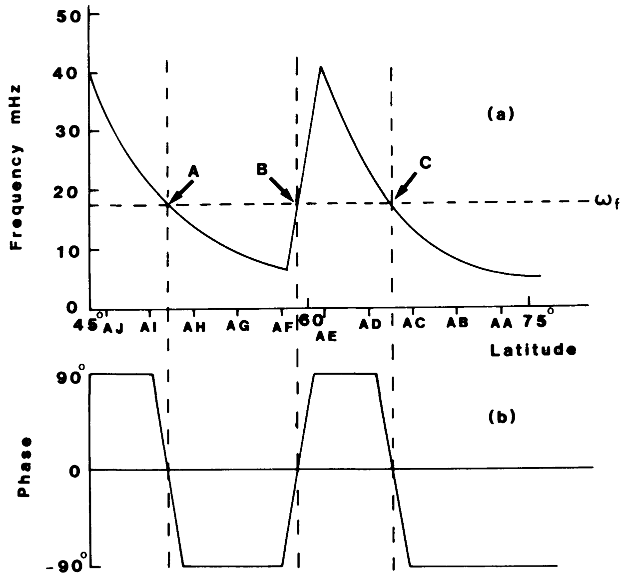

Fig. 3. **a.** A schematic representation of the variation of geomagnetic pulsation eigenfrequencies with latitude. **b**. The idealized predicted variation of phase with latitude for a forcing wave corresponding to ω_f in **a** when steady state conditions have been achieved. (Orr and Hanson, 1981). The ten equispaced station locations AA to AJ are used in the model studies presented in Figs. 11 and 12

a) At the beginning of an event, when the transient term is important, the phase change between the three respresentative flux tubes is not large.

b) Later, when the transient has died away and steady-state conditions obtain it is seen that for the three cases given, when

(i) $\omega_f > \omega_n$ ($=10$ mHz), the driving term is in anti-phase with the response,

(ii) $\omega_f = \omega_n$ ($=20$ mHz), the phase difference is $\pi/2$, and

(iii) $\omega_f < \omega_n$ ($=30$ mHz), the driving term is in phase with the response.

Thus, across a resonance, a phase difference of π is established; the phase decreasing with increasing latitude across plasmaspheric and plasmatrough resonances, and being in the opposite sense across a resonance at the plasmapause (Waldock et al. 1983).

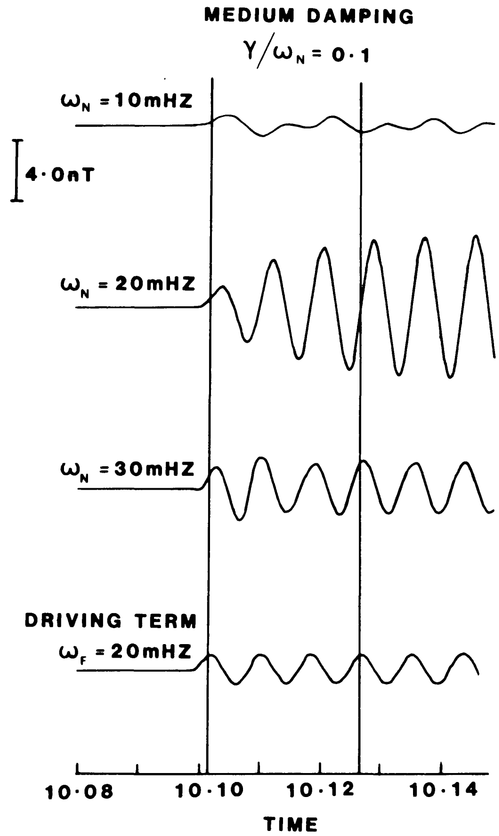

Fig. 4. Solutions of Eq. (1), see text, for a damping factor $\gamma/\omega_n = 0.1$ and a source driving frequency of 20 mHz. The *top three panels* show the signals associated with toroidal *L*-shells with eigenfrequencies of 10, 20 and 30 mHz

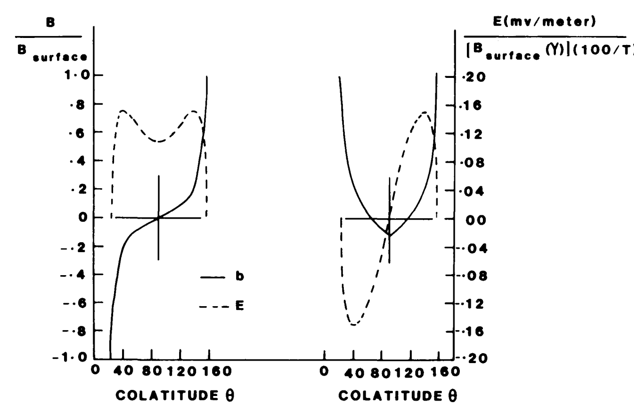

Fig. 5. Solution of the toroidal equation for the wave electric field (*dotted line*) and the wave magnetic field (*solid line*). The *left hand panel* is for the fundamental mode and the *right hand panel* is the second harmonic (Cummings et al., 1969) with $m = 3$

Standing wave magnetic and electric fields in the magnetosphere with a superconducting ionosphere

In this section we summarise some of the properties of ULF standing waves in the magnetosphere with the assumption that the E region of the ionosphere is a perfect conductor. A qualitative description of many of the characteristics of such waves can be obtained from the elastic string model of Sugiura and Wilson (1964). Cummings et al. (1969) calculated eigenperiods for toroidal and guided poloidal modes and also the amplitudes of the magnetic and electric wave vectors along the dipolar magnetic field lines. Examples of solutions of the toroidal wave equation for the fundamental and second harmonic with $m = 3$ are given in Fig. 5. The boundary conditions are such that the electric field is zero at the ionosphere and in the case of the second (and all even) harmonics an electric field sensor will detect zero field on a satellite at the geomagnetic equator. Similarly, a magnetometer at the geomagnetic equator will be at a node in the magnetic wave field for the fundamental and all odd harmonics; in this case, to detect the standing wave field an electric field sensor is required.

The electric and magnetic wave fields are everywhere in phase quadrature in this model of a standing wave. When a spacecraft is displaced from the geomagnetic equator and it is possible to meaure (or infer) both the electric and magnetic field oscillations, then Cummings et al. (1978) and Singer et al. (1982) have shown that from a consideration of the phase differences between the electric and magnetic

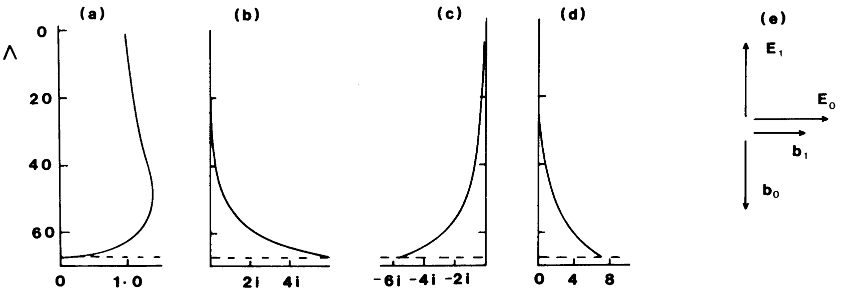

Fig. 6a–e. The zero-order and first-order electric and magnetic fields for the fundamental as a function of geomagnetic latitude, Λ, on the field line $L=6.71$ for $m=3$ (Walker, 1980). The scaled field quantities are plotted horizontally and are respectively **a** E_0; **b** $10^3\ \sigma_p\ n_{eq}^{-1/2}\ E_1$; **c** $10^{-3}\ \mu^{-1}\ n_{eq}^{1/2}\ b_0$; **d** $10^7\ \mu^{-1}\ \sigma_p\ b_1$ and **e** indicates the relative phases between the wave components

fields it is possible to distinguish between odd and even harmonics. In the fundamental mode the magnetic field leads the electric field by 90° when the spacecraft is north of the geomagnetic equator. For the same spacecraft location a second-harmonic standing wave would give the magnetic field lagging the electric field by 90°.

Standing wave magnetic and electric fields in the magnetosphere and on the ground for the case of an ionosphere with finite electrical conductivity

The model outlined in the previous section has been useful in the estimation of the eigenperiods of standing waves that can be supported within the magnetosphere, and also in the prediction of the variation of electric and magnetic field strengths along the geomagnetic field lines near the equatorial plane. However, the model is seriously in error in the ionosphere and on the ground. For a superconducting ionosphere the electric field would always be zero there, in contradiction to the electric fields of up to 50 mV m^{-1} observed in the E region by high-latitude radars [for example, Greenwald et al. (1978) and Walker et al. (1979)]. In addition, a superconducting ionosphere would shield all magnetic field variations from ground magnetometers.

Newton et al. (1978), Allan and Knox (1979a) and Walker (1980) have considered the effect of losses in the ionosphere brought about by finite ionospheric conductivities.

Figure 6a and c shows the zero order electric and magnetic fields for the fundamental mode as a function of geomagnetic latitude; this is the variation depicted in Fig. 5 for infinite ionospheric conductivity. Walker (1980) showed the effect of finite ionospheric conductivity explicitly, Fig. 6b and d which give the first- order electric and magnetic fields, respectively. These first-order fields have been evaluated assuming that the damping is light. Complex eigenvalues, $k=k_0+ik_1$, have been computed from the axisymmetric toroidal equation where $k_1 \ll k_0$ and the solutions have been expanded to first order in k_1/k_0. Figure 6e emphasizes the phase relationships between the fields just above the ionosphere.

For typical parameters, b_1 is everywhere small compared with b_0 and E_1 may have significant values in and above the ionosphere but decreasing to zero well before the equatorial plane is reached. The first-order fields, E_1, b_1, which have arisen as a consequence of finite ionospheric Pedersen conductivity, when combined with the zero-order fields give a Poynting flux of value $(1/\mu_0)\ (E_0 b_1 - E_1 b_0)$ which heats the ionosphere.

The magnetic field detected at the ground is determined by the Hall conductivity.

Hughes and Southwood (1976a, b) have undertaken de-

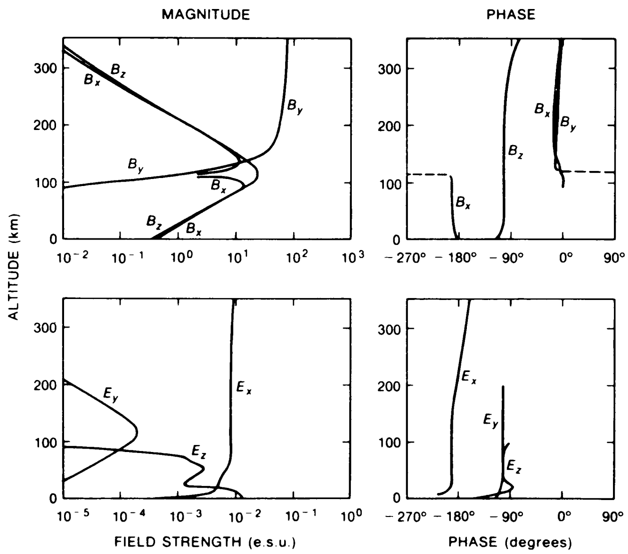

Fig. 7. The variation of magnetic and electric fields in the ionosphere and atmosphere due to incident transverse wave for $\omega=0.1\ s^{-1}$, $k_x={}^1/25\ km^{-1}$, $k_y=0\ km^{-1}$: a model calculation for daytime sunspot maximum conditions (Hughes and Southwood, 1976a)

tailed numerical solutions where the wave fields have a horizontal spatial and time variation of the form $\exp(ik_x x+ik_y y+i\omega t)$ and a complex conductivity tensor, representing the Hall, Pedersen and direct conductivities, has been used in the altitude range from the ground to 2,000 km. Figure 7 is one example showing the variation in amplitude and phase of the six wave components with altitude. Modelling of this type allows the magnetospheric wave fields to be mapped through the ionosphere to ground level.

In this example E_x and b_y are the most important components in the magnetosphere. The action of the E_x wave field on the anisotropic ionosphere is to cause a Pedersen current J_x to flow, which produces a local b_y which shields the magnetospheric signal from the ground. However, E_x, in general, also generates a Hall current, J_y in the ionosphere which gives a horizontal component b_x which may be detected by a magnetometer at the Earth's surface.

Allan and Knox (1979b) and Allan (1982) have considered the variation of amplitude and phase of the electric and magnetic fields (E_v and b_Φ) along a geomagnetic field line for a number of different boundary conditions at the ionospheres. Figure 8 is an example where the height-integrated Pedersen conductivity in the northern ionosphere, a_p^n, is 10 *S* and in the southern ionosphere, a_p^s, is 3 S. In this case, the wave depicted in Fig. 8 retains some of the classical standing-wave characteristics of the half-wave fun-

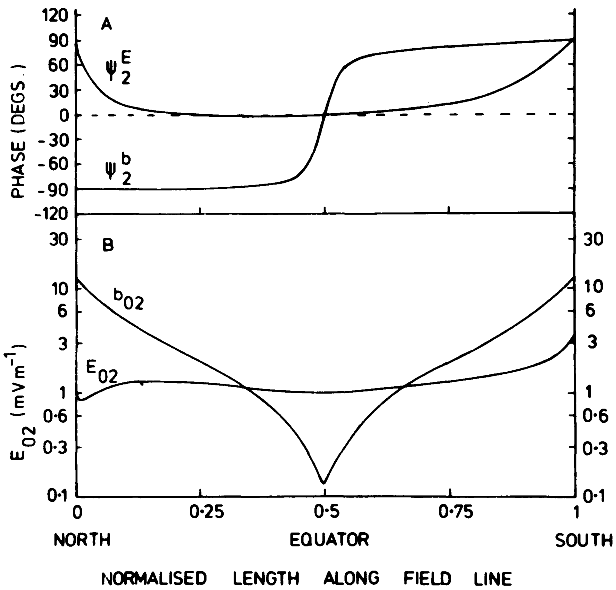

Fig. 8. Phases (*A*) and magnitudes (*B*) of b_Φ and E_v at $L=4$ for the half-wave fundamental with the height-integrated Pedersen conductivities equal to 10 S and 3 S in the northern and southern ionospheres respectively, at $t=0$ (Allan and Knox, 1979b)

damental shown in Fig. 5 where the ionospheres are superconductors. For example, between the conjugate ionospheres the magnetic field has a phase difference of 180°, whereas the electric field is approximately in phase. Over the central portion of the geomagnetic field line the phase differences between E_v and b_Φ are approximately $\pm 90°$, which is indicative of the domination of the standing-wave solution in this region. Important differences are as follows:

i) The electric field is finite at both ionospheres, being larger where the ionospheric conductivity is smaller.

ii) b_Φ does not have a node at the equatorial plane, but there is a nearby minimum.

iii) The phase of b_Φ changes continuously through the equatorial plane.

iv) As the ionospheres are approached, the phase differences between E_v and b_Φ depart more and more from quadrature and tend towards 0° and 180°. The field components in these regions have a dominant travelling-wave character and evaluation of the Poynting vector shows that electromagnetic energy is being directed into both ionospheres (see also Fig. 6e).

Green and Hamilton (1981) have studied Pi 2 pulsations recorded over a 21-month period at conjugate observatories St. Anthony and Halley; for these stations at $L \approx 4.2$ it was expected that an odd-mode relationship with $\sim 0°$ phase difference for the *H* components and $\sim 180°$ for the *D* components would hold. These relationships were well satisfied when both ionospheres were under night conditions. However, for asymmetric ionospheric conductivities there was a deterioration in the similarity between conjugate signals compared with the symmetric cases, probably due to different damping in the north and south.

Green and Hamilton concluded that for the occasions when the ionospheres were asymmetric there was no statistical evidence for the excitation of quarter-wave resonances.

In the actual ionosphere (unlike the idealised cases discussed above) there are likely to be significant conductivity gradients within the same hemisphere; for example, the contrast between the day-night boundaries in the ionosphere and also the high-conductivity auroral arc regions compared with adjacent poorly conducting regions.

Ellis and Southwood (1983) have considered the reflection of Alfvén waves incident on ionospheres with discontinuities in the Hall and Pedersen conductivities. They show the importance of the orientation of the wave electric field vector with respect to the boundary defining the conductivity contrast, and also that field-aligned currents can be set up above the conductivity contrast. These field-aligned current sheets act as sources for subsidiary surface waves.

The theory of non-uniform ionospheres and their response to hydromagnetic waves has been extended by Glaßmeier (1984); he has undertaken model calculations for particular wave electric field and ionospheric conductivity distributions and demonstrated that conductivity gradients can change the 90° rotation (predicted for uniform ionospheres) between the magnetic field below and above the ionosphere. Also, double-peaked total ionospheric electric field distributions can occur, generated by a single peak incident electric field distribution.

Ground-satellite correlations

Correlations have been sought between hydromagnetic waves in the magnetosphere and geomagnetic pulsations measured at observatories on the earth for some 20 years (Patel and Cahill, 1964). Several of the events recorded remain a challenge, requiring new theoretical work for their interpretation; for example, the pure compressional wave detected on ATS-1 which appears as a dominantly transverse wave on the ground (Lanzerotti and Tartaglia, 1972) and the giant pulsation event recorded on the ATS-6 magnetometer and particle experiments and on a ground network of magnetometers (Hillebrand et al. 1982).

The Dodge satellite at 6.25 earth radii established the 20–25 mHz band as an important one for wave activity in the magnetosphere and found similar spectral peaks from ground magnetometers close to the satellite meridian in the *L* range 6.9 to 1.5 (Patel et al., 1979). Kokubun (1980) has provided a valuable review of satellite-ground correlation work up to 1980 and Hughes (1983) has extended this with a review of highlights from over 300 listed papers published between 1978 and 1982.

In the remainder of this section we look at an example of hydromagnetic waves detected on the geostationary satellite ATS-6 (Gough et al., 1983) and the magnetic wave signature recorded at ten ground observatories ranging in latitude from 43° to 67°. Figure 9 shows the magnetic field variation in three components at ATS-6 between 0800 and 1200 UT on 29 July 1975, together with the ground station KEV in northern Scandinavia which is close to the intersection of the ATS-6 geomagnetic field line with the Earth's surface. The event, which is recorded at ATS-6 in the time interval 0930 to 0950 UT with dominant period of approximately 55 s, was observed clearly at the wide range of stations mentioned above.

In the following 2 h the waves detected on the ground and at the spacecraft were notable for the significant differences they displayed. On the spacecraft large amplitude transverse waves with a dominent period of about 120 s were measured. These 120-s waves were not detected on the ground. This may well be an example of a hydromagnetic wave in space which is highly localized in the direc-

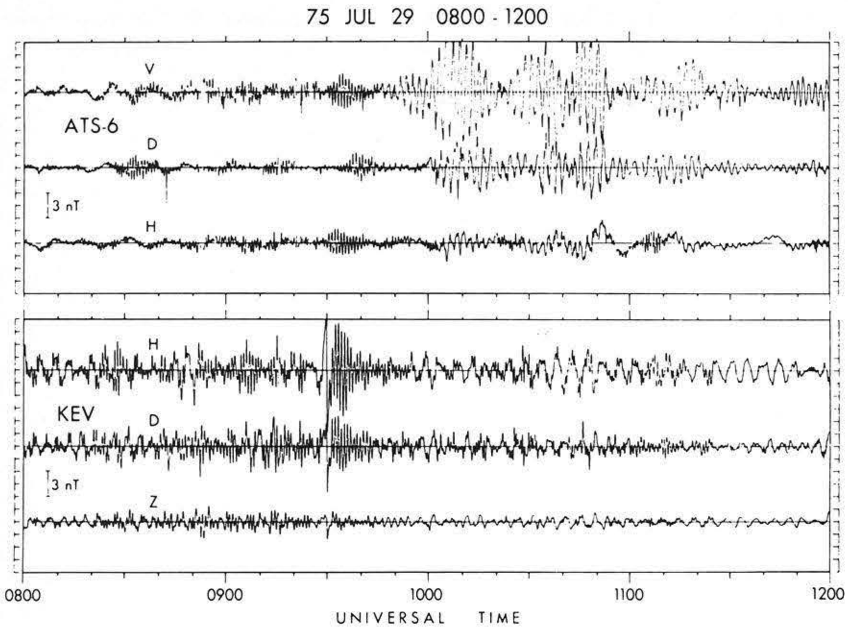

Fig. 9. Three components of magnetic field variations from ATS-6 during the interval 0800–1200 UT on 29 July 1975 with simultaneous records from KEV in northern Scandinavia which is close to the intersection of the geomagnetic field line through ATS-6 and the Earth's surface (Gough et al. 1983)

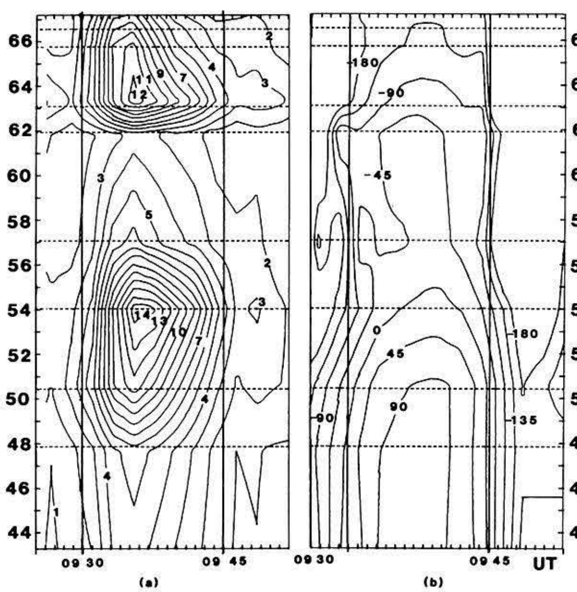

Fig. 10. The variation of H component **a** amplitude and **b** phase for the Pc 4 event detected at ten ground observatories from 43°–67° in latitude in the time interval 0930–0950 UT in Fig. 9 (Gough et al., 1983)

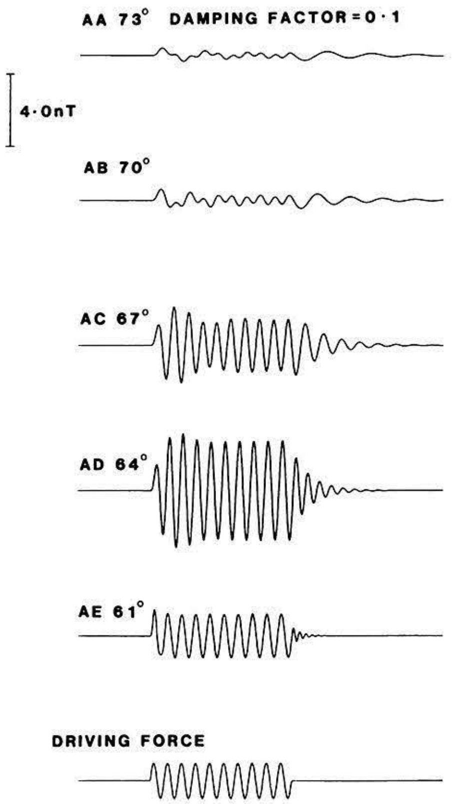

Fig. 11. The modelled pulsations from the five stations in the array associated with the plasmatrough (see Fig. 3) with a damping factor of 0.1 (Gough and Orr, 1984)

tions transverse to the ambient geomagnetic field; Hughes and Southwood (1976a) demonstrated that the ionosphere-neutral atmosphere system strongly attenuates signals with large cross-field wave numbers (k_x, $k_y > 1/50\ \text{km}^{-1}$). Some of the high-latitude ground observatories in the interval 1000–1200 UT detected Pc 5 waves with average period of 275 s, but these longer-period waves were of negligible amplitude at the spacecraft position.

The ground signature for the Pc 4, 55-s wave period event is summarized in Fig. 10. The technique of complex demodulation (Beamish et al., 1979; Webb, 1979) has been used to evaluate the H (north-south) amplitude of the magnetic wave component at the dominant 55-s period (Fig. 10a) and the relative H phase (Fig. 10b). The results are presented in the form of contour maps in which contours of equal amplitude and phase are plotted in latitude and time.

The figures point to a double resonance within the magnetosphere. The higher-latitude resonance has an amplitude maximum centred on $63^1/_2°$ with an H phase change of

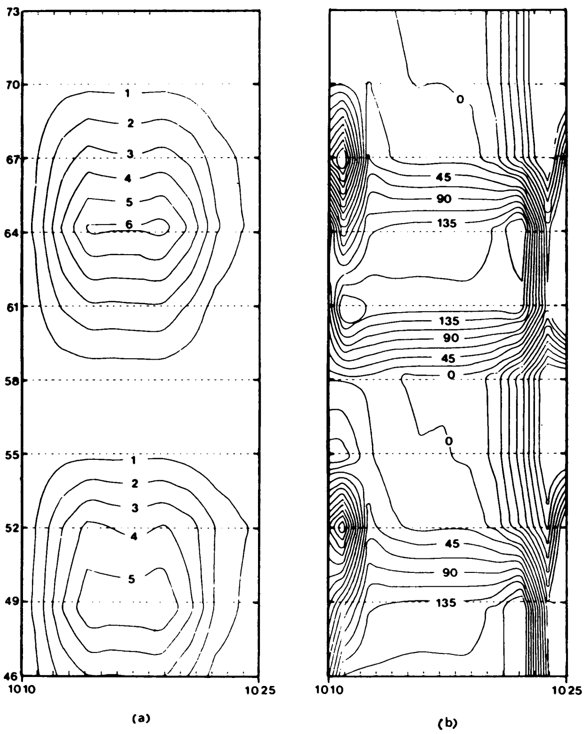

Fig. 12a. The amplitude and **b** the phase contours for the modelled pulsations shown in Fig. 11, together with the lower latitude plasmaspheric stations (Gough and Orr, 1984)

approximately 126°. The lower latitude resonance has its maximum amplitude enhancement at 54° with a phase change of 162° across it. The higher latitude resonance is likely to be associated with standing waves on flux tubes permeated with plasmatrough plasma, whereas the lower latitude resonance is within the plasmasphere.

Finally, we draw attention to some modelling studies by Gough and Orr (1984) which reproduce some of the characteristics of geomagnetic pulsations observed by arrays of ground magnetometers. The model of forced, damped oscillations has been outlined earlier in the third section. Consider a source driving frequency, ω_f, operative on the magnetosphere when the eigenfrequency variation with latitude is similar to that depicted in Fig. 3 and the damping factor γ/ω_n is 0.1. Figure 11 illustrates the ten oscillations of the driving force and the response of the five plasmatrough flux tubes AA to AE. Figure 12 shows the result of applying complex demodulation to the predicted waveforms at the ten equispaced stations, AA to AJ, from 73° to 46° in latitude. In this case, two amplitude peaks are identified on Fig. 12a corresponding to positions A and C on Fig. 3, and Fig. 12b shows large phase changes across each of the resonances and also between the stations AE and AF straddling the plasmapause.

Concluding remarks

Some characteristics of hydromagnetic waves in the magnetosphere and their signature on the ground have been mentioned. The search for ground-satellite correlations is important but not easy.

We have seen that spacecraft, even when they are located on a flux tube corresponding to a field line resonance, will detect zero electric or magnetic wave fields if the spacecraft is positioned at a node in the standing Alfvén wave. On other occasions large amplitude hydromagnetic waves, highly localized in the magnetosphere and measured on satellites, have no ground signature due to the screening of the ionosphere. A further complication for spacecraft on high-latitude geomagnetic field lines is the uncertainty in mapping the field line to ground level. Future progress will come from co-ordinated multiple satellite observations measuring both waves and charged particles, together with closely spaced networks of ground observations.

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Received November 15, 1983; Revised April 18, 1984; Accepted May 2, 1984