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Nonlinear inversion of local seismic travel times for the simultaneous determination of the 3D-velocity structure and hypocentres – application to the seismic zone Vrancea

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Abstract. A new method for solving the full nonlinear problem of simultaneous inversion for 3D structure and hypocentres (SSH method) has been developed. In comparison with the simplified linear inversion procedure of Aki and Lee (1976), the method proposed here exhibits several extensions and improvements:

a) The seismic forward problem is solved exactly by a newly developed 3D ray tracing method. This procedure has been conceived as a shooting method. The 3D heterogeneous medium is parametrized by rectangular blocks.

b) The full nonlinear inverse problem is solved. The method works iteratively and was conceived in terms of the Levenberg – Marquardt (LM) algorithm. In each iteration step the seismic forward problem is solved with the ray tracing procedure and a linear inversion is performed.

c) In order to improve the stability and uniqueness of the inverse solution, à priori information on the model space may be used in the inversion process. For example, a known crustal structure may be exploited for the retrieval of the deeper lithosphere.

The SSH method has been tested on various theoretical 1D and 3D models and its capabilities and limitations are discussed. The results demonstrate the existence of a tradeoff between hypocentral depths, origin times and seismic velocity, particularly when the horizontal dimensions of the recording network are small relative to the depths of the earthquakes.

The SSH method has been applied to real earthquake data of the Vrancea region, Romania. About 50 crustal and intermediate depth events were used in the inversion process. First, the crustal events were inverted to infer the crustal structure of the region. This crustal information was then used as an a priori constraint in the inversion of the intermediate depth events. The 3D-velocity model found for the Vrancea region reduces the RMS value of the travel-time residuals from 0.5 to 0.3 s. The most essential feature of the model is a high-velocity anomaly of about 4%–6% beneath the eastern Carpathian foredeeps, extending from about 80 to 160 km depth. This appears to be in agreement with results of teleseismic studies of other authors and may be understood in terms of the plate tectonic concept proposed for the Vrancea region. Due to lack of spatial resolution and high standard errors, the results obtained above should, nevertheless, be taken with some care and have to be substantiated by further investigations with higher quality data.

Key words: 3D-inversion – Lateral heterogeneities – Vrancea region

Introduction

The determination of lateral velocity heterogeneities of the lithosphere by inversion of seismic travel-time data has become of major importance in seismology in recent years. There are essentially two different methods of inversion used to date for the retrieval of the 3D structure of the lithosphere. One method of inversion (ACH method), which is known as the method of Aki et al. (1977), uses teleseismic arrival-time data to infer the structure under a seismic array (see e.g. Koch [1983a; b] for a numerical study and improvements of this method). The other method of inversion, which is the subject of the present paper, has been developed for the inversion of arrival-time data from local earthquakes. This procedure is, consequently, restricted to the retrieval of the 3D structure of the lithosphere in seismically active regions. In contrast to the ACH method, the hypocentres themselves are now unknown and a function of the unknown velocity structure. Thus, one is faced with the problem of simultaneous structure and hypocentre determination (SSH method).

The theoretical bases of the SSH method have been laid for a vertical inhomogeneous medium (1D medium) by Crosson (1976a, b). For a 3D seismic medium a SSH method has been formulated by Aki and Lee (1976) and has been widely applied to different seismically active regions of the world for the determination of lateral velocity heterogeneities in the upper crust (Aki and Lee, 1976; Hawley et al. 1981) or in the lithosphere down to 250-km depth (Horie and Aki, 1982; Roecker, 1982; Takanami, 1982).

Despite its numerous applications, the SSH method of Aki and Lee (1976) has some severe limitations inherent in its construction which may considerably bias the solution obtained. First of all, it is a linear inverse method and, thus, only valid in the presence of small lateral velocity perturbations. On the other hand, lateral velocity inhomogeneities up to 10% contrast have been found, particularly for seismically active subduction zones (Spencer and Gubbins, 1980; Huppert and Froehlich, 1981; Roecker, 1982). In such a case the linear approach may lead to a systematic bias in the solution. In fact, for teleseismic arrival-time data, numerical simulations by Koch (1983a, b) show systematic errors greater than 20% in the reconstruction of lateral velocity perturbations with 5%–6% contrast. These errors could be eliminated with a nonlinear iterative inversion procedure using exact 3D ray tracing. The strong nonlinearity of the SSH problem has also been pointed out by Pavlis and Booker (1983).

Another important approximation in the SSH method of Aki and Lee (1976) is the oversimplified procedure for calculating the ray paths in the different blocks of the model. In fact, Aki and Lee (1976) use a homogeneous initial model, i.e. the ray path segments for the different blocks are summed up along a straight line between source and receiver. As Aki and Lee (1976) admit, this primitive block-sampling may bias the precision of the calculated hypocentres (see also Engdahl and Lee, 1976). On the other hand, as a consequence of the strong nonlinear coupling between hypocentres and velocity structure, the latter may also be strongly biased.

In the present paper a new SSH method will be proposed. Its essential characteristics are:

a) It uses an exact 3D ray tracing program, developed especially for this purpose, in order to overcome the unreliable block-sampling and allowing for better hypocentre and velocity determination.

b) It is conceived as a fully nonlinear inversion procedure and thus tries to eliminate the systematic bias in the linear inverse solution of Aki and Lee (1976). In this sense, the method proposed here appears to be similar to the one proposed by Hawley et al. (1981) for the retrieval of crustal structure, but uses a different ray tracing which avoids a priori smoothing of the lateral velocity structure and so giving a higher spatial resolution of the model. An iterative improvement of the linear inverse solution has also been obtained by Horie and Aki (1982) and Takanami (1982) for the structure under the Japanese arc using a simplified ray tracing technique of Thurber and Ellsworth (1980).

c) It can be applied in such a way that à priori information on the velocity structure of the lithosphere can be handled conveniently to reduce the instabilities and the non-uniqueness in the solution. In fact, the use of explicit à priori information in inversion theory appears to be the most efficient method for a unique retrieval of the seismic structure of the earth (Jackson, 1979; Koch, 1983a).

The nonlinear SSH method developed here will be applied to earthquake data of the Vrancea region, Romania, to retrieve the 1D and 3D structure of this tectonically complicated zone. In fact, the Vrancea region has been put into the concept of plate tectonics (McKenzie, 1972; Fuchs et al., 1979). That is, the occurrence of seismic activity, which is confined to a rather small volume at intermediate depths of about 100–160 km, appears to be a consequence of the subduction of the continuation of the Black Sea plate under the Eurasian plate. Thus, one is led to the assumption that there must be strong lateral velocity inhomogeneities in this region. Some seismic structure investigations, using teleseismic (Vinnik and Lenartovich, 1975; Hovland and Husebye, 1982) and local (Koch, 1982) arrival-time data, essentially show positive lateral velocity anomalies in the Eastern Carpathian foredeeps. This seems to substantiate the plate tectonic hypothesis mentioned above, but further quantitative studies in this field must be continued to elucidate the, as yet, unclear picture of this region.

The present paper is organized as follows. In the next section the essential mathematical foundations of the nonlinear SSH method are formulated. An exact 3D ray tracing program will be presented for solving the seismic forward problem. In the following section the results of some numerical computations are given, which demonstrate the potentials and limitations of the nonlinear SSH method proposed here. The stability and uniqueness of the solution will be discussed. Finally, the SSH method will be applied to retrieve the 1D and 3D seismic structure of the Vrancea region, Romania.

Theory of the SSH method

Formulation of the nonlinear inverse problem

In the following, a short outline of the mathematical foundations of the nonlinear SSH method is given. A derivation of the SSH method for a vertically inhomogeneous medium (1D case) can be found in Crosson (1976a, b). The formulation for a 3D medium in terms of optimization theory is almost identical to the 1D case. However, it will be shown that the formulation is different in the manner in which the Fréchet derivatives of the travel-time function are calculated with respect to the model parameters.

The principle of the 3D SSH method can be summarized as follows. The earth volume to be modelled is divided in layers. Each layer is divided in blocks which need not have the same size in the different layers as it is the case with the original SSH method of Aki and Lee (1976). This modification proved to be computationally very advantageous, as will be shown later.

The theoretical arrival-time vector is a nonlinear function of the hypocentral coordinates and the velocity parameters

$$\mathbf{t}_{th} = \mathbf{F}(\mathbf{H}, \mathbf{v}) \tag{1}$$

where $\mathbf{t}_{th} = (t_1, t_2, \ldots, t_n)$ and $n = p*q$ is the maximum possible number of arrival-time data. q is the number of events and p is the number of stations. $\mathbf{H}(x_{01}, y_{01}, z_{01}, t_{01}, \ldots, x_{0q}, y_{0q}, z_{0q}, t_{0q})$ is the vector of the hypocentral parameters for the q events, i.e. its dimension is $4*q$. $v = (v_1, v_2, \ldots, v_l)$ is the vector of the l parametrized velocities, l is the number of blocks in the model. Thus, one has $m = 4q + 1$ unknown model parameters to be determined by the nonlinear inversion of Eq. (1). For the observed arrival times a similar relationship to Eq. (1) also applies, but now with an error vector $\mathbf{e}$ including measurement errors and errors resulting from an insufficient parametrization of the model

$$\mathbf{t}_{0b} = \mathbf{F}(\mathbf{x}) + \mathbf{e} \tag{2}$$

with $\mathrm{x} = (\mathbf{H}, \mathbf{v})$.

The purpose now is to minimize the error $\mathbf{e}$ in Eq. (2) in the least squares sense:

$$||\mathbf{t}_{0b} - \mathbf{F}(\mathbf{x})||^2 = ||\mathbf{e}||^2 \rightarrow \text{minimum} \tag{3}$$

The solution of Eq. (3) by methods of optimization theory (e.g. Gill et al., 1981) can be obtained through linearization of the nonlinear function $\mathbf{F}(\mathbf{x})$ around a starting value $\mathbf{x}^{(0)}$, differentiating Eq. (3) with respect to $\mathbf{x}$ and equating the resulting expression to zero. In this way one gets the classical normal equations for the correction vector $\Delta\mathbf{x}$

$$(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T \cdot \Delta\mathbf{x} = \mathbf{r}. \tag{4}$$

$\mathbf{A}$ is the Jacobian matrix of the Fréchet derivatives of the travel-time function F with respect to the model vector $\mathbf{x}$, i.e. $\mathbf{A}=\mathbf{F}$ and $\mathbf{r}=\mathbf{t}_{0b}-\mathbf{F}(\mathbf{x}^{(0)})$, i.e. the travel-time residual between the observed data $\mathbf{t}_{0b}$ and the theoretical data computed with the initial model vector $\mathbf{x}^{(0)}$.

Because of the linearization of Eq. (3), the new vector $\mathbf{x}^{(1)}=\mathbf{x}^{(0)}+\Delta\mathbf{x}$ will not locate the minimum of Eq. (3), but will only define a new starting vector for the next iteration. In each iteration the normal equations (4), with the new Jacobian matrix calculated for $x^{(i+1)}$, have to be solved. The iterations are stopped when either $\Delta\mathbf{x}$ or the residual sum of squares $s(\mathbf{x})=\mathbf{r}^T\mathbf{r}$ is smaller than a prescribed bound.

The solution of the linear system of equations (4) is by no means trivial and comprises all the pitfalls of linear inverse theory. In fact, because of errors in the data and insufficient model specifications (bad distribution of events and stations), the inverse problem is highly 'illposed'. This means that the solution of Eq. (4) is unstable and nonunique and possesses high covariances. To remove the instabilities and to reduce the covariances, one has to regularize the inverse problem (Tykhonov and Arsenine, 1976), which, on the other hand, introduces a degradation of the spatial resolution. There is a trade-off between these parameters (e.g. Backus and Gilbert, 1968; 1970; Aki and Richards, 1980), or a statistical bias of the inverse estimator (Hoerl and Kennard, 1970) and finally a degradation of the fit of the model to the observed data. For an extensive review and application of regularization theory for the recovery of the 'ill-posedness' of the geophysical inverse problem, see Koch (1983a).

In terms of nonlinear inversion theory, instabilities in $\Delta\mathbf{x}$ in Eq. (4) will cause overshooting of the new starting vector over the minimum searched and thus prevent the iteration converging to the minimum of Eq. (3). For improvement, Levenberg (1944) and Marquardt (1963) proposed damping the correction vector x in Eq. (4) by adding a constant k on the diagonal of the normal matrix $\mathbf{A}^T\mathbf{A}$

$$(\mathbf{A}^T\mathbf{A}+k\mathbf{I})^{-1}\mathbf{A}^T\Delta\mathbf{x}=\mathbf{r}. \tag{5}$$

It can be shown (e.g. Marquardt, 1963; Gill et al., 1981) that k will not only shorten the step vector $\Delta\mathbf{x}$, but also turn it towards another search direction which will often lead to a better convergence to the minimum of Eq. (3).

Among the different algorithms proposed so far for the optimal choice of k in Eq. (5) (see Beck and Arnold, 1977; Gill et al., 1981), the original version of Marquardt (1963) appears to be the most convenient to use in practice. Here, k is only subjected to the condition for the residual sum

$$s(\mathbf{x}^{(i+1)}) < s(\mathbf{x}^{(i)}), \tag{6}$$

i.e. k must be chosen in such a way that the step vector from one iteration to the following ensures convergence. This is accomplished by increasing k within an iterative loop until Eq. (6) is satisfied. This algorithm (see Koch [1983a] for details) is used in the present paper and proves to give generally good convergence after 2–4 iterations.

The solution of the seismic forward problem (3D ray tracing)

According to Eq. (1), the nonlinear inversion procedure requires the knowledge of the nonlinear travel-time function $\mathbf{F}(\mathbf{x})$. Thus, the solution of the seismic forward problem must be found by 3D ray tracing.

The 3D ray tracing procedure used in the present paper is exact in the sense that it eliminates the approximations in the SSH method of Aki and Lee (1976). The ray tracing method has been adapted to block parametrization, i.e. in each block the velocity is assumed to be constant. From each seismic source, rays with variable azimuths and take-off angles are shot towards the respective station. The initial ray direction is modified in an iterative process, until the ray hits the station within a prescribed vicinity (<100 m was found to be sufficient for obtaining travel-time errors <0.05 s). The calculation of the ray paths and travel times in the different blocks is performed by applying Snell's law at the block boundaries (see Koch [1983a, b] for further details).

The ray tracing procedure proposed here is sort of an extension of the ray tracing method of Thurber and Ellsworth (1980), in the sense that the travel times along the true ray paths are calculated. In the method of Thurber and Ellsworth, the computed ray paths represent only average paths through the lateral heterogeneous structure and are assumed to be straight lines in one layer. Thus, lateral refraction at the vertical block boundaries is neglected. Numerical simulations showed that, for lateral velocity perturbations of 5%, travel-time differences up to 0.1 s exist between the travel times calculated along the true ray paths and the ones computed along the laterally averaged paths of Thurber and Ellsworth.

The present ray tracing program is computationally very efficient. Numerical problems may arise when a ray is critically refracted or totally reflected at the vertical block boundaries. As a result, ray geometrical shadow zones are created at the earth's surface. Due to wave seismic effects such shadow zones will barely be seen in the real earth, so that seismic stations situated in a numerical shadow zone will also record a seismic onset. Numerical simulations showed that due to these effects about 10% of the rays are lost for the inversion process. To retrieve these rays the travel time is calculated approximately along a laterally unrefracted ray path. In such a case the ray tracing procedure corresponds to the method of Thurber and Ellsworth (1980).

Another way to overcome the ray geometrical effects, due to vertical block discontinuities, consists of a horizontal smoothing of the velocity field, e.g. by bicubic spline functions (ČFervený et al., 1977; Hawley et al., 1981). This introduces a degradation of the lateral structural resolution and, moreover, makes the ray tracing procedure more time consuming as the differential equations for the propagation of seismic rays in a 3D medium must be solved in this case (Červený et al., 1977).

Computation of the Fréchet derivatives

The nonlinear inversion procedure requires that the Jacobian matrix $\mathbf{A}=\nabla\mathbf{F}$ of the Fréchet derivatives in Eq. (4) is computed in each iterative step. For the SSH problem, $\mathbf{A}$ has the following form (Crosson, 1976a)

$$\mathbf{A}=\left(\begin{array}{cccc|c} \mathbf{C}_1 & 0 & \cdots & 0 & \\ 0 & \mathbf{C}_2 & & & \\ \vdots & & \ddots & & \mathbf{B} \\ 0 & & & \mathbf{C}_q & \end{array}\right) \tag{7}$$

where $\mathbf{C}_i$ are the submatrices of the partial derivatives of the arrival times with respect to the four hypocentral parameters x, y, z, t of the seismic event i. $\mathbf{C}_i$ is the matrix generally used for the classical problem of hypocentre location in a laterally homogeneous earth (Lee and Lahr, 1972; Buland, 1976).

The matrix **B** in Eq. (7) represents the partial derivatives of the arrival time t_{ij} with respect to the block velocities v_s of the model. For an element b of **B** in column s and row r $[r=(i-1)^*p+j]$ one has

$$b_{rs}=\partial t_{ij}\partial v_s=-L/v_s^2 \tag{8}$$

where L is the length of the ray path in block r calculated by ray tracing. Thus, the Fréchet derivatives with respect to the velocity parameters can be computed analytically.

The same is no longer true for the partial derivatives with respect to the hypocentral parameters, x, y, z. Only in the 1D, vertically inhomogeneous medium can the $\mathbf{C}_i$ be computed analytically (Crosson, 1976a). In 3D media one has to approximate the partial derivatives in Eq. (8) by a finite difference quotient (DQ). Here the one-sided DQ has been used, which requires only one additional computation of the travel-time function for the hypocentre shifted by an increment h. Thus one gets, for example for x,

$$\partial t_{ij}/\partial x_{0i}\approx\Delta t_{ij}/\Delta x_{0i}=[t_{ij}(x_{0i}+h)-t_{ij}(x_{0i})]/h \tag{9}$$

and similar equivalent expressios for the DQ with respect to y and z.

The approximation of the partial derivatives by a finite DQ, Eq. (9), depends strongly on the choice of the increment h (Gill et al., 1981). If one chooses h too large, considerable errors of linearization form in Eq. (9), whereas with a very small h the time increment Δt_{ij} for the shifted hypocentre will be very small and numerical rounding errors arise. A comparison of the finite DQ with the analytically computed, exact partial derivative in a 1D medium showed (Koch, 1983a) that, for 20 m $<h<$ 100 m, these approximation errors can be neglected.

The matrix **B** in Eq. (7) introduces a coupling of the different submatrices $\mathbf{C}_i$ of the hypocentre determination problem, which, in the case of $\mathbf{B}=0$ (no structure determination), could be solved independently of each other. The coupling of the $\mathbf{C}_i$ by **B** makes the matrix **A** in Eq. (7) very sizeable. Despite its high sparsity, the effective number of unknown parameters is exclusively limited by the amount of computer storage available. A different approach to break off the coupling between hypocentres and seismic velocities, and thus to reduce the size of the matrix **A** in Eq. (7), has been proposed by Pavlis and Booker (1980) for a 1D medium by spectral decomposition of the data set.

Numerical method of inversion

The numerical solution of Eq. (5) can be performed by construction of the generalized Inverse (g-Inverse or Lanczos-Inverse) of the matrix **A** (see e.g. Lanczos, 1961; Aki and Richards, 1980). The singular value decomposition of **A** may be written as

$$\mathbf{A}=\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^T \tag{10}$$

where **S** is the matrix of the eigenvalues (singular values) of **A** and ordered in a decreasing series, and **U**, **V** are the corresponding eigenvectors. Putting Eq. (10), into Eq. (5) and solving for $\Delta\mathbf{x}$ one gets, with the definition of the g-Inverse $\mathbf{A}^+=\mathbf{V}\,\mathbf{S}^{-1}\,\mathbf{U}^T$,

$$\Delta\mathbf{x}=\mathbf{V}\cdot\mathbf{S}(\mathbf{S}^2+k\mathbf{I})^{-1}\mathbf{U}^T\mathbf{r}. \tag{11}$$

Thus the parameter k has the effect of filtering out the very small eigenvalues of **A** (which in fact are the numerical manifestation of the ill-posedness of the inverse problem) and so preventing the solution $\Delta\mathbf{x}$ from blowing up and overshooting. This filtering or tapering process may be compared with the sharp 'cut-off' strategy proposed by some authors (Wiggins, 1972; Jackson, 1972) where all eigenvalues smaller than a prescribed bound are eliminated completely from the g-Inverse. Several investigations showed (e.g. Marquardt, 1970) that the tapering of the spectrum by the parameter k is more flexible to use than the 'cut-off' strategy and thus mostly preferred in nonlinear optimization theory.

The numerical computations showed, however, that the spectral decomposition of **A** is very time consuming. The direct solution of the system, Eq. (5), proved to be numerically much more efficient. The normal matrix $(\mathbf{A}^T\mathbf{A}+k\mathbf{I})$ is triangularized by applying Householder transformations (QR-decomposition) (e.g. Lawson and Hanson, 1974) and subsequent back-substitution. Moreover, this direct approach has the advantage that, for the classical least squares problem ($n>m$), only the $m\times m$ matrix $\mathbf{A}^T\mathbf{A}$ and not the $n\times m$ matrix **A** has to be stored in the central core. Thus, problems with a bigger number of unknowns can be solved.

Numerical computations

Method

Here, the results of the reconstruction of some numerical test models by the nonlinear SSH method will be presented. Having the application of the SSH method to real earthquake – data from the Vrancea region, Romania in mind, the test models were chosen in such a way as to simulate the conditions of this region fairly well. These are: the rather confined seismic activity at 100–160 km depth, the possible presence of a subduction plate (McKenzie, 1972; Fuchs et al., 1979) and the seismic station array used for the recording of the earthquake data (Fig. 1).

Most of these events were recorded at a maximum of 10–15 stations. This is about the number of recordings to be used in the numerical simulations. Technical drop outs of seismic stations that exist in reality were simulated by a random generator.

The numerical model experiments were performed in the following way. An earth volume with a specified seismic structure (layers, blocks and seismic velocities) was chosen initially. Then, randomly distributed hypocentres were generated within this earth volume and theoretical arrival times were computed. To simulate noise in the real data, normally distributed travel-time errors with a standard deviation $\sigma=0.1$–0.2 s were added.

For the application of the SSH method, a starting solution for the hypocentral and velocity parameters is required. The starting hypocentres were obtained by applying the standard hypocentre localization program, HYPO, of Lee and Lahr (1972) for a laterally homogeneous earth model.

The reconstruction of 1D models

In Fig. 2 the results of the reconstruction of three vertically inhomogeneous models are shown. The number of iterations is $n=4$. Numerous numerical computations showed that fairly good agreement between the reconstructed model

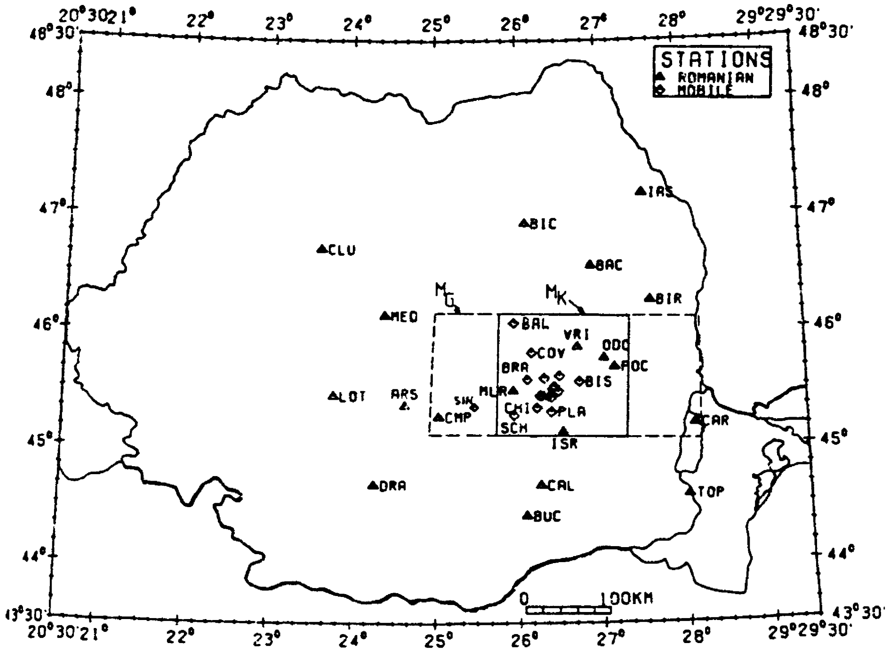


Fig. 1. Map of Romania, showing the seismic station array used for the location of the Vrancea earthquakes. The *two rectangles* delineate the horizontal extensions of two earth models, which will be discussed later

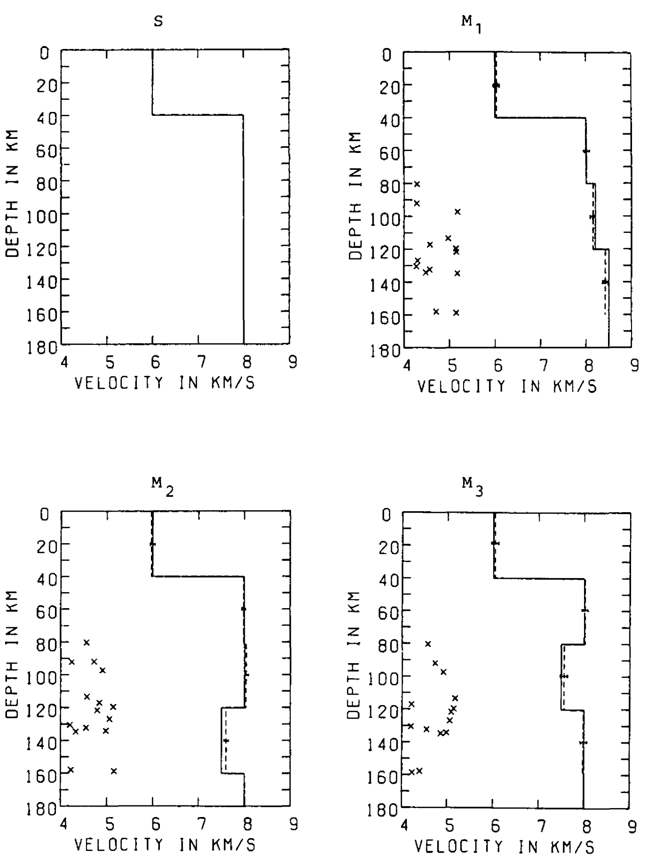


Fig. 2. Results of the nonlinear inversion for three 1D-velocity models M_1, M_2, M_3. *S* is the starting velocity model, also used for the initial hypocentre location with the HYPO program (Lee and Lahr, 1972). The original model is represented by the *full line* and the reconstructed one by the *dashed line*. The *error bars* indicate the standard deviations of the inverted layer velocity. The *crosses* represent the depths of the theoretical hypocentres used in the simulation procedure

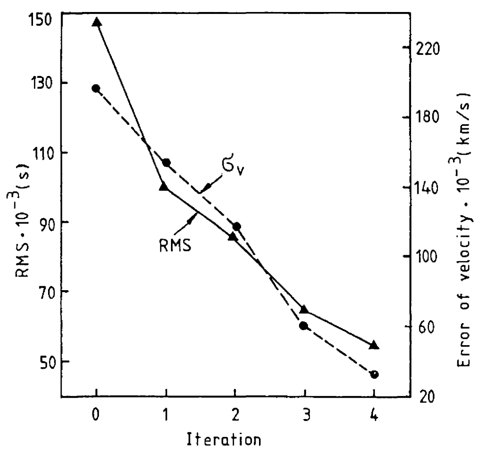
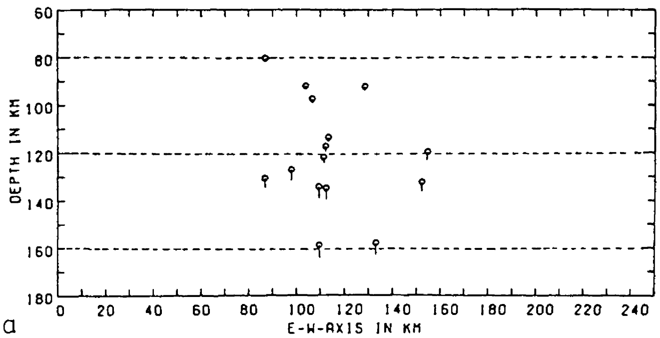


Fig. 3. The RMS of the residuals (*left ordinate*) and the reconstruction of the velocity (*right ordinate*) as a function of the number of iteration steps for model M_1 in Fig. 2

and the original one (errors $<1\%$) could generally be found after 3–4 iterations. Moreover, the computations showed that, by careful choice of the LM-parameter *k* in Eq. (5), the final solution is almost independent of the starting model, thus demonstrating the excellent convergence properties of the LM-algorithm.

This can be seen in Fig. 3 for model M_1 of Fig. 2, where the convergence behaviour fo the SSH inversion is shown as a function of the number of iterations. Figure 3 clearly shows the high nonlinearity of the inverse problem, since a significant improvement in the reconstruction can be achieved by further iteration of the inverse solution, found by one step linear inverse theory ($n=0$).

Because of the coupling of the hypocentres with the velocity structure, one can also assume a shift of the SSH hypocentres, relative to the starting ones. For the model



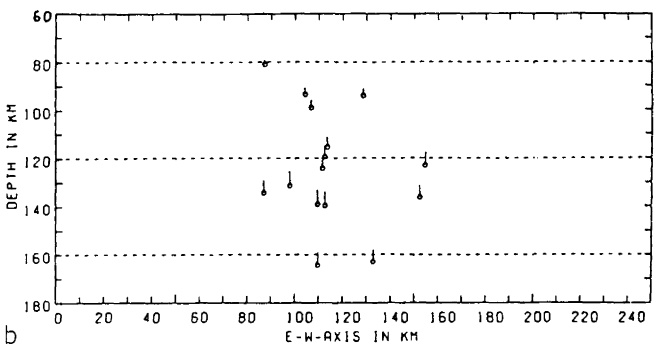


Fig. 4. Cross-sections in the EW-Z-plane showing the hypocentres used in the inversion of model M_1. *Top:* The *circles* mark the true hypocentres and the *lines* the shifts of the hypocentres computed with the HYPO program using the wrong starting model S (Fig. 2). *Bottom:* Shift of the hypocentres in the SSH inversion with respect to the starting hypocentres. The *circles* now denote the starting hypocentres, i.e. correspond to the endpoints of the lines in Fig. 4 top. The endpoints of the lines mark the hypocentres obtained, simultaneously with model M_1 in Fig. 2, by the SSH procedure

M_1, these shifts are demonstrated in Fig. 4. The two diagrams are cross-sections in the EW-Z-plane and show the distribution of the hypocentres. In the top diagram the circles represent the true hypocentres, whereas the endpoints of the lines mark the hypocentres located with HYPO using the starting model S in Fig. 2. It can be seen that, due to the velocity of S being too low relative to the true velocity of M_1, the HYPO hypocentres are shifted systematically about 3–5 km deeper. Along with this, there is an advance of the origin time of up to 0.5 s. On the other hand, due to the lateral homogeneity of the model, the epicentres do not show significant deviations from the true ones.

Figure 4b now shows the shifts of the SSH hypocentres (endpoints of the lines) with respect to the starting hypocentres (circles). That is, the circles in Fig. 4b are situated at the endpoints of the lines of Fig. 4a. The SSH method corrects most of the hypocentres located at too great a depth in Fig. 4a to a shallower depth, thus nearer to the true ones, where the true errors in depth are between 1–2 km. Since this is also approximately the statistical error in depth, the SSH method gives a significant improvement in hypocentre location.

The results for the models M_2 and M_3 are quite similar to those of M_1. As a consequence of the higher average velocity of the starting model S with respect to the velocities of M_2 and M_3 (see Fig. 2), the hypocentres located with HYPO are situated at shallower depths than the true ones. In this case the SSH method was able to correct the erroneous starting hypocentres in the direction of the deeper, true ones.

A detailed investigation of the statistical properties of the solution (i.e. correlation, covariance and resolution matrices) showed, however, that the deeper hypocentres in particular are not very well resolved. This means that the computed depth of the hypocentre is highly correlated both with the origin time and with the seismic layer velocity of the model. There is a trade-off for these three parameters, i.e. they can be varied one against the other without a significant change in the fit of the model to the observed data. This shows the strong ill-posedness of the SSH problem, which can be remedied by regularization and use of a priori information on the model space (Jackson, 1979). A more detailed discussion of this problem can be found in Koch (1983).

A significant break in the coupling between these three parameters can be obtained by additional use of S phases, which better constrain the hypocentral depths (Lee and Lahr, 1972; Buland, 1976), and by use of arrival-time data from a seismic array with a larger aperture. In the latter case the seismic rays propagate less steeply through the model, thus giving more independent information on the hypocentres and on the velocity structure. The foregoing simulations were performed with a relatively small station array, similar to the one used fo the recording of the real Vrancea earthquakes (Fig. 1). Therefore, the practical application of the SSH method to these events will not be free of problems.

The reconstruction of 3D models

In this section the results of the reconstruction of a seismic model will be presented, which approximately simulates the hypothesized plate-tectonic structure of the Vrancea region.

For the simulation of a high-velocity subducting plate, the test model of Fig. 5a was used. For simpler modelling, the plate was assumed to be sinking vertically, as proposed by Fuchs et al. (1979) for the Vrancea region. The velocities are $v = 8.5$ km/s within the plate and $v = 8.0$ km/s outside.

For the starting model in the inversion, the lateral model S in Fig. 2 was also used in the reconstruction. Similarly, the hypocentres computed with HYPO using this starting model are taken as the starting hypocentres in the SSH method.

As in the case of Fig. 4, the systematical mislocations, obtained with HYPO and the lateral homogeneous velocity model, are discussed first (Fig. 6a). It can be observed that the hypocentres located with HYPO are systematically shifted to greater depths and in the direction of the subducting plate. This is an important result because it shows that if one tries to retrieve the geometry of a subduction zone by looking solely at the distribution of the located seismic events (which in fact was the principal way in which Benioff zones were inferred), one will get a subduction which appears to be steeper than it is in reality (see also Engdahl, 1973; Huppert and Froehlich, 1981). Thus, in the presence of lateral heterogeneities, a systematic mislocation occurs when the earthquakes are located using a laterally homogeneous model. This mislocation can only be remedied by using 3D ray tracing (Engdahl and Lee, 1976).

The final hypocentres obtained with the SSH method exhibit a somewhat smaller mislocation (Fig. 6b). The cor-

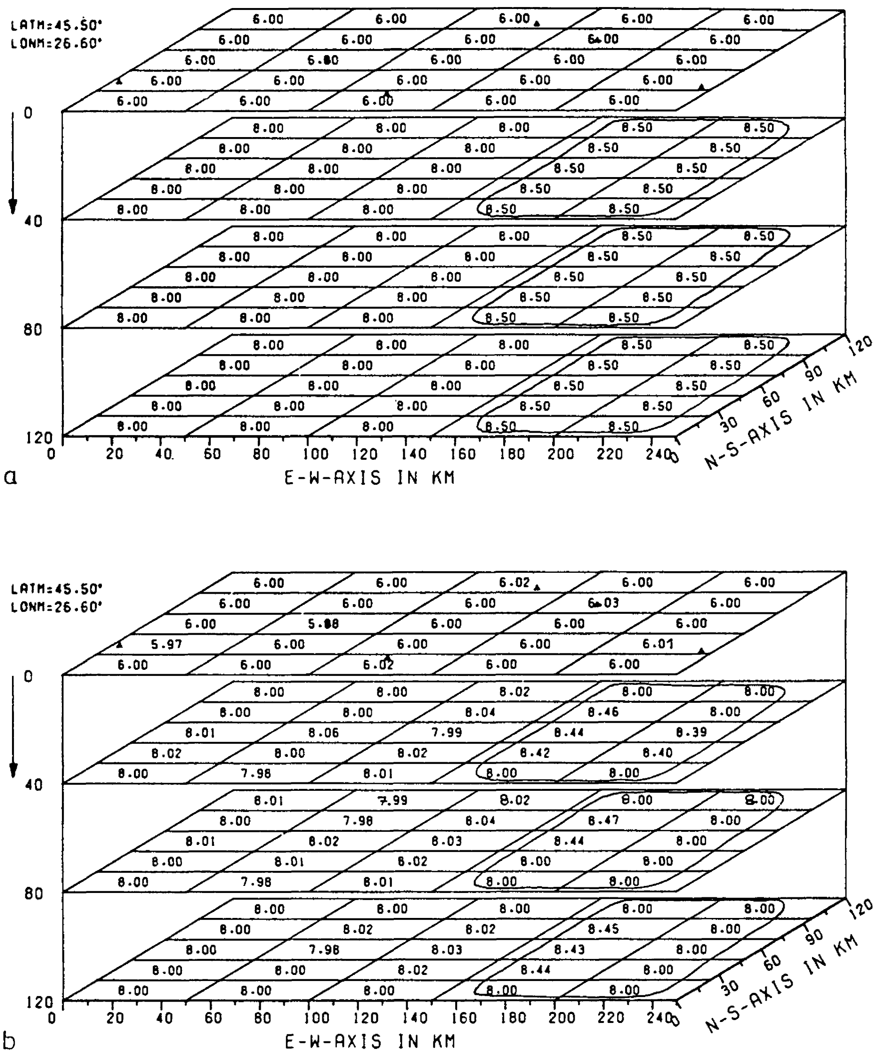


Fig. 5a and b. a 3D-velocity model to approximately simulate the subduction zone hypothesized for the Vrancea region. The location of the earth model corresponds to the dashed rectangle in Fig. 1. The velocities chosen for the model are given within the blocks. The high-velocity zone associated with the subducting plate is marked with a *line*. b The reconstruction of the model of Fig. 5a

rections of the epicentres are significant, but the errors in the depths are still quite large. This is due to the strong trade-off between hypocentral depth and origin time. The arrival-time errors due to the over-estimated depths of the computed hypocentres of about 5 km are compensated by an advance of the origin time of about 0.5 s. The results showed that the 'trade-off' could be somewhat decreased by further iterations, but these improvements proved to be statistically insignificant. Therefore, with the present data it is not possible to remove the inherent 'trade-off' between hypocentral depth and origin time.

Better results could also be achieved by using S phases in the arrival-time data. On the other hand, this requires a precise knowledge of the v_p/v_s ratio of the earth volume under study. The v_p/v_s ratio is known to vary significantly in lateral direction in the Vrancea region, which would cause additional intricacies (Koch, 1982).

In Fig. 5b the final results for the reconstruction of the velocities, in comparison with the original ones of Fig. 5a, are shown. As a consequence of the sparse station coverage of the model, not all blocks are resolved by seismic rays. The initial velocities (i.e. 6.00 and 8.00 km/s) were assigned to these blocks. Figure 5b shows that the original model and particularly the anomalies in the subducting plate have been reconstructed fairly well. The standard deviations of the velocity anomalies vary between 0.01 and 0.04 km/s, depending on the location of the corresponding blocks. On the other hand, the diagonal elements of the resolution matrix (Koch, 1983a), in general, have values between 0.7 and 0.9 for blocks within the centre of the model, but values of only 0.6 for the two blocks located in the most eastern part of the second layer of the model.

For a more detailed computation of the geometrical resolution of the structure obtained, a 3D extension of the method of Backus and Gilbert (Backus and Gilbert, 1967; 1968; 1970) has been applied to the present model. The results showed (Koch, 1983a) that the spread of the resolved velocity in both the lateral and vertical direction is about 1.5–2.0 times larger than the block size for the eastern part of the model, resulting in a substantial leaking of the resolved anomalies from one block into adjacent ones. The reason for the strong smoothing of the velocity structure is the poor station coverage in the Eastern Carpathian foredeeps, where in fact there is only one station, CAR (see Fig. 1). These results thus show that a unique estimation of the geometry of the hypothesized subduction plate will barely be possible with the presently existing station network.

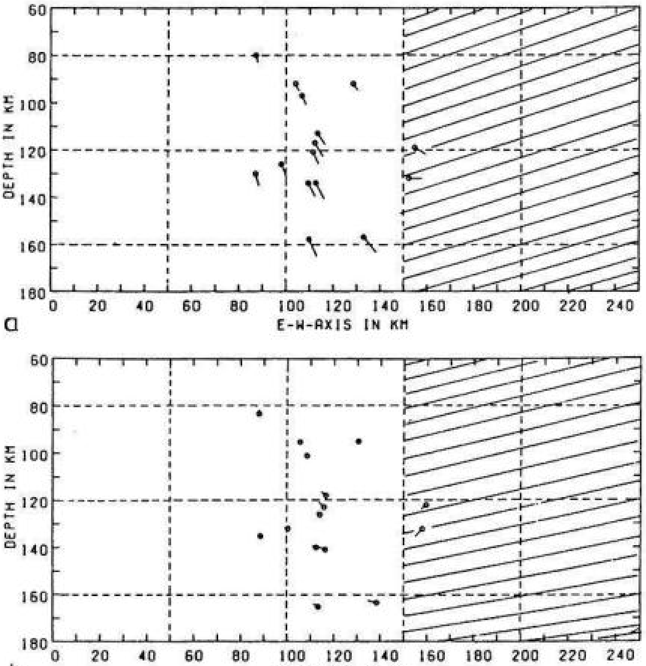


Fig. 6a and b. As Fig. 4, but now for the 3D-velocity model of Fig. 5a. **a** The shifts of the hypocentres computed with HYPO relative to the true ones (*circles*). **b** The shifts of the SSH hypocentres relative to the starting hypocentres (*circles*). The *shaded region* delineates the high-velocity subduction zone

Application of the SSH method to earthquake data of the seismic zone Vrancea, Romania

Evaluation of the data

In the following sections the SSH method will be applied to real earthquake data to infer the 1D- and 3D-seismic structure of the Vrancea region, Romania. The data used in the inversion may be divided into three different groups:

1) Aftershocks of the big seismic event of 4 March 1977, recorded at a seismic network installed temporarily for the two months following the main event (see Fuchs et al. [1979] for details). About 15 crustal and 15 intermediate depth events, for which average residuals (RMS values) of 0.5–0.8 s were obtained with the localization program HYPO (Lee and Lahr, 1972), are used in the SSH inversion. The events were generally recorded at about 8–12 stations.

2) Microevents (Surduc-events) which were recorded at a temporarily installed seismic network for three months in 1979 (Jung, 1983). Approximately 20 crustal- and a similar number of intermediate-depth events were detected. As a consequence of denser station coverage (with 10–15 arrival times per event) and better technical recording, this data set has higher reliability than the set in 1). For these events the RMS values are 0.2–0.3 s.

3) Seismic events of intermediate depths as recorded by the Romanian seismic station network in the years 1964–1981. Although this data set comprises about 50 events, its reliability must be taken with reservation as the accuracy of the arrival times is only moderate. The RMS values for these events range between 0.5 and 1.0 s.

In Fig. 7 the epicentral distribution of the seismic events is shown (see Koch [1982] for a discussion of the seimicity and the results of preliminary travel-time investigations). These epicentres have been relocated with the HYPO program, using the vertically inhomogeneous velocity model of Fuchs et al. (1979).

Inversion of the crustal events

1D-inversion. First, the results of the inversion of the crustal events with the SSH method, in order to retrieve the 1D- and 3D-crustal structure of the Vrancea region, will be presented.

Figure 8 (left) shows the refined vertically inhomoge-

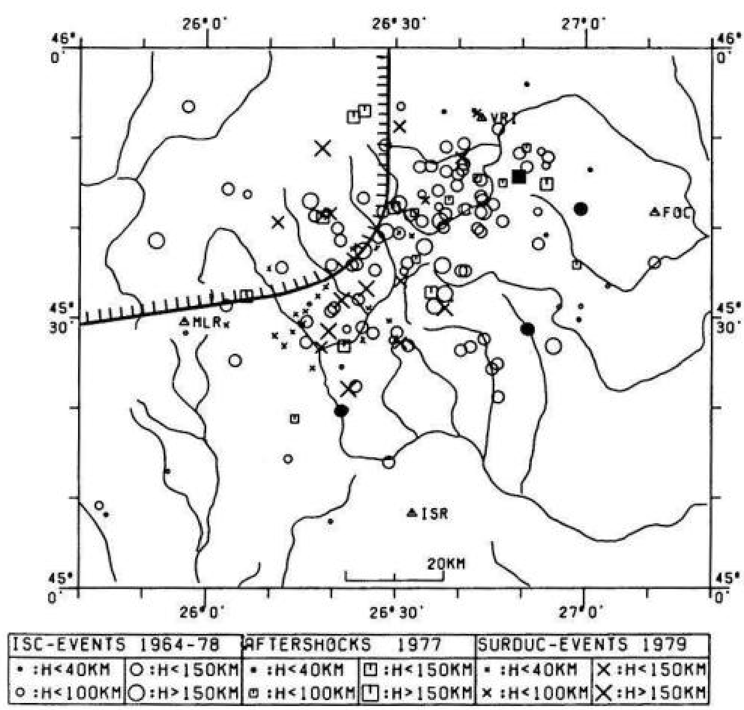


Fig. 7. The epicentral distribution of the seismic events of the three data sets used in the SSH inversion to retrieve the structure of the Vrancea region

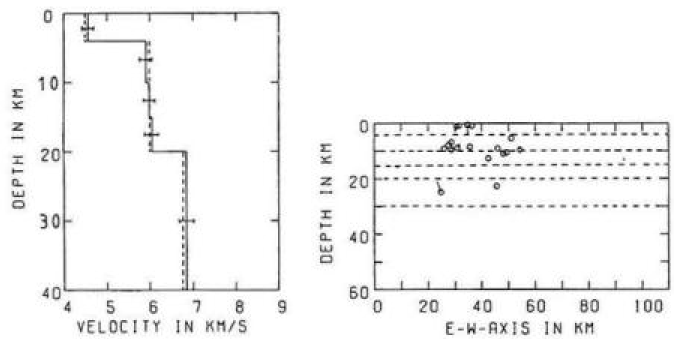


Fig. 8. Results of the SSH inversion of the crustal events for the refinement of the 1D-velocity structure of the crust of the Vrancea region. *Left:* The final velocity model obtained (*solid line*) with standard errors. The *hatched line* denotes the initial velocity model. *Right:* EW-Z-cross-section showing the shifts of the final SSH hypocentres relative to the initial ones (*circles*)

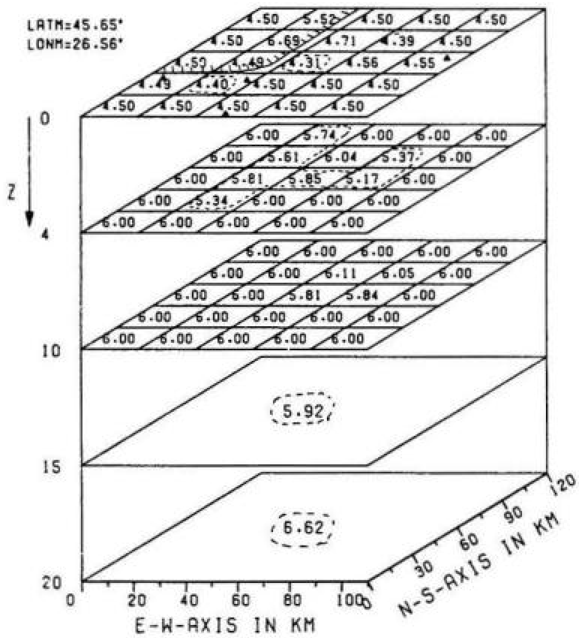


Fig. 9. 3D model of the crust of the Vrancea region. The horizontal extensions of the model correspond to the rectangle M_K in Fig. 1. Zones of negative anomalies are delineated with a *hatched line*. The *thick hatched line* approximately marks the crest of the Carpathian chain

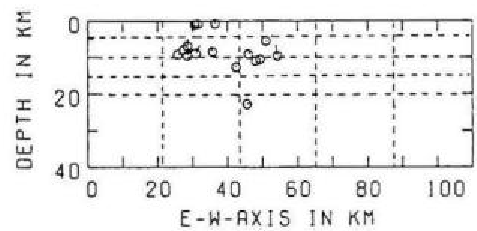


Fig. 10. Cross-section in the EW-Z-plane, showing the shifts of the hypocentres, obtained simultaneously with the crustal structure of Fig. 9, relative to the initial ones (*circles*)

neous (1D) velocity model for the crust of the Vrancea region. There appears to be no significant refinement of the standard model of Fuchs et al. (1979) possible with the present data. The RMS value of the fit of the model to the data could be improved by about 0.1 s with the final velocity model and the final hypocentres obtained (Fig. 8 right). Nevertheless, there remain about 0.4 s unexplained by the data. The reasons for the high RMS values are mainly systematic errors in assigning the correct seismic phases, but may also be due to the impossibility of a 1D-velocity model to account for travel-time residuals due to the lateral velocity heterogeneities present in this region.

3D-inversion. In this section we will present a 3D-velocity model for the crustal structure of the Vrancea region, as it resulted from various inversion runs, with a varying number of layers and blocks. As a consequence of the decreasing coverage of the model by seismic rays with increasing depth, the horizontal block dimensions were chosen to be larger in the deeper layers than in the shallower ones (Fig. 9). In comparison with the original SSH method of Aki and Lee (1976), which works with a fixed number of blocks in the different layers, this modification proved to be very advantageous. In fact, it allows the number of blocks to be chosen according to the resolution by seismic rays and to reduce it, for example in those parts of the model which are barely hit by rays anyway.

The 3D-velocity model finally obtained (Fig. 9) reduces the RMS value for the travel-time residual from 0.5 s to about 0.3 s. A statistical *F*-test (Draper and Smith, 1966) showed this improvement to be significant with more than 95% certainty over the initial vertically inhomogeneous velocity model.

In the following, the different blocks will be subscripted by (i, j) going from the SW- corner of the model in an EW and NS direction, respectively.

The most striking feature of the 3D model is the decrease of the seismic velocity in the second layer with respect to the starting model ($v = 6.0$ km/s). This is particularly true for the two resolved blocks (4,3) and (4,4) in the eastern part of the model. To some extent these anomalies project into the third layer. These negative velocity anomalies, with standard errors of 0.03–0.05 km/s and diagonal elements of the resolution matrix between 0.6 and 0.7, may be interpreted as the thick sedimentary layers in the eastern Carpathian foredeeps which, according to refraction seismology, increase up to a depth of 15 km under the station FOC (Sollugub, 1969).

The shift of the SSH hypocentres with respect to the original hypocentres computed with HYPO is shown in Fig. 10 in the EW-Z-cross-section. Whereas the epicentral shift appears to be negligible, the depths of the hypocentres are generally shallower than the initial ones (circles). This shows the strong nonlinear coupling of the velocity structure and the hypocentre relocation.

Inversion of the intermediate-depth events

The 3D model for the crustal structure of the Vrancea region will be partly used to retrieve a model for the deeper lithosphere by inverting the arrival-time data of the intermediate-depth events. This means that the crustal structure will be used as an à priori constraint in the inversion for the structure of the lithosphere, which reduces the degree of nonuniqueness in the inverted model (Jackson, 1979).

In the following sections, two 3D models will be proposed which are different in their horizontal dimensions and are denoted by M_G and M_K, (see Fig. 1). This proves

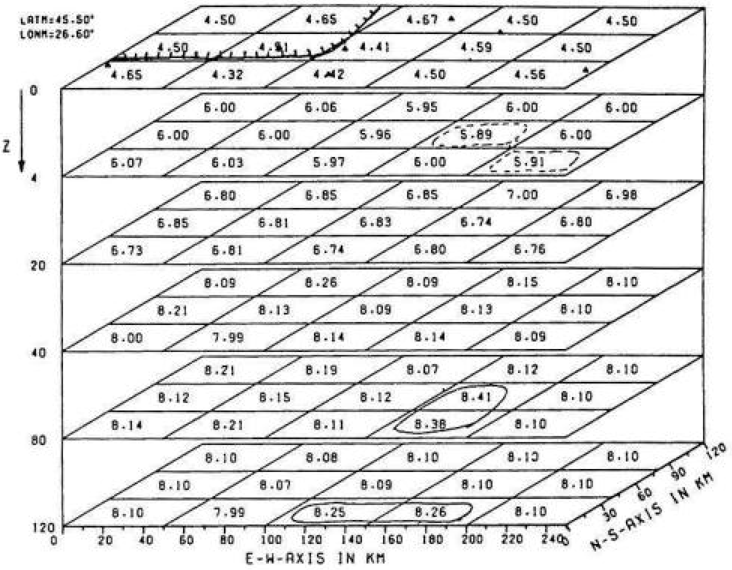


Fig. 11. 3D structure of the lithosphere of the Vrancea region obtained by inversion of the intermediate-depth events. The horizontal extensions of the model correspond to the rectangle M_G in Fig. 1. Zones of positive velocity anomalies and of negative ones are delineated with a *solid* or a *dashed line*, respectively. The *triangles* mark the locations of the Romanian seismic stations (see Fig. 1). The *hatched line* delineates the crest of the Carpathian chain

to be advantageous for optimally exploiting the different data sets used in the inversion process and taking into account the limited computer storage available. Thus, the larger model M_G was obtained by using mainly the data sets 1) and 3) which were recorded at stations covering a larger area also outside the Vrancea region. Model M_G thus mainly reveals the coarse structure of this region. The smaller model M_K is based on data set 2), which was recorded at a dense station array within the Vrancea region itself (Jung, 1983), and parts of data set 2). Model M_K therefore gives a finer resolution of this part of the lithosphere than model M_G.

Before performing the 3D-inversion, several numerical experiments were executed, to get a refined 1D-velocity model, starting from the vertically inhomogeneous velocity model of Fuchs et al. (1979). The results were negative. Because of the rather small aperture of the array, relative to the depths of the events in the Vrancea region which go down to 160 km depth, there appears to be a strong trade-off between hypocentral depths, origin times and layer velocities. Thus, a better 1D-velocity model than that proposed by Fuchs et al. (1979) could not be retrieved uniquely with the present data.

We will first discuss the larger model M_G (see Fig. 1). Figure 11 shows the 3D model obtained after several numerical test inversions with varying layers and blocks. The 3D model in Fig. 11 reduces the original RMS value significantly from 0.5 s in the laterally homogeneous starting model of Fuchs et al. (1979) to 0.3 s. The essential features of the 3D-model in Fig. 11 can be summarized as follows:

The blocks (4,2) and (5,1) in the eastern part of layer 2 show a slightly reduced velocity compared with the starting model. Although these blocks have not been constrained a priori, due to lack of crustal information from Fig. 9, they show the same tendency as in Fig. 9. Thus, once more, the effect of the sedimentary layers can be seen in the solution.

In layer 5 (80–120 km), a positive velocity anomaly of about 4%–5% can be observed in blocks (4,1) and (4,2). The standard errors of the velocities are between 0.2 and 0.3 km/s, whereas the diagonal elements of the resolution

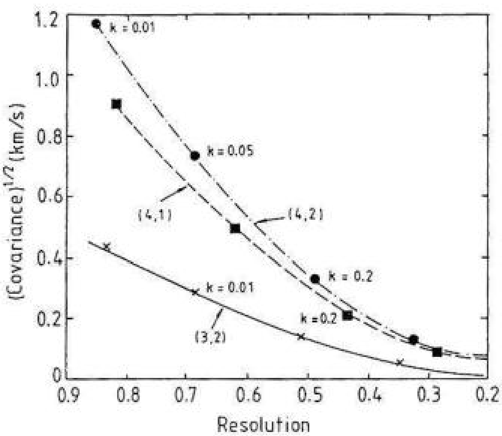


Fig. 12. Trade-off curves between the diagonal elements of the resolution matrix and the standard deviations for three blocks in layer 5 of Fig. 11 as a function of the regularization parameter k

matrix range between 0.4 and 0.6, depending on the choice of the damping parameter k in Eq. (5). In fact, from a more detailed investigation of the results, there appears to be a strong trade-off between the covariance and the resolution of the solution, so that an optimal point on the trade-off curve (Aki and Richards, 1980) is difficult to find. This can be seen from Fig. 12 where the trade-off curves between diagonal elements of the resolution matrix and the standard deviations are shown for three blocks in layer 5. Since no distinguished knee in the trade-off can be observed, the choice of an optimal k to reduce the standard deviations without decreasing the resolution too much is somewhat arbitrary. Nevertheless, for $k = 0.2$ a good compromise appears to be obtained. This value of k proved to be appropriate also in ensuring convergence in the LM-procedure according to Eq. (6).

Despite the moderate quality of the inverse solution, these positive velocity anomalies can be identified as the zone of the hypothesized subducting plate (McKenzie, 1972; Fuchs et al., 1979). This appears to be coincident

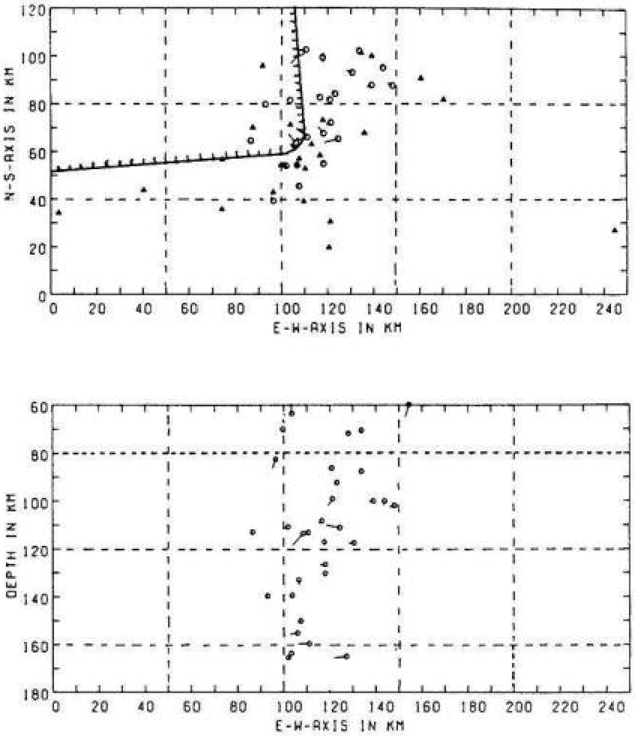


Fig. 13. Shifts of the intermediate-depth hypocentres, inverted simultaneously with the 3D structure of Fig. 11, relative to the initial ones (*circles*)

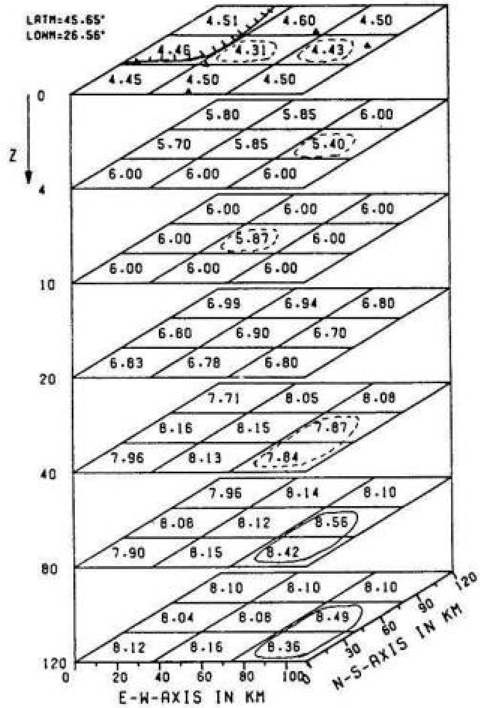


Fig. 14. 3D-structure for the model M_K (see Fig. 1) using the crustal structure in Fig. 9 as an a priori constraint in the inversion process

with the results of teleseismic investigations of Vinnik and Lenartovich (1975), Hovland and Husebye (1982), and Oncescu et al. (1983) who inferred positive velocity anomalies under the eastern Carpathian region.

Figure 13 shows the shift of the intermediate-depth hypocentres inverted simultaneously with the 3D structure of Fig. 11 relative to the original ones. One can observe a systematic westward drift of the final SSH hypocentres, of about 5–10 km, away from the zone of high velocity. A comparison of this drift to the drift obtained for the theoretical plate model in Fig. 6b shows the same trend. Thus, this appears to corroborate the high-velocity zone found in Fig. 11. Furthermore, it shows the systematic errors were introduced in the relocation of the hypocentres of the Vrancea region when using a laterally homogeneous velocity model, instead of using ray tracing in the true laterally heterogeneous structure (e.g. Engdahl, 1973).

The situation is somewhat similar for the second model M_K (see Fig. 1) used for the retrieval of the 3D structure of the Vrancea region (see Fig. 14). In the inversion for this model, the upper crust has been constrained a priori with the crustal model of Fig. 9, thus reducing the degree of nonuniqueness of the model. Nevertheless, as a consequence of a complete lack of events between 40 and 80 km depth, there is still considerable ambiguity in the final model which requires further investigations. The essential features of the 3D model in Fig. 14, which significantly reduces the original RMS value from 0.5 s to 0.3 s, can be summarized as follows:

In layer 5 (40–80 km depth), the blocks (3,1) and (3,2) are characterized by negative velocity anomalies of 0.2–0.3 km/s. Since the standard deviations are larger than 0.1 km/s, with diagonal elements of the resolution matrix of about 0.5 (which may be a result of the lack of coverage of this depth range with seismic events), no statistical inference can be made. Nevertheless, as these negative anomalies have been obtained in different inversion experiments, one physical explanation would be to assume this depth range to be the zone of low viscosity postulated by Fuchs et al. (1979) to explain its aseismicity.

The layers 6 (80–120 km) and 7 (120–160 km) show high-velocity anomalies of 8.4–8.6 km/s in the blocks (3,1) and (3,2), which correspond to a velocity contrast of 4%–6%. In favour of an improved spatial resolution with diagonal elements of about 0.6, the regularization parameter in Eq. (5) might have been chosen too low resulting, according to Fig. 12, in slightly exaggerated velocity perturbations and covariances or standard errors, the latter ranging between 0.2 and 0.3 km/s. Nevertheless, with the reservation of these high errors, the general trend is the same as in the model M_G in Fig. 11. Consequently, this high-velocity region appears to outline the hypothesized subducting plate in the Vrancea region (McKenzie, 1972; Fuchs et al., 1979). However, with the present data it is not possible to delineate exactly the geometry of the subducting plate and, in particular, to answer the question of whether the plate is steeply inclined, as in other Benioff zones (Engdahl, 1973; Huppert and Froehlich, 1981), or even sinking vertically as a detached slab, as proposed by Fuchs et al. (1979). In fact, calculation of the 3D-spread functions with a 3D extension of the Backus and Gilbert method (Backus and Gilbert, 1967; 1968; 1970) showed that, for the denoted blocks (3,1) and (3,2), the block volume effectively resolved is about 1.5–2 times larger than the initially chosen block

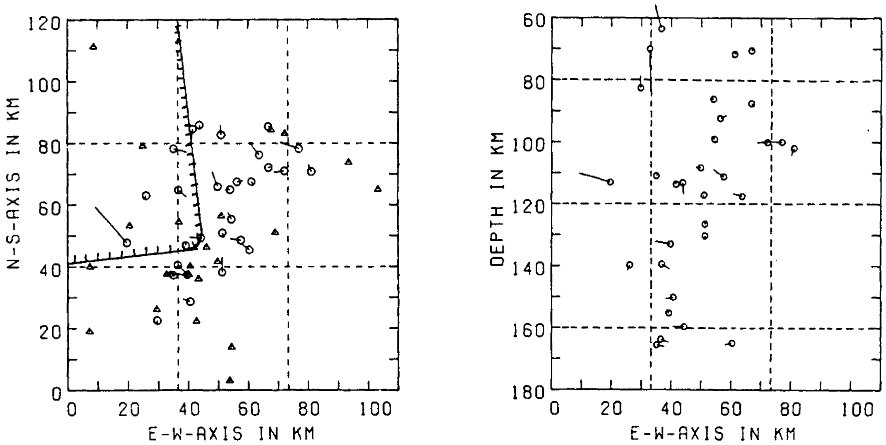


Fig. 15. Similar to Fig. 13, but now showing the shifts of the hypocentres obtained simultaneously with the 3D structure of the model M_K in Fig. 14

sizes in Fig. 13. Thus, there is substantial leaking of the block velocities into the adjacent blocks, prohibiting a precise delineation of the high-velocity subduction zone.

Finally, Fig. 15 shows the shift of the SSH hypocentres relative to the original ones. One can observe, in most cases, a shift in westward direction and also to lower depths, similar to the results of Fig. 13. Thus, from the hypocentral distribution alone, the inclination of the subducting slab appears to be less steep than the one inferred by Fuchs et al. (1979) from the distribution of the hypocentres located with a laterally homogeneous velocity model (see also Engdahl [1973]). These results demonstrate, once more, the systematic errors which are made when one uses a laterally homogeneous earth model for the relocation of the Vrancea earthquakes instead of 3D ray tracing, taking the strong lateral velocity heterogeneities of this region into account.

Summary

In the present paper, a new nonlinear 3D SSH method has been developed which overcomes the systematic approximations inherent in the original SSH method of Aki and Lee (1976), i.e. simplified block-sampling and no ray-tracing. To remedy these approximations, an exact 3D ray tracing has been developed. The method, which was conceived as a shooting method, evaluates the travel times along the true ray paths in the heterogeneous medium specified by rectangular blocks. Snell's law is applied at the block interfaces. With the help of the ray-tracing procedure, the full nonlinear SSH problem is solved. The method developed here is based on the Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1963) which is famous for its excellent stability and convergence properties. The inverse method is an iterative one where, in each iteration step, a linear inverse problem has to be solved.

A substantial decrease of the degree of nonuniqueness and of the instabilities in the geophysical inverse problem results from the use of à priori information on the model space (Jackson, 1979). Thus, the present SSH method has been conceived to handle efficiently à priori constraints in the model, e.g. crustal structure known from explosion experiments.

The SSH method has been tested on various 1D and 3D test models which were essentially established in such a way as to represent fairly well the seismic conditions of the Vrancea region, Romania. The results are encouraging, but clearly demonstrate the intricacies in the inverse solution due to the trade-off between hypocentre depths, origin times and seismic velocities. This is especially true for a station array with small horizontal extensions relative to the depths of the seismic events, as is the case for the Vrancea region.

With 3D ray tracing we investigate the effect of the often hypothesized subduction plate (McKenzie, 1972; Fuchs et al., 1979) on the relocation of the seismic events when these are relocated with a standard location procedure and a laterally homogeneous earth model. The results demonstrate a systematic mislocation which, on the other hand, can be substantially remedied with the 3D SSH method and tracing rays through the true heterogeneous seismic structure. However, due to the trade-off between hypocentral depths and origin times it is not possible to determine the hypocentres without bias.

The SSH procedure has been applied to real crustal- and intermediate-depth events of the Vrancea region recorded during two campaigns, 1977 (Fuchs et al., 1979) and 1979 (Jung, 1983), and to seismic events recorded with the fixed Romanian seismic network during 1964–1981 (Koch, 1982). Because of the limited computer storage capacity, the crustal- and intermediate-depth events were inverted separately. First, the crustal events were inverted for the retrieval of the 3D-crustal structure. This information was then used as an à priori constraint in the inversion of the intermediate-depth events, thus reducing the degree of nonuniqueness in the lithospheric structure and achieving better vertical resolution.

The 3D seismic structure of the lithosphere of the Vrancea region inferred in this way decreases the original RMS value for the travel-time residuals in the laterally homogenous earth model from 0.5 s to about 0.3 s. In spite of this statistically significant improvement, the computed standard errors and the widths of the resolution kernels are still too large to uniquely corroborate the 3D-velocity model retrieved. Its essential feature is a zone of high velocity with 8.4–8.6 km/s, extending from 80 to 160 km depth under the eastern Carpathian foredeeps. This is also in agreement with results of inversion of teleseismic travel-time data of Hovland and Husebye (1982) and of Oncescu et al. (1984) and could substantiate the plate tectonic concepts for the Vrancea region (McKenzie, 1972; Fuchs et al., 1979). Moreover, the model shows a slight decrease of the

seismic velocity to 7.9–7.8 km/s between 40 and 80 km depth, which could support the idea of a low-viscosity zone proposed by Fuchs et al. (1979) to account for the aseismic nature of this depth range.

In conclusion, further investigations are needed for an unambigous substantiation of the results obtained in the present paper for the velocity structure of the seismic zone of Vrancea. Future recordings from the new Romanian seismic station network, which gives a better coverage of the important eastern Carpathian foredeeps, will certainly give further insight into the seismic structure of this complicated region (e.g. Oncescu et al. 1984).

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