

# Evaluation of the effectiveness of theoretical model calculation in determining the plasmopause structure

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**Abstract.** The relative position of the VLF/ELF emission region with respect to the plasmopause is of essential importance in studying their generation and propagation mechanism. On occasions when whistler data are not available, providing extensively the experimental determination of the plasmopause, we are obliged to rely on the theoretical model calculation or, alternatively, on the empirical formulas.

The present paper deals with the evaluation of the effectiveness of the use of a theoretical model calculation in estimating the plasmopause location with reference to its comparison with in-situ electron density measurements and empirical formulas, during a specific geomagnetic storm. It is concluded that the temporal evolution with the present theoretical calculation, under a more acceptable convection electric field model, would yield a sufficiently reliable value for the plasmopause configuration rather than the empirical formulas. It can be used in the study of wave-particle interactions when whistler data are not available and also in the study of the erosion of the plasmasphere itself.

**Key words:** Plasmopause – Plasmasphere – Magnetosphere – Convection – VLF/ELF emissions

## Introduction

The plasmopause is known to play an important role in wave-particle interactions or in the generation of VLF/ELF and ULF emissions (Kaiser et al., 1977). When one studies the generation and propagation mechanism of VLF/ELF emissions based on ground-based measurements, the relative position of the emission occurrence region with respect to the plasmopause is of great importance (Kaiser and Bulough, 1975; Gendrin, 1975; Foster et al., 1976; Hayakawa et al., 1977; Hayakawa et al., 1981) and we normally deduce the plasmopause location by using whistler data simultaneously observed (Corcuff, 1975). However, on some occasions when the whistler data or in-situ measurement for the plasmopause are not available, we are obliged to make use of empirical formulas obtained at specific local times as a function of magnetic activity (Rycroft and Thomas, 1970; Carpenter and Park, 1973) or alternatively to theoret-

ically estimate the plasmopause structure based on the temporal  $K_p$  variation during the period preceding the observation.

The present paper is concerned with the calculation of the theoretical evolution of the plasmopause structure such as the erosion of the plasmasphere for a specific geomagnetic storm of 16–19th December, 1971, under a more realistic model for the convection electric field than the uniform field model previously adopted by Grebowsky (1970) and Chen and Wolf (1972). Then this temporal evolution of the theoretical plasmopause configurations is compared with the in-situ density measurement aboard the S<sup>3</sup>-A satellite (Maynard and Cauffman, 1973) and with empirical formulas, in order to evaluate the effectiveness of the use of the present theoretical model calculation in approximating the actual plasmopause location.

## Theoretical calculation of the plasmopause structure in a time-dependent convection electric field

The most important quantity in determining the plasmopause configuration and particle trajectory is the model of convection electric field. A semi-empirical model for the convection electric field has been proposed by Volland (1973) and Stern (1974, 1975), who assumed that the electrostatic field could be described by a scalar potential  $\phi_E$  of the following form.

$$\phi_E = A R^\gamma \sin \phi - 91.5 R_e/R, (\text{kV}) \quad (1)$$

$$E = -\nabla \phi_E \quad (2)$$

where  $\phi$  is the local time measured from the midnight sector,  $R$  the radial distance from the Earth and  $R_e$  the Earth's radius. The first and second terms in Eq. (1) represent the convection and corotating electric field, respectively. the parameter  $\gamma$  in the convection electric field indicates the degree of screening of the field from the inner magnetosphere.  $\gamma=1$  means a uniform electric field, which was adopted in the previous plasmopause calculation by Grebowsky (1970) and Chen and Wolf (1972).  $\gamma=2$  is adopted in the present paper as the optimum value, based on the theoretical evidence that the convection electric field is partially shielded from the inner magnetosphere (Volland, 1973; Jaggi and Wolf, 1973; Southwood, 1977) and also on the comparison with a variety of observational results on particle injections (Ejiri et al., 1978; Kivelson et al., 1978; Kaye and Kivelson, 1979). The amplitude factor  $A$

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for the convection field in Eq. (1) should be expressed as a function of  $K_p$  index and we have utilized the expression by Maynard and Chen (1975), whose relationship of  $A$  with  $K_p$  index is given in the following, based on the observational data aboard S<sup>3</sup>-A satellite.

$$A = \frac{0.045}{(1 - 0.159 K_p + 0.0093 K_p^2)^3} \quad (\text{kV}/R_e^2). \quad (3)$$

The magnetic field is assumed to be that of a centred dipole model. The trajectories of energetic particles in these field configurations can be traced by a method similar to that developed by Ejiri (1978), on which the calculation of plasmopause structure is essentially based.

Much more complete theoretical models have in fact been constructed (e.g. Blanc, 1983; Spiro et al., 1981). These more elaborate physical models exhibit some features that are not present in the simple Volland-Stern model, but do seem to be present in the observations. Of course, they are too complicated to be useful for the purpose of this paper; namely, an establishment of a simple, easy-to-use method of estimating the plasmopause position for a magnetospheric event. The Volland-Stern model remains the best semi-empirical formula for estimating the electric field in the inner magnetosphere.

The method of calculating the temporal evolution of the plasmopause is based on the original idea by Grebowsky (1970) and Chen and Wolf (1972), which traces the trajectory of cold electrons (energetic electrons with zero energy) backward in time from the universal time of interest. Since the flux tubes are depleted of plasma when they open to interplanetary space and are effectively filled with plasma when they are closed, the density in a closed flux tube at a specific time will depend roughly on the total time the flux tube has been closed and on the dayside of the Earth where solar ionization produces the plasma. By computing this "dayside closure time" at all ( $L$ , LT) coordinates, we can deduce the plasmopause structure in the ( $L$ , LT) space, on the criterion that the dayside closure time is more than 5 days and the field lines corresponding to  $L$  greater than 10 are open. The previous authors (Grebowsky, 1970; Chen and Wolf, 1972; Grebowsky et al., 1974) adopted the spatially invariant ( $\gamma = 1$ ) convection electric field, whose magnitude is varied in step with  $K_p$ . While, in the present paper, we choose the more acceptable model for the convection electric field, as discussed previously.

### Empirical formulas for the plasmopause location

Two main formulas for the estimation of the plasmopause location have been proposed by Rycroft and Thomas (1970)

and Carpenter and Park (1973). Rycroft and Thomas (1970) derived the following empirical formula for the plasmopause location at a specific L.T. sector (L.T. = 0 h, midnight), based on the measurement of the midlatitude trough of the electron density on spacecrafts.

$$L_p = 5.64 - (1.09 \pm 0.22) \sqrt{K_p}, \quad (4)$$

where  $K_p$  is the value at the observation time. So this formula is seen to include no information of the past history of the geomagnetic activity.

On the other hand, using the statistics of the plasmopause estimation by means of ground-based knee whistlers, Carpenter and Park (1973) deduced the following empirical formula of the plasmopause at dawn (L.T. = 6 h).

$$L_p = 5.7 - 0.47 K_{pm}, \quad (5)$$

where  $K_{pm}$  is the maximum  $K_p$  value during 12 h preceding the observation time, and their formula includes, in part, a hysteresis effect as is clearly involved in the theoretical plasmopause.

Higel and Lei (1984) have recently made a quite thorough study of the plasmopause characteristics based on the statistical investigation using the GEOS density measurements and, furthermore, they have discussed the previous empirical formulas of the plasmopause locations, as discussed here. Readers are advised to consult their paper for a detailed review.

### Theoretical plasmopause structure during the geomagnetic storm (16–19th December 1971) and its comparison with in-situ density measurements and the empirical formulas

Figure 1 illustrates the temporal evolution of the geomagnetic activity measured by  $K_p$  index during the storm of 16–19th December, 1971 and the interval preceding it. Figure 2(a)–(j) shows the successive variations of the theoretically calculated plasmopause configuration with an interval of 6 h. At 18 h on the 16th (times are all in U.T.) the plasmopause is found to be nearly circular with its  $L$  value around 4.0 (Fig. 2a). The shape begins to change noticeably after 6 h (i.e. 0 h on the 17th in Fig. 2b) such that the plasmasphere exhibits a bulge in the afternoon sector. As the bulge rotates from the afternoon to dusk sector (Fig. 2d), the bulge becomes a thin plasma-tail. At 18 h on the 17th and 0 h the 18th (Fig. 2e and f) when we have the maximum  $K_p$  value of 7, the plasma-tail appears at the earlier L.T. sector of 14–15 h and wraps round the main body of the plasmasphere (as in Fig. 2g–j) and corotates with the Earth.

For the geomagnetic storm we are dealing with, the

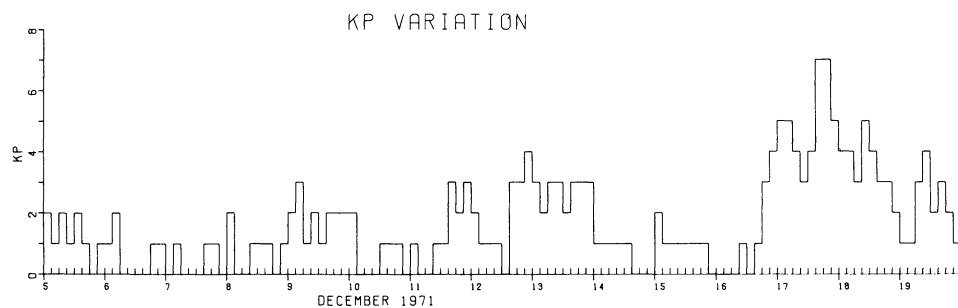
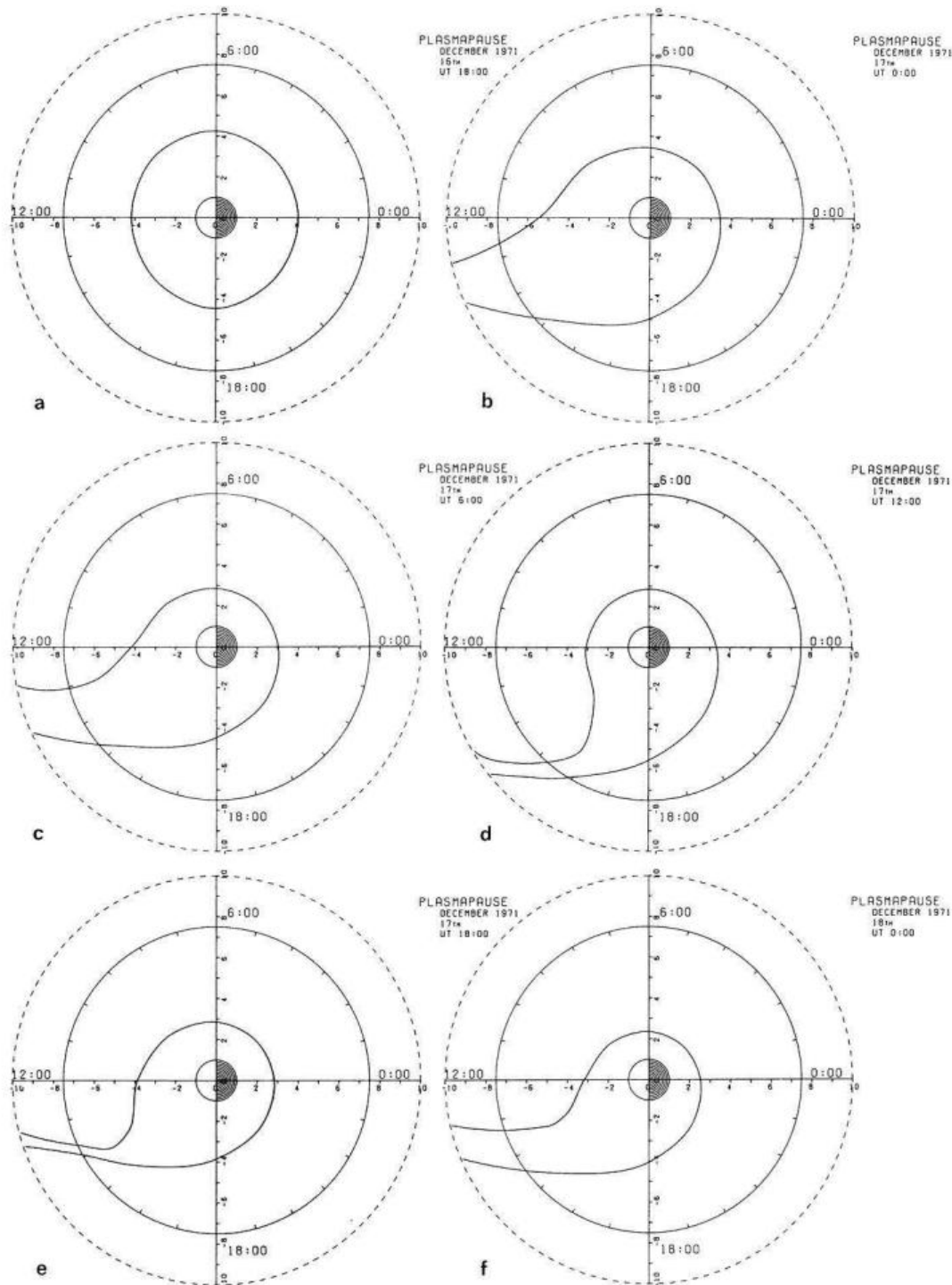


Fig. 1. Temporal evolution of  $K_p$  index. There are geomagnetic storms during the period from 16th to 19th December, 1971



**Fig. 2a-j.** Temporal evolution of theoretically calculated plasmopause structure with an interval of 6 hours from 18 h on 16th (U.T.) (a) to 0 h on 19th (j)

data on the equatorial electron density are available from the S<sup>3</sup>-A satellite (Maynard and Cauffman, 1973). The double floating probe measurement designed to measure dc electric fields is also used as a crude plasmopause detector. Figure 3 presents the comparison of the plasmopause location determined by the S<sup>3</sup>-A satellite (in full lines) and by the theoretical model calculation (in chain lines) at the two local times, L.T. = 18 h, dusk and L.T. = 0 h, midnight. One can find two different values of the plasmopause at some

times on the full lines, which may be the consequence of the two-step structure as discussed by Chappell (1972) or the existence of the detached plasma. While the similar two values on the chain line result from the plasma-tail as found in Fig. 2. The following features have emerged from the comparison.

A comparison of the plasmopause at the dusk sector indicates that the *L* value of the theoretical plasmopause is, on some occasions, larger than the experimental one

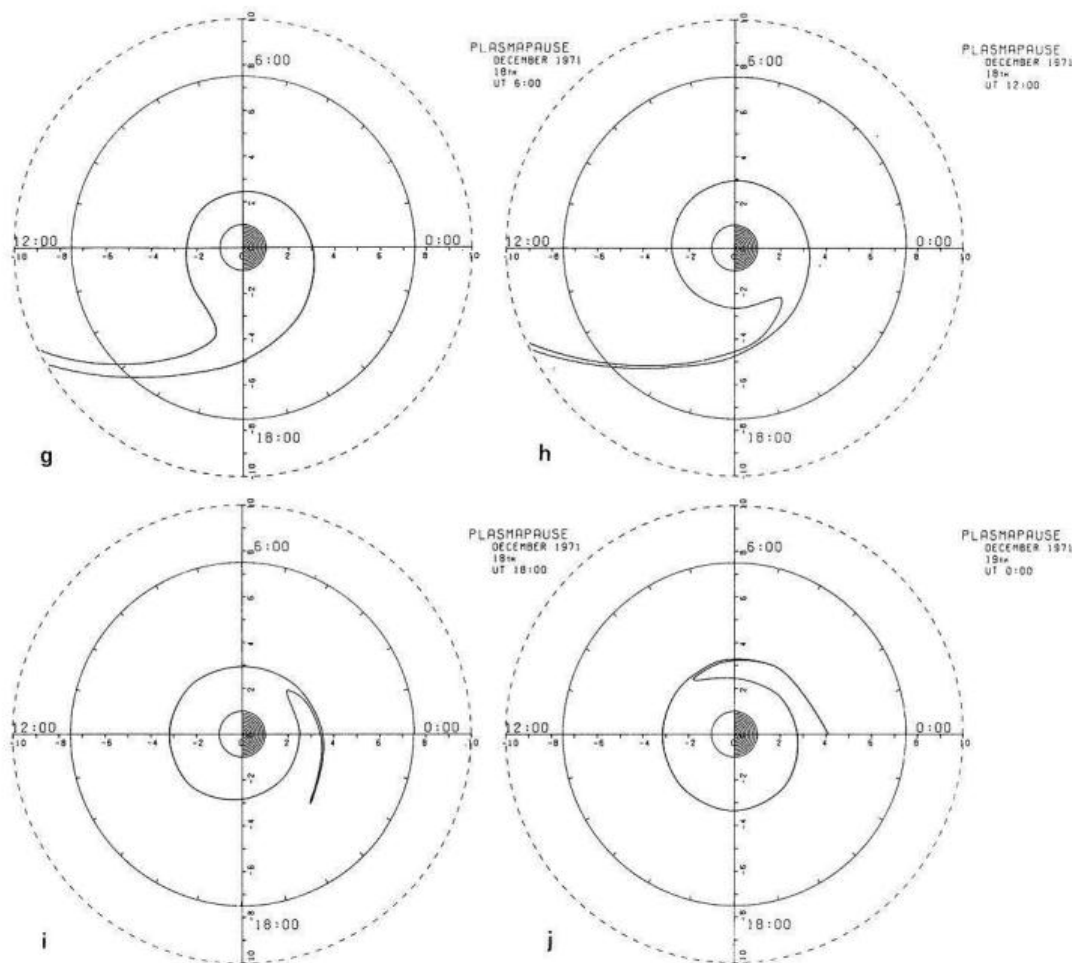


Fig. 2g-j.

and we have the reverse situation on other occasions. However, considering that dusk is the sector exhibiting the most complicated behaviours, we can say that the theoretical model calculation and the in-situ result are, as a general tendency, in good agreement. The discrepancy is not larger than  $1.0 R_e$  and it is normally about  $0.5 R_e$ , except at the orbit numbers of 103 and 104.

A comparison of the temporal variation of the plasma-pause locations in the midnight sector indicates that both of the two plasma-pause locations show an excellently parallel variation. This very nature is very important, suggesting that the theoretical model calculation will yield a very reliable measure of the plasma-pause location in the midnight sector. The experimental value is always larger than the theoretical one by no more than  $1.0 R_e$  and normally about  $0.5 R_e$ . Hence, in this example, the experimental plasma-pause at midnight can be estimated by adding  $\sim 0.5 R_e$  to the theoretical one.

The theoretical model calculation is now compared with the empirical formulas. Figure 4 illustrates the comparison with the formula, Eq. (4), by Rycroft and Thomas (1970) in the midnight sector (L.T.=0 h) indicated by two full lines. The chain line refers to the theoretical model calculation. The empirical formula, itself, has an uncertainty range as given by Eq. (4). At the two times; 18 h on the 18th and 0 h on the 19th, the theoretical plasma-pause of the

main body of the plasmasphere is found to the outside the uncertainty range; however, at other times, the theoretical plasma-pause is found to be nearly within that range. An excellent parallel nature is recognizable for midnight between the theory and in-situ result in Fig. 3. So, if we find a highly parallel variation between the empirical and observational plasma-pause, then we would expect, consequently, a parallel relationship between the theoretical and empirical plasma-pause in Fig. 4. However, the parallel nature becomes much less obvious, since we take, in Fig. 4, the smaller  $L$  values for the last two times corresponding to the plasma-pause of the main body of the plasmasphere. This means that the use of the present theoretical model calculation would provide a more reliable plasma-pause location at the midnight sector than the empirical formula.

Next, the dawn-side plasma-pause is compared (Fig. 5) between the theoretical estimation (in chain line) and the empirical formula, Eq. (5), by Carpenter and Park (1973) (in full line). Very good agreement is obtained in the period of the main and early recovery phase of the storm (i.e. 18 h on 17th to 6 h on the 18th). Otherwise, their empirical value seems to be  $\sim 0.5 R_e$  larger than the theoretical value. The general impression is that the parallel nature of the theoretical plasma-pause with the empirical one seems to be improved in Fig. 5, which might be associated with the inclusion of a hysteresis effect in their formula.

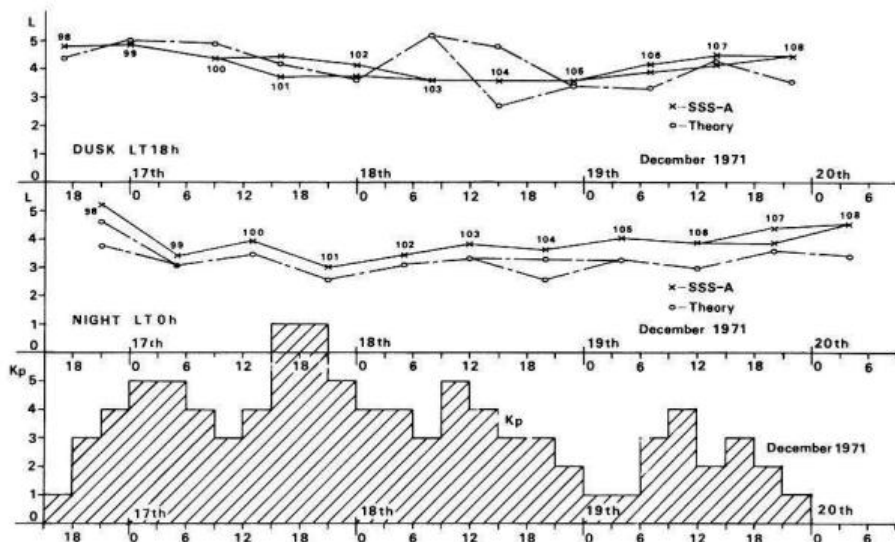


Fig. 3. Comparison of the theoretically calculated plasmopause with the in-situ electron density measurement by S<sup>3</sup>-A satellite (after Maynard and Cauffman, 1973) at two different local times. The full line linked by crosses refers to the satellite measurement, and the chain line linked by open circles indicates the theoretical plasmopause

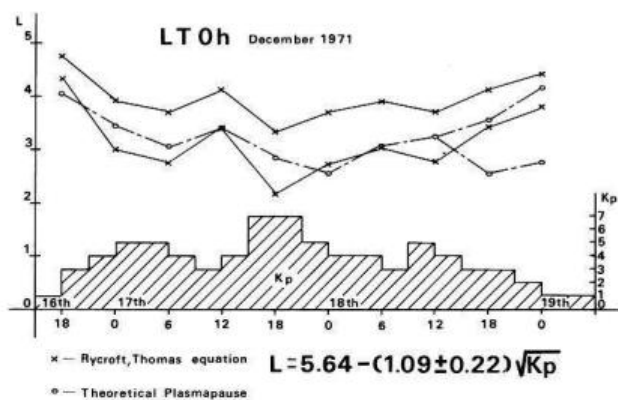


Fig. 4. Comparison of the theoretically calculated plasmopause with the empirical formula at the midnight sector. The chain line connected by open circles corresponds to the theoretical plasmopause and the two full lines connected by crosses refer to the boundaries of the empirical formula by Rycroft and Thomas (1970)

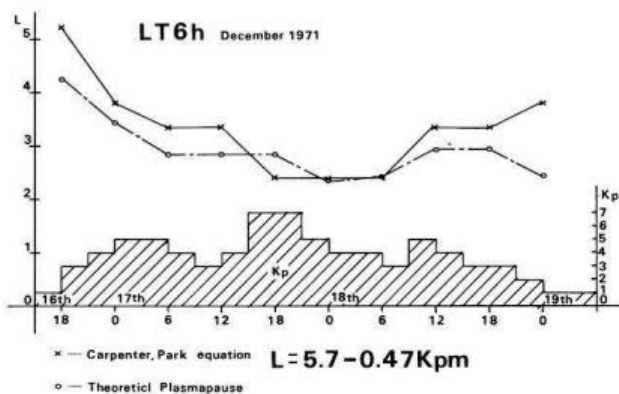


Fig. 5. Comparison of the theoretical plasmopause with the empirical formula at the dawn sector. The chain line has the same meaning as in Fig. 4, while the full line connected by crosses indicate the Carpenter and Park (1973)'s formula with taking into account a hysteresis effect

### Concluding remark

The plasmopause configuration can be theoretically calculated under the more realistic and acceptable convection electric field model than the previous spatially invariant model. This approach is useful in describing the erosion of the plasmasphere during a magnetospheric event. The present study is based on a comparison of the theoretical plasmopause with the in-situ density measurement for one specific geomagnetic storm only. However, the study seems to show that the theoretical calculation, taking into account the past history of  $K_p$  index, provides a considerably reliable plasmopause location and hence it can be utilized in specifying the plasmopause location in VLF/ELF emission studies even when the simultaneous whistler data are not available.

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