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Earth's Flattening Effect on the Tidal Forcing Field*

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Abstract. A small part of the tidal forcing field whose contribution is omitted in the conventional spherical harmonic development of the tidal potential is caused by the flattening of the earth. It is a homogeneous tidal field of magnitude of about 1 ngal superposed on the commonly known tidal forcing field. The conventional tidal forcing field can be completely described by the spatial variation of the gravity field of the tide-generating body within the space occupied by the earth. The advantage of this description is that any reference to the Earth's motion with respect to the tide-generating body (called revolution without rotation) can be avoided.

Key words: Ellipticity of the earth – Tidal forces – Earth tides

Introduction

Some decades ago Jung (1955) pointed out that the tidal forcing field of the Moon contains a small part which is not included in the conventional derivation of this field. Bartels (1957) noted this point but did not consider it in detail. Recently Wahr (1979; 1981) insisted again on the existence of this part, surely without knowing Jung's paper. A reconsideration of Jung's article showed that some modifications have to be introduced and that a clarification is needed.

Tidal Forcing Field

In every spatially extended physical system upon which an inhomogeneous forcing field is acting, relative forces are induced resulting from the inhomogeneity of the forcing field in the space occupied by the system. For example the Moon, for simplicity regarded as a point mass, exerts a corresponding inhomogeneous gravitational field on the Earth. Introducing an inertial coordinate system with origin S at the unaccelerated common centre of mass, this gravitational field shall be described by $\mathbf{g}_M(\tilde{\mathbf{r}})$ where $\tilde{\mathbf{r}}$ is the position vector. The difference between the gravitational field at an arbitrary point $P(\tilde{\mathbf{r}})$ and the centre of mass $O(\tilde{\mathbf{r}}_0)$ of the earth is a relative gravitational field resulting from subtraction of the constant field $\mathbf{g}_H = \mathbf{g}_M(\tilde{\mathbf{r}}_0)$ from the spatially varying field $\mathbf{g}_M(\tilde{\mathbf{r}})$

$$\mathbf{b}_d(\tilde{\mathbf{r}}) = \mathbf{g}_M(\tilde{\mathbf{r}}) - \mathbf{g}_M(\tilde{\mathbf{r}}_0) = \mathbf{g}_M(\tilde{\mathbf{r}}) - \mathbf{g}_H. \quad (1)$$

It can be computed from a potential $V_d(\tilde{\mathbf{r}})$

$$\mathbf{b}_d(\tilde{\mathbf{r}}) = \nabla V_d \quad (2)$$

with

$$V_d(\tilde{\mathbf{r}}) = V(\tilde{\mathbf{r}}) - V_H(\tilde{\mathbf{r}}), \quad (3)$$

where

$$\mathbf{g}_M(\tilde{\mathbf{r}}) = \nabla V \quad (4)$$

and

$$V_H(\tilde{\mathbf{r}}) = \tilde{\mathbf{r}} \cdot \mathbf{g}_M(\tilde{\mathbf{r}}_0) + C_0. \quad (5)$$

$V(\tilde{\mathbf{r}})$ is the lunar gravitational potential, $V_H(\tilde{\mathbf{r}})$ the potential of the homogeneous field $\mathbf{g}_H$ which has to be subtracted in Eq. (1), and C_0 is a free constant which will be given a suitable value by Eq. (9).

These fields can also be expressed in a geocentric spherical coordinate system (r, ψ, γ) with its origin at the Earth's centre of mass O and its axis pointing to the Moon, see Figs. 1a, 2. In this system P is characterized by its radius vector $\mathbf{r}$ and the fields given by the Eqs. (1)–(5) can be calculated by the conventional development of $V(\mathbf{r})$ in spherical harmonics (Bartels, 1957)

$$V(\mathbf{r}) = V(r, \psi) = \frac{GM_L}{c} \sum_{n=0}^{\infty} \left(\frac{r}{c}\right)^n P_n(\cos\psi), \quad (6)$$

where G is the gravitational constant, M_L the lunar mass, c the distance between the centres of mass of the Earth and the Moon. Introducing the unit vector $\hat{\mathbf{s}}$ pointing from O to the Moon (Fig. 1b) and regarding

$$\tilde{\mathbf{r}} = \tilde{\mathbf{r}}_0 + \mathbf{r} \quad (7)$$

$$V_H(\tilde{\mathbf{r}}) = \frac{GM_L}{c^2} \hat{\mathbf{s}} \cdot \tilde{\mathbf{r}} + C_0 = \frac{GM_L}{c^2}(r\cos\psi - \tilde{r}_0) + C_0, \quad (8)$$

so that for

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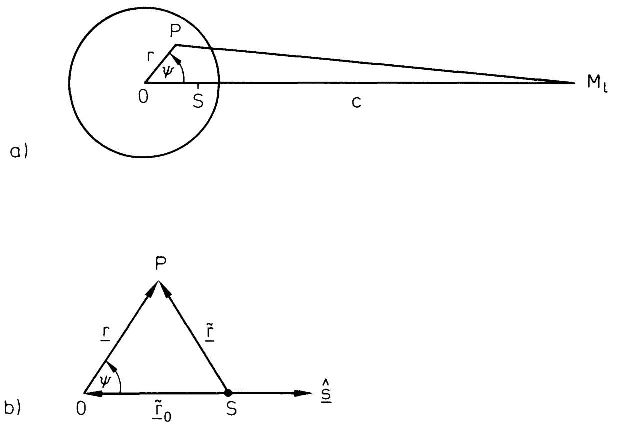

Fig. 1. a Spherical coordinate system r, ψ centred with respect to the axis OM_L, O centre of mass of the earth, S common centre of mass of earth and moon; M_L lunar mass; c distance OM_L. **b** Radius vector $\tilde{\mathbf{r}}$ and $\mathbf{r}$ of P in the inertial system with origin S and the geocentric system with origin O respectively; $\tilde{\mathbf{r}}_0$ radius vector of O in the inertial system; $\hat{\mathbf{s}}$ unit vector in direction OM_L

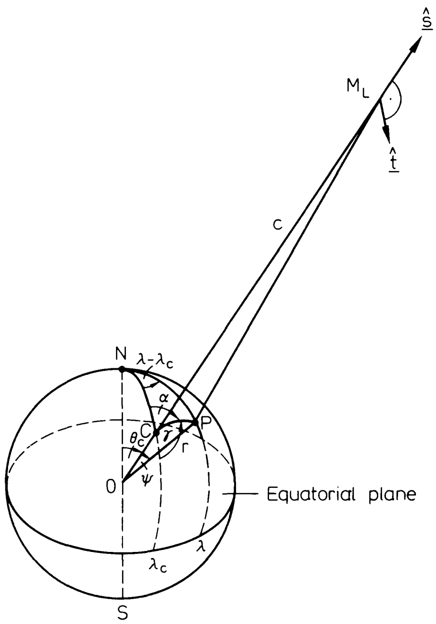

Fig. 2. Spherical coordinates r, ψ, γ of a mass point of the earth at P in the system centred with respect to the axis OM_L and corresponding coordinates r, θ, λ and c, θ_c, λ_c of P and lunar mass M_L with respect to a geocentric coordinate system with NS-axis, α coazimuth of γ. Unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{t}}$ designate r- and θ-direction at the moon's place $\mathbf{r}=\mathbf{c}$ or at the corresponding surface point C respectively

$$C_0=\frac{GM_L}{c^2}(\tilde{r}_0+c) \tag{9}$$

$$V_d(r,\psi)=V(r,\psi)-V_H(r,\psi)=\frac{GM_L}{c}\sum_{n=2}^{\infty}\left(\frac{r}{c}\right)^n P_n(\cos\psi). \tag{10}$$

This potential is known as the forcing tidal potential of the Moon and has been derived here without reference to the Earth's orbital motion, known as "revolution without rotation", and without any assumptions about its physical properties such as density, rigidity or elasticity.

However, the difference in the gravitational field of the moon between a point P of the earth and its centre of mass O is *not* the lunar tidal forcing field. As initially mentioned, tidal forces are acting with respect to the centre of mass of a physical system which is exposed to an inhomogeneous forcing field generating an acceleration of the centre of mass in an inertial frame. The subtraction of this orbital acceleration from the forcing field yields the relative forces, i.e. the tidal forcing field, in the accelerated system associated with the centre of mass. Hence, the tidal forcing field is the vector difference between the gravitational field of the tide-generating body at the observation point and the orbital acceleration of the Earth's centre of mass with respect to the unaccelerated common centre of mass of the Earth and the tide-generating body, i.e. with respect to the inertial frame. In the inertial system with origin S the lunar tidal forcing field $\mathbf{b}(\tilde{\mathbf{r}})$ is therefore

$$\mathbf{b}(\tilde{\mathbf{r}})=\mathbf{g}_M(\tilde{\mathbf{r}})-\mathbf{g}_0 \tag{11}$$

where

$$\mathbf{g}_0=\frac{1}{M_E}\int\rho_E(\tilde{\mathbf{r}})\,\mathbf{g}_M(\tilde{\mathbf{r}})\,d^3\tilde{\mathbf{r}}$$

is the orbital acceleration of the Earth's centre of mass in the Earth-Moon system relative to S, M_E is the mass of the earth and $\rho_E(\tilde{\mathbf{r}})$ the density of the Earth at $\tilde{\mathbf{r}}$. The lunar tidal forcing field therefore by definition does not contribute to the orbital motion of the Earth with respect to the Moon which is completely determined by $\mathbf{g}_0$. If the Earth were rigid no relative accelerations with respect to its centre of mass could occur in response to the tidal forcing field and the orbital motion of the Earth would remain unchanged. However, the response of the Earth to the tidal forcing field actually has an influence on $\mathbf{g}_0$ and therefore on the orbital motion of the Earth. For example the dissipation of ocean tide energy is a major factor for orbital variation in the Earth-Moon system. But also the elastic response has, at least in principle, an effect on the orbital acceleration of the Earth by its flattening effect.

The two fields $\mathbf{b}_d(\tilde{\mathbf{r}})$ and $\mathbf{b}(\tilde{\mathbf{r}})$ given by Eqs. (1) and (11) are identical only if

$$\mathbf{g}_0=\mathbf{g}_M(\tilde{\mathbf{r}}_0). \tag{12}$$

It will be shown that this relation would be valid if the Earth were spherically symmetric.

The acceleration $\mathbf{g}_0$ of the Earth's centre of mass is given by

$$\mathbf{g}_0=-\frac{M_L}{M_E}\mathbf{g}_E(\mathbf{c}) \tag{13}$$

where $\mathbf{g}_E(\mathbf{c})$ is the gravitational field of the Earth at the place $\mathbf{r}=\mathbf{c}$ of the moon which is assumed to be a point mass.

The gravitational field of the Earth is known from measurements of orbits of artificial satellites and can be expressed by its gravitational potential $U(\mathbf{r})$

$$\mathbf{g}_E(\mathbf{r})=\nabla U. \tag{14}$$

Neglecting longitude dependent terms in $U(\mathbf{r})$ (Heiskanen and Moritz, 1967)

$$U(r,\theta)=\frac{GM_E}{r}\left[1-\sum_{n=2}^{\infty}J_n\left(\frac{a}{r}\right)^n P_n(\cos\theta)\right] \tag{15}$$

where (r,θ,λ) is a geocentric spherical coordinate system centred with respect to the Earth's axis of main inertia, the constant a is the Earth's equatorial radius and J_n are the coefficients of the expansion in spherical harmonics.

At the position of the Moon ($r=c$, $\theta=\theta_c$) the gravitational field of the Earth is

$$\begin{aligned}\mathbf{g}_E(\mathbf{c})&=\nabla U|_{\substack{r=c\\ \theta=\theta_c}}\\ &=-\frac{GM_E}{c^2}\left\{\left[1-\sum_{n=2}^{\infty}(n+1)J_n\left(\frac{a}{c}\right)^n P_n(\cos\theta_c)\right]\hat{\mathbf{s}}\right.\\ &\qquad\left.+\sum_{n=2}^{\infty}J_n\left(\frac{a}{c}\right)^n\frac{dP_n}{d\theta}\bigg|_{\theta=\theta_c}\hat{\mathbf{t}}\right\}\end{aligned} \tag{16}$$

where θ_c is the lunar colatitude and $\hat{\mathbf{s}}$ and $\hat{\mathbf{t}}$ are the unit vectors in r- and θ-direction at $r=c$, $\theta=\theta_c$. Figure 2 shows the geometric configuration.

The first part $-\frac{GM_E}{c^2}\hat{\mathbf{s}}$ is the gravitational field of a spherically symmetric Earth of the same mass M_E acting at the position of the Moon. The rest is contributed by the non-radially symmetric part of the Earth's mass distribution.

If all coefficients J_n disappear

$$J_n=0, \quad n=2,3,4\ldots \tag{17}$$

$$\mathbf{g}_E(\mathbf{c})=-\frac{GM_E}{c^2}\hat{\mathbf{s}} \tag{18}$$

and with (13)

$$\mathbf{g}_0=\frac{GM_L}{c^2}\hat{\mathbf{s}}=\mathbf{g}_M(\tilde{\mathbf{r}}_0). \tag{19}$$

Hence, only if the Earth were spherically symmetric would the acceleration of its centre of mass be equal to the gravitational field acting at its centre of mass. Actually, because of the flattening of the Earth $\mathbf{g}_0\neq\mathbf{g}_M(\tilde{\mathbf{r}}_0)$. Therefore, the two fields (1) and (11) are not identical.

The tidal field (11) can be split into two parts from which the effect of the flattening becomes obvious:

$$\begin{aligned}\mathbf{b}(\tilde{\mathbf{r}})&=\mathbf{g}_M(\tilde{\mathbf{r}})-\mathbf{g}_0=\mathbf{g}_M(\tilde{\mathbf{r}})-\mathbf{g}_M(\tilde{\mathbf{r}}_0)+\mathbf{g}_M(\tilde{\mathbf{r}}_0)-\mathbf{g}_0\\ &=\mathbf{b}_d(\tilde{\mathbf{r}})+\mathbf{b}(\tilde{\mathbf{r}}_0)=\mathbf{b}_d(\tilde{\mathbf{r}})+\mathbf{b}_H.\end{aligned} \tag{20}$$

The first part $\mathbf{b}_d(\tilde{\mathbf{r}})$ is the relative gravitational field introduced by Eq. (1) which by Eq. (10) represents the conventional tidal forcing field whereas the second part is the specific tidal force at the Earth's centre of mass $\tilde{\mathbf{r}}=\tilde{\mathbf{r}}_0$ which is non-vanishing for a flattened Earth. To the conventional tidal forcing field $\mathbf{b}_d(\tilde{\mathbf{r}})$ is added the homogeneous field $\mathbf{b}_H=\mathbf{b}(\tilde{\mathbf{r}}_0)$ if the flattening of the Earth is taken into account.

From (2), (11), and (16) it follows for the tidal forcing field expressed in geocentric coordinates

$$\begin{aligned}\mathbf{b}(\mathbf{r})&=\frac{GM_L}{c}\nabla\sum_{n=2}^{\infty}\left(\frac{r}{c}\right)^n P_n(\cos\psi)\\ &\quad+\frac{GM_L}{c^2}\sum_{n=2}^{\infty}J_n\left(\frac{a}{c}\right)^n\\ &\quad\cdot\left\{(n+1)P_n(\cos\theta_c)\hat{\mathbf{s}}-\frac{dP_n}{d\theta}\bigg|_{\theta=\theta_c}\hat{\mathbf{t}}\right\}.\end{aligned} \tag{21}$$

The first expression is the usual tidal forcing field which really is the relative gravitational field (1) and which is completely independent of the physical properties of the Earth. The second part is the expression for $\mathbf{b}_H$. It depends on the density distribution of the Earth and vanishes if this density distribution is spherically symmetric. From (21) it is evident that the flattening of the Earth makes a contribution to the tidal forcing field of the Moon. The physical explanation for this effect is given by Eq. (16). The gravitational field of the Earth acting on the Moon depends on the aspherical part of the density distribution of the Earth and by the action-reaction principle the acceleration of the Earth's centre of mass $\mathbf{g}_0$ also depends on this part of the Earth's density distribution. Since $\mathbf{g}_0$ is involved in the tidal forcing field this field, too, must depend on the mass distribution of the Earth. A corresponding contribution from the Earth's flattening to the solar tidal forcing field is smaller by $(M_s/M_L)\cdot(c/c_s)^4\sim 10^{-3}$ where M_s is the solar mass and c_s is $1\,AU$.

From (13) and (16)

$$\begin{aligned}\mathbf{g}_0&=\frac{GM_L}{c^2}\left\{\hat{\mathbf{s}}-\sum_{n=2}^{\infty}\left(\frac{a}{c}\right)^n J_n\right.\\ &\quad\left.\cdot\left[(n+1)P_n(\cos\theta_c)\hat{\mathbf{s}}-\frac{dP_n}{d\theta}\bigg|_{\theta=\theta_c}\hat{\mathbf{t}}\right]\right\}.\end{aligned} \tag{22}$$

Hence, the motion of the Earth is disturbed by its nonspherical mass distribution. These disturbances are however small compared to the gravitational disturbances caused by the sun (Kaula, 1968, p. 176) resulting from the change of the gravitational field of the sun by the variation of the distance between the geo-lunar mass centre S and the sun. This effect is of the order of $2GM_s a/c_s^3$ which is about $c/(aJ_2)\sim 6\cdot 10^4$ times greater than the aspherical mass distribution effect in (22) for $n=2$. Therefore the effect of the Earth's aspherical mass distribution is not revealed in orbital motion, but it shows up in precession and nutation.

For the development of the tidal potential $\hat{\mathbf{s}}\cdot\mathbf{r}$ and $\hat{\mathbf{t}}\cdot\mathbf{r}$ have to be calculated. From Fig. 2 follows:

$$\begin{aligned}\hat{\mathbf{s}}\cdot\mathbf{r}&=r\cos\psi\\ \hat{\mathbf{t}}\cdot\mathbf{r}&=-r\sin\psi\cos\alpha.\end{aligned} \tag{23}$$

Following from (10) and (20) the complete expression for the tidal forcing potential is given by

$$\begin{aligned}V(r,\psi)&=V_d(r,\psi)+\mathbf{r}\cdot\mathbf{b}_H\\ &=\frac{GM_L}{c}\left\{\sum_{n=2}^{\infty}\left(\frac{r}{c}\right)^n P_n(\cos\psi)+\frac{r}{c}\sum_{n=2}^{\infty}J_n\left(\frac{a}{c}\right)^n\right.\\ &\quad\cdot\left[(n+1)P_n(\cos\theta_c)\cos\psi\right.\\ &\quad\left.\left.+\frac{dP_n}{d\theta}\bigg|_{\theta=\theta_c}\sin\psi\cos\alpha\right]\right\}.\end{aligned} \tag{24}$$

The last sum of (24) is caused by the non-spherical mass distribution of the Earth and relates to the homogeneous field $\mathbf{b}_H$ in (20) whereas the first part is the usual tidal forcing potential.

With

$$\cos\theta = \cos\theta_c \cos\psi + \sin\theta_c \sin\psi \cos\alpha \tag{25}$$

$$\cos\psi = \cos\theta\cos\theta_c + \sin\theta\sin\theta_c\cos(\lambda-\lambda_c) \tag{26}$$

$$P_n(\cos\psi) = P_n(\cos\theta)\,P_n(\cos\theta_c) + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta)\,P_n^m(\cos\theta_c)\cdot\cos m(\lambda-\lambda_c) \tag{27}$$

where $P_n^m(\cos\theta)$ are the associated Legendre polynomials, Eq. (24) can be expressed in geocentric coordinates r, θ, λ

$$\begin{aligned} V(r,\theta,\lambda) = {} & \frac{GM_L}{c}\sum_{n=2}^{\infty}\left(\frac{r}{c}\right)^n \\ & \cdot\left\{P_n(\cos\theta)\,P_n(\cos\theta_c) + 2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}\right. \\ & \left.\cdot P_n^m(\cos\theta)\,P_n^m(\cos\theta_c)\cdot\cos m(\lambda-\lambda_c)\right\} \\ & + \frac{GM_L}{c}\frac{r}{c}\sum_{n=2}^{\infty}J_n\left(\frac{a}{c}\right)^n \\ & \cdot\left\{(n+1)\,P_n(\cos\theta_c)\,[\cos\theta\cos\theta_c\right. \\ & + \sin\theta\sin\theta_c\cos(\lambda-\lambda_c)] + \left.\frac{dP_n(\cos\theta)}{d\theta}\right|_{\theta=\theta_c} \\ & \left.\cdot[\cos\theta\sin\theta_c - \sin\theta\cos\theta_c\cos(\lambda-\lambda_c)]\right\}. \end{aligned} \tag{28}$$

By introducing Cartesian coordinates it can be verified that the second sum in (28) determined by the coefficients J_n represents the potential of the homogeneous field $\mathbf{b}_H$.

Numerical Results

For the determination of the flattening effect on the tidal forcing field only the term $n=2$ will be considered in (16) because $J_n/J_2 \lesssim 10^{-2}$ for $n>2$ (Kaula, 1968). With

$$\mathbf{g}_M(\tilde{\mathbf{r}}_0) = \frac{GM_L}{c^2}\hat{\mathbf{s}} \tag{29}$$

$$\begin{aligned} \mathbf{b}_H &= \mathbf{g}_M(\tilde{\mathbf{r}}_0) - \mathbf{g}_0 = \frac{GM_L}{c^2}\hat{\mathbf{s}} + \frac{M_L}{M_E}\mathbf{g}_E(\mathbf{c}) \\ &= \frac{GM_L}{c^2}\left(\frac{a}{c}\right)^2\frac{3}{2}J_2[(3\cos^2\theta_c - 1)\hat{\mathbf{s}} + \sin 2\theta_c\hat{\mathbf{t}}]. \end{aligned} \tag{30}$$

The corresponding term in the tidal potential is given by Eq. (24)

$$\mathbf{r}\cdot\mathbf{b}_H = \frac{GM_L}{c}\left(\frac{a}{c}\right)^2\frac{r}{c}\frac{3}{2}J_2 \cdot[(3\cos^2\theta_c - 1)\cos\psi - \sin 2\theta_c\sin\psi\cos\alpha]. \tag{31}$$

With $a/c = 0.016593$, $M_E/M_L = 81.30$, and GM_E, a and J_2 from the IAG 1980 system of constants (Müller, 1980)

$$\frac{3}{2}\frac{GM_L}{c}\left(\frac{a}{c}\right)^3 J_2 = 9.462\,10^{-5}\ \mathrm{m^2/s^2}. \tag{32}$$

This is about 10% of the $n=4$ term of the normal tidal potential

$$\frac{GM_L}{c}\left(\frac{a}{c}\right)^4 = 9.688\,10^{-4}\ \mathrm{m^2/s^2}. \tag{33}$$

The magnitude of the corresponding force per unit mass given by Eq. (30) is

$$\frac{3}{2}\frac{GM_L}{c^2}\left(\frac{a}{c}\right)^2 J_2 = 1.48\,10^{-11}\ \mathrm{m/s^2} \approx 1\ \mathrm{ngal} \tag{34}$$

that is about 3% of the magnitude of the vertical force per unit mass for $n=4$

$$4\frac{GM_L}{c^2}\left(\frac{a}{c}\right)^3 = 62.3\ \mathrm{ngal}. \tag{35}$$

The first part of the homogeneous field $\mathbf{b}_H$ in (30)

$$\mathbf{b}_s = \tfrac{3}{2}J_2\frac{GM_L}{c^2}\left(\frac{a}{c}\right)^2(3\cos^2\theta_c - 1)\hat{\mathbf{s}} \tag{36}$$

always points away from the Moon since $\cos^2\theta_c < \frac{1}{3}$ for all possible values of the lunar declination $\delta_c = \frac{\pi}{2} - \theta_c$. The second part

$$\mathbf{b}_t = \tfrac{3}{2}J_2\frac{GM_L}{c^2}\left(\frac{a}{c}\right)^2\sin 2\theta_c\hat{\mathbf{t}} \tag{37}$$

changes its sign for $\theta_c = \frac{\pi}{2}$ i.e., when the Moon is in the equatorial plane of the Earth. As $\mathbf{b}_H$ is a homogeneous field it represents a constant tidal forcing field acting simultaneously on the whole Earth at a specific time t. It is of course varying with time.

The periodicities can be determined from the corresponding part of the potential expressed in geocentric coordinates. From the Eqs. (28) and (31) this is

$$\begin{aligned} \mathbf{r}\cdot\mathbf{b}_H = {} & \frac{GM_L}{c}\left(\frac{a}{c}\right)^2\frac{r}{c}\frac{3}{2}J_2\{(3\cos^2\theta_c - 1)(\cos\theta\cos\theta_c \\ & + \sin\theta\sin\theta_c\cos(\lambda-\lambda_c)) \\ & - \sin 2\theta_c[\cos\theta\sin\theta_c - \sin\theta\cos\theta_c\cos(\lambda-\lambda_c)]\}. \end{aligned} \tag{38}$$

The time variations are caused by the variations of c, θ_c and λ_c in Eq. (38). The major inherent periodicities are lunar-daily from $\cos(\lambda-\lambda_c)$, lunar-third-monthly from the products of $\cos\theta_c$ and $\sin\theta_c$ with $\cos^2\theta_c$ and $\sin 2\theta_c$, and monthly from c^{-4}, $\cos\theta_c$ and $\sin\theta_c$.

At present it appears hopeless to search for the described effect in the tidal gravity records because it has an amplitude of only about 1 ngal. It should however be noticed that a homogeneous tidal field may raise deformations in a radially stratified Earth and especially a forced tidal motion of the central core of the Earth into an eccentric position, and it might be suspected that this effect could have significant dynamical consequences. This suspicion must however be discarded because the displacement will be of the order of 1 µm as can be shown by considering the balance between the homogeneous tidal and the repelling gravitational force acting on a solid inner core in a fluid outer core with constant but different densities and an assumed density difference of 2 g/cm^3.

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