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On the influence of ionospheres with non-uniform conductivity distribution on hydromagnetic waves

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Abstract. A theoretical model is proposed which describes the influence of non-uniform ionospheric height-integrated conductivity distributions on ULF-pulsations. The assumption is made that the field-aligned currents carried by the wave-field are closed by polarisation currents in the magnetosphere and by the irrotational part of the height-integrated ionospheric currents. Current continuity at the magnetosphere-ionosphere boundary provides for a differential equation governing the reflected electric field for arbitrary non-uniform conductivity distributions. Model calculations for simple, but realistic conductivity and electric field distributions show that local shifts of the ionospheric field maximum against that of the magnetic field below the ionosphere as well as double-peak distributions of the electric field can occur. Strong electric field anomalies i.e. significant deviations of the electric field distribution as compared with the uniform case occur in conductivity gradient zones and fall off rapidly outside. The previously predicted 90° rotation between the magnetic field below and above the ionosphere does not hold generally because the rotation angle depends strongly on the conductivity gradients.

Key words: Geomagnetic pulsations – Magnetosphere-ionosphere coupling – Ionospheric reflection coefficient – Ionospheric rotation effect.

Introduction

During the last decade much effort, theoretical and experimental, has been made to investigate the nature of geomagnetic pulsations in the pc 4–5 range. One of the specific problems which has been tackled is the influence of the ionosphere on the propagation of hydromagnetic waves coming from the magnetosphere and being recorded at the earth's surface as geomagnetic pulsations. Early attempts to solve this problem have been made by Dungey (1963) and Nishida (1964). Dungey (1963) has been able to show that the part of the disturbance field $\mathbf{b}$ which has a vertical current, i.e. $(\nabla \times \mathbf{b})_Z \neq 0$, is strongly shielded by the ionosphere while that part with $(\nabla \times \mathbf{b})_Z = 0$ is observable at the ground. Nishida (1964) studying the effect of the ionosphere on storm sudden commencements showed that the magnetic field of an incident wave is rotated during passage through the ionosphere. A computed solution of the problem using a realistic distribution of the ionospheric conductivity distribution with height has been given by Inoue (1973) and Hughes (1974). Hughes (1974) and later Hughes and Southwood (1976) followed the line given by Dungey (1963) and were successful in showing that the magnetic field of the incident wave is strongly shielded by the magnetic field effect of the Pedersen current flowing in the ionosphere while the disturbance magnetic field below the ionosphere is mainly due to the ionospheric Hall currents. Thus the polarisation ellipse undergoes a rotation by 90° when passing the ionosphere (Hughes, 1974).

In all the above referenced work it has been assumed that the ionospheric conductivity distribution is uniform in planes perpendicular to the ambient magnetic field. This is far from being a realistic assumption especially in the auroral zone or at the terminator between sunlit and dark regions in the ionosphere where strong conductivity gradients may occur (see for example, Vickrey et al., 1981).

Recently Saka et al. (1982) gave evidence that polarisation characteristics of low-latitude pulsations are severely changed at sunrise i.e. when strong conductivity gradients occur. They report that the magnetic *D*-component is much smaller than the *H*-component before sunrise and is increased at sunrise to values comparable with the *H*-component. Doupnik et al. (1977) and Walker et al. (1979) were able to predict the *H*-component of ground-observed pc 5 pulsations from measured values of the ionospheric electric field and the associated ionospheric currents but failed in reproducing the *D*-component. This failure may be due to the conductivity gradients occurring in the auroral zone.

The aim of the present analysis is to work out a theoretical basis necessary to investigate the influence of ionospheres with horizontally non-uniform conductivity distribution on the reflection of MHD-waves (see also Glaßmeier, 1983) and on ULF-pulsations at the ground. A similar attempt to tackle the problem has recently been undertaken by Ellis and Southwood (1983). However, their approach allows only the study of the effect of conductivity discontinuities while the present work holds for arbitrary distributions of the heigh-integrated ionospheric conductivity.

First the model and its basic equations are discussed while the next two sections are devoted to the ionospheric reflection coefficient and the apparent rotation angle of the wave polarisation ellipse between regions above and below the ionosphere. To demonstrate the compatibility of results of the model used with those of previous studies the continuation of the wave magnetic field away from the ionosphere will then be discussed and the generation of a fast-mode wave (see also Hughes, 1974) at the upper boundary of the ionosphere due to the reflection process will be demonstrated. Results of numerical analyses for some realistic distributions of the conductivity and of the electric field of the incident wave are shown and their consequences for ground-based observations of geomagnetic pulsations are discussed.

The model and basic equations

The coordinate system adopted for the following calculations is a rectangular one with its x-axis directed to the north, the y-direction pointing towards east and the z-direction positively downwards. In the coordinate system the ionosphere will be represented by a highly conducting sheet lying in the horizontal plane at a height $-h$ and with integrated Hall and Pedersen conductivities Σ_H and Σ_P, respectively. Above this infinitely thin sheet ionosphere we assume a uniform semi-infinite hydromagnetic region as a model-magnetosphere while the region below the ionosphere is regarded as a semi-infinite non-conducting atmosphere.

In the ionosphere the electric field $\mathbf{E}$ of an Alfvén-wave generates a sheet current $\mathbf{I}(x, y)$ with

$$\mathbf{I}=\Sigma_P\,\mathbf{E}-\Sigma_H\,\mathbf{E}\times\mathbf{e}_B \tag{1}$$

where both, Σ_P and Σ_H, are arbitrary functions of the horizontal coordinates and $\mathbf{e}_B$ is a unit vector parallel to the ambient magnetic field $\mathbf{B}_0$ being normal to the plane ionosphere i.e. parallel to the z-axis. Displacement currents have been neglected in (1). Using a general theorem of vector analysis $\mathbf{I}$ can be split up into two current systems (for a discussion of the applicability of the Helmholtz theorem to planar vector fields see Duschek and Hochrainer, 1961)

$$\mathbf{I}=\mathbf{I}_{irr}+\mathbf{I}_{sf} \tag{2}$$

with $\mathbf{I}_{irr}$ being the irrotational part and $\mathbf{I}_{sf}$ being the source free part of the sheet current system $\mathbf{I}$. The following relations hold for $\mathbf{I}$, $\mathbf{I}_{irr}$, and $\mathbf{I}_{sf}$:

$$\begin{aligned}
&\nabla\cdot\mathbf{I}=\nabla\cdot\mathbf{I}_{irr}=\Sigma_P\,\nabla\cdot\mathbf{E}-(\nabla\Sigma_H\times\mathbf{E})_Z+\nabla\Sigma_P\cdot\mathbf{E}\\
&(\nabla\times\mathbf{I})_Z=(\nabla\times\mathbf{I}_{sf})_Z=\Sigma_H\,\nabla\cdot\mathbf{E}+(\nabla\Sigma_P\times\mathbf{E})_Z+\nabla\Sigma_H\cdot\mathbf{E}\\
&\nabla\cdot\mathbf{I}_{sf}=0\\
&(\nabla\times\mathbf{I}_{irr})_Z=0.
\end{aligned} \tag{3}$$

For the derivation of the above equations it has been assumed that $(\nabla\times\mathbf{E})_Z=-i\omega b_Z\cong 0$. Such an assumption is justified if the incident wave is in the transverse-mode i.e. there is no b_Z-component in the wave magnetic field. The assumption $(\nabla\times\mathbf{E})_Z=0$ is also justified in the more general case when $b_Z\neq 0$. Assuming $b_Z\sim 10\,\mathrm{nT}$ and $\omega\sim 2\pi/300\,\mathrm{s}^{-1}$ which is typical for a pc5 pulsation $|(\nabla\times\mathbf{E})_Z|$ is estimated to be of the order $10^{-10}\,\mathrm{V/m^2}$. This must be compared with values of expressions like $\nabla\cdot\mathbf{E}$ which, assuming a horizontal scale length $\sim 100\,\mathrm{km}$ and $E_{x,y}\sim 20\,\mathrm{mV/m}$ (cf. Walker et al., 1979) are of the order $10^{-7}\,\mathrm{V/m^2}$. Thus $(\nabla\times\mathbf{E})_Z=0$ is an also suitable approximation in the more general case. In the hydromagnetic region above the ionosphere the transverse wave electric field $\mathbf{E}$ is related to a polarisation current density $\mathbf{j}_P$ (cf. Boström, 1972; Chen, 1974) by

$$\mathbf{j}_P=\frac{1}{\mu_0 v_A^2}\cdot\frac{\partial\mathbf{E}}{\partial t} \tag{4}$$

where v_A is the Alfvén-velocity. This polarisation current is not necessarily source free and the wave therefore carries a significant vertical current (cf. Hughes and Southwood, 1976; Greenwald and Walker, 1980). Using Ampère-Maxwell's law

$$(\nabla\times\mathbf{b})_Z=\mu_0 j_{\parallel} \tag{5}$$

and

$$(\nabla\times\mathbf{E})_{\perp}=-\frac{\partial b_{\perp}}{\partial t} \tag{6}$$

where the subscript $\perp$ denotes the component transverse to the ambient field $\mathbf{B}_0$, one gets the relation

$$\frac{\partial}{\partial z}(\nabla\cdot E)=-\mu_0\frac{\partial}{\partial t}j_{\parallel} \tag{7}$$

assuming $E_z=0$, or, with a time dependence $\sim\exp(i\omega t)$

$$j_{\parallel}=\nabla\cdot\left(\frac{i}{\mu_0\,\omega}\,\frac{\partial\mathbf{E}}{\partial z}\right). \tag{8}$$

Thus we have found a relation between the wave-field associated field-aligned current and the transverse wave electric field $\mathbf{E}$.

The ionospheric reflection coefficient

Just above the thin sheet ionosphere the field-aligned current distribution carried by the wave must fit with that due to the irrotational part of the height-integrated ionospheric currents because of current continuity reasons. Because

$$\nabla\cdot\mathbf{I}=\nabla\cdot\mathbf{I}_{irr}=j_{\parallel} \tag{9}$$

and with (3) and (8) one has at the height of the ionosphere

$$\Sigma_P\,\nabla\cdot\mathbf{E}-(\nabla\Sigma_H\times\mathbf{E})_Z+\nabla\Sigma_P\cdot\mathbf{E}=\nabla\cdot\left(\frac{i}{\mu_0\,\omega}\,\frac{\partial}{\partial z}\,\mathbf{E}\right). \tag{10}$$

Taking into account that $\mathbf{E}=\mathbf{E}_I+\mathbf{E}_R$ where $\mathbf{E}_I$ and $\mathbf{E}_R$ are the electric fields of the incident and the reflected wave, respectively, and assuming a variation along z by $\exp(ik_z z)$ with $k_z>0$ or $k_z<0$, depending on the propagation direction of the down- or upgoing wave, one finds from (10)

$$\begin{aligned}
&\Sigma_P\,\nabla\cdot(\mathbf{E}_I+\mathbf{E}_R)-(\nabla\Sigma_H\times(\mathbf{E}_I+\mathbf{E}_R))_Z+\nabla\Sigma_P\cdot(\mathbf{E}_I+\mathbf{E}_R)\\
&=\Sigma_W\;\nabla\cdot(\mathbf{E}_I-\mathbf{E}_R)
\end{aligned} \tag{11}$$

where a wave conductivity, $\Sigma_W=1/\mu_0 v_A$, has been defined (cf. Maltsev et al., 1974; Mallinckrodt and Carlson, 1978). Rearrangement of this equation gives

$$
\begin{aligned}
&(\Sigma_P+\Sigma_W)\,\nabla\cdot\mathbf{E}_R+\nabla\Sigma_P\cdot\mathbf{E}_R-(\nabla\Sigma_H\times\mathbf{E}_R)_Z\\
&=(\Sigma_W-\Sigma_P)\,\nabla\cdot\mathbf{E}_I-\nabla\Sigma_P\cdot\mathbf{E}_I+(\nabla\Sigma_H\times\mathbf{E}_I)_Z
\end{aligned}
\tag{12}
$$

which is a differential equation for the determination of $\mathbf{E}_R$ if $\mathbf{E}_I$, Σ_H, Σ_P, and Σ_W are given quantities.

If the ionospheric conductivity is uniform (12) reduces to the simple relation earlier derived by Maltsev et al. (1974) (cf. also Hughes, 1974; Ellis and Southwood, 1983):

$$
\mathbf{E}_R=\frac{\Sigma_W-\Sigma_P}{\Sigma_W+\Sigma_P}\,\mathbf{E}_I. \tag{13}
$$

Two situations may be regarded for which the solution of Eq. (12) is straight forward. Let us first assume that $(\nabla\Sigma_{P,H}\times\mathbf{E}_I)_Z=0$. As may be easily proved the incident and the reflected electric field for this specific case are related by Eq. (13) but now Σ_P depending on the spatial coordinates. The physical reason for the relationship found is that the Hall currents are now source free and the Pedersen currents are now curl free as in the uniform case (cf. Eqs. 3).

Equation (13) is also the solution of (12) if $\nabla\Sigma_P\cdot\mathbf{I}=0$ and the ratio Σ_H/Σ_P is everywhere constant. In this case the divergence of the Hall- and Pedersen currents due to the non-uniform conductivity distribution cancel each other and the sum of the second and third term on the left side in Eq. (11) vanishes as may be easily shown. Then (12) reduces to the equation valid in the uniform case and this again yields (13).

For more general cases it is useful to represent the reflected electric field just above the ionosphere as the gradient of a scalar function, $\mathbf{E}_R=(\nabla\phi)_\perp$, which is possible because of $(\nabla\times\mathbf{E}_R)_Z=0$. Then it follows from (12)

$$
\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\delta\,\frac{\partial\phi}{\partial x}+\varepsilon\,\frac{\partial\phi}{\partial y}=c(x,y) \tag{14}
$$

with

$$
\begin{aligned}
\delta&=\left(\frac{\partial\Sigma_P}{\partial x}+\frac{\partial\Sigma_H}{\partial y}\right)\Big/(\Sigma_W+\Sigma_P)\\
\varepsilon&=\left(\frac{\partial\Sigma_P}{\partial y}-\frac{\partial\Sigma_H}{\partial x}\right)\Big/(\Sigma_W+\Sigma_P)\\
c(x,y)&=((\Sigma_W-\Sigma_P)\,\nabla\cdot\mathbf{E}_I\\
&\quad-\nabla\Sigma_P\cdot\mathbf{E}_I+(\nabla\Sigma_H\times\mathbf{E}_I)_Z)/(\Sigma_W+\Sigma_P).
\end{aligned}
\tag{15}
$$

For any realistic distribution of the Hall and Pedersen conductivities and of the electric field of an incident wave (14) must be solved numerically.

The ionospheric rotation effect

In earlier analyses about the influence of the ionosphere on hydromagnetic waves by Nishida (1964), Inoue (1973) and especially Hughes (1974) it has been pointed out that magnetic fields due to a transverse wave incident from the magnetosphere are seen rotated through 90° when observed on the ground. Because this apparent rotation has been regarded theoretically by the above mentioned authors only in the case of a uniform ionospheric conductivity distribution, the problem must be treated again for a non-uniform conductivity.

As may be seen from the above considerations concerning the reflection process at the ionosphere $\mathbf{I}_{irr}$ serves as a closure current of the polarisation and field-aligned currents of the wave, and the current system consisting of the magnetospheric currents and $\mathbf{I}_{irr}$ constitute a poloidal current system which produces no magnetic effect below the ionosphere (cf. Boström, 1964; Vasyliunas, 1970; Fukushima, 1976). Only the source free current $\mathbf{I}_{sf}$ contributes to the magnetic field at the ground, and just below the ionosphere $\mathbf{I}_{sf}$ is related to the atmospheric magnetic field $\mathbf{b}_A$ by (cf. Chapman and Bartels, 1940)

$$
\mathbf{I}_{sf}=-\frac{2}{\mu_0}\,\mathbf{b}_A\times\mathbf{e}_B \tag{16a}
$$

from which one has

$$
(\nabla\times\mathbf{I}_{sf})_Z=\frac{2}{\mu_0}(\nabla\cdot\mathbf{b}_A)_T \tag{16b}
$$

where 'T' denotes that only the derivatives transverse to the ambient magnetic field $\mathbf{B}_0$ are regarded.

Above the ionosphere the field-aligned current density $\mathbf{j}_\parallel$ is related to the magnetic field $\mathbf{b}_M$ of a transverse mode wave and by virtue of Ampère-Maxwell's law (5) and $\nabla\cdot\mathbf{I}_{irr}=j_\parallel$ the following relation holds

$$
\nabla\cdot\mathbf{I}_{irr}=\frac{1}{\mu_0}(\nabla\times\mathbf{b}_M)_Z \tag{17a}
$$

and because $\mathbf{b}_M$ is the magnetic field of an Alfvén-wave i.e. $(\nabla\cdot\mathbf{b}_M)_T=0$ holds, $\mathbf{I}_{irr}$ is everywhere perpendicular to $\mathbf{b}_M$ in the horizontal plane just above the thin sheet ionosphere:

$$
\mathbf{I}_{irr}=\frac{1}{\mu_0}\,\mathbf{b}_M\times\mathbf{e}_B. \tag{17b}
$$

Schematically the situation is sketched in Fig. 1. It should be noted that $\mathbf{b}_M$ is not the total magnetic field just above the ionosphere. Also the source free part of

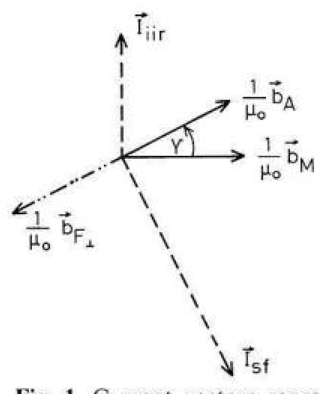

Fig. 1. Current vectors representing the source free, $\mathbf{I}_{sf}$, and the irrotational, $\mathbf{I}_{irr}$, part of an ionospheric height integrated sheet current system together with associated magnetic field vectors just below, $\mathbf{b}_A$, and just above, $\mathbf{b}_M$, the ionosphere. $(\mathbf{b}_F)_\perp$ represents the magnetic field just above the ionosphere due to $\mathbf{I}_{sf}$

the height-integrated ionospheric current system, $\mathbf{I}_{sf}$, contributes to the magnetic field above the ionosphere. As is shown in the following section $\mathbf{I}_{sf}$ is associated with a fast mode wave whose amplitude decreases rapidly away from the ionosphere and thus need not to be regarded when discussing the rotation between the atmospheric magnetic field and the magnetospheric magnetic field far away from the ionosphere.

It follows from (16a) and (17b) that both $\mathbf{b}_A$ and $\mathbf{b}_M$ are everywhere perpendicular to $\mathbf{I}_{sf}$ and $\mathbf{I}_{irr}$, respectively (see also Fig. 1). Thus the angle γ by which $\mathbf{b}_A$ is rotated with respect to $\mathbf{b}_M$, counted counterclockwise if viewed from above, is the angle by which $\mathbf{I}_{sf}$ is rotated with respect to $-\mathbf{I}_{irr}$ counted counterclockwise if viewed from above. Due to this the determination of the ionospheric rotation angle is reduced to the determination of the irrotational and source free parts of the total ionospheric current system.

The height-integrated ionospheric current, $\mathbf{I}$, can easily be computed after the reflected electric field has been determined (see above). The irrotational part of $\mathbf{I}$ is then given by

$$\mathbf{I}_{irr}(\mathbf{r})=\frac{1}{2\pi}\iint_S \frac{(\nabla\cdot\mathbf{I})(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\,dS \tag{18a}$$

(there exist certain, usually fullfilled, conditions on the kernel for the integral to exist which are different for a plane vector field than for a vector field in space; for details reference is made to Duschek and Hochrainer, 1961) and for $\mathbf{I}_{sf}$ one has

$$\mathbf{I}_{sf}=\mathbf{I}-\mathbf{I}_{irr}. \tag{18b}$$

From (18a) and (18b) the rotation angle γ can easily be computed.

In the case of a uniform ionosphere or if there are no gradients of the conductivity perpendicular to $\mathbf{E}$, $\mathbf{I}_{irr}$ corresponds to the Pedersen current and $\mathbf{I}_{sf}$ to the Hall current and thus $\mathbf{b}_A$ is rotated with respect to $\mathbf{b}_M$ by 90° counterclockwise if viewed from above.

For the uniform case it follows from (16b) and (17a) using the upper two equations of (3)

$$(\nabla\cdot\mathbf{b}_A)_T\sim(\nabla\times\mathbf{b}_M)_Z. \tag{19}$$

This relation is similar to the ones earlier used by Hasegawa and Lanzerotti (1978) to explain the ionospheric rotation effect.

The above considerations show that results of the present work are consistent with earlier results by Nishida (1964), Inoue (1973) and Hughes (1974).

The fast mode

As pointed out above $\mathbf{I}_{sf}$ also contributes to the magnetic field above the ionosphere and according to Eq. (16) we have just above the ionosphere:

$$(\mathbf{b}_F)_\perp=\frac{\mu_0}{2}\mathbf{I}_{sf}\times\mathbf{e}_B \tag{20a}$$

and

$$(\nabla\cdot\mathbf{b}_F)_T=-\frac{\mu_0}{2}(\nabla\times\mathbf{I}_{sf})_Z. \tag{20b}$$

Because of (20) $(\nabla\times\mathbf{b}_F)_Z=0$ at $z=-h$ and the wave launched at the ionosphere due to $\mathbf{I}_{sf}$ and with the magnetic field $\mathbf{b}_F$ carries no field-aligned current i.e. the polarisation currents flowing are source free. Some insight into the nature of this wave is gained by introducing a vector potential $\mathbf{A}$ with $\mathbf{b}_F=\nabla\times\mathbf{A}$ so that one gets $(\nabla\times\nabla\times A)=0$ or, using the Couloumb gauge,

$$\Delta A_Z=0. \tag{21}$$

Assuming a spatial variation of $\mathbf{A}$ and $\mathbf{b}_F$ with $\exp(i(k_x x+k_y y)+k_z z)$ one can easily see from (21) that $\mathbf{b}_F$ is decreases exponentially away from the ionosphere i.e. it is a surface wave.

From the Henry-Faraday law one has

$$(\nabla\cdot\mathbf{b}_F)_T=\frac{i}{\omega}\frac{\partial}{\partial z}(\nabla\times\mathbf{E}_F)_Z \tag{22}$$

and because of (20b) and $(\nabla\times\mathbf{I}_{sf})\neq0$ $(\nabla\times\mathbf{E}_F)_Z$ does not vanish and thereby $b_{F_z}\neq0$ above the ionosphere. This shows that $\mathbf{b}_F$ and $\mathbf{E}_F$ must be the magnetic and electric field, respectively, of a fast mode wave (see also Hughes and Southwood, 1976).

Due to the continuity of the tangential component of the electric field across the magnetosphere-ionosphere boundary $\mathbf{E}_F$ contributes to the total ionospheric electric field. From (20b) and (22) one has

$$\mathbf{I}_{sf}=i\frac{2}{\mu_0\,\omega}\frac{\partial}{\partial z}\mathbf{E}_F \tag{23}$$

or, if $\nabla\Sigma_{P,H}=0$, i.e. $\mathbf{I}_{sf}$ corresponds to the Hall current, and using (23)

$$E_{F_x}^K=-i\frac{\mu_0\,\omega}{2\sqrt{k_x^2+k_y^2}}\Sigma_H E_y^K \tag{24a}$$

$$E_{F_y}^K=i\frac{\mu_0\,\omega}{2\sqrt{k_x^2+k_y^2}}\Sigma_H E_x^K. \tag{24b}$$

With typical values of

$$\Sigma_H\sim10\,\mathrm{S},\quad \sqrt{k_x^2+k_y^2}\sim10^{-6}\,\mathrm{m}^{-1}$$

and $\omega\sim1/300\,\mathrm{s}^{-1}$ one has $|\mathbf{E}|\sim40\cdot|\mathbf{E}_F|$, and thus $\mathbf{E}_F$ can be neglected when compared with $\mathbf{E}$.

The transverse component of the magnetic field of the fast mode, $(\mathbf{b}_F)_\perp$ cannot be neglected just above the ionosphere when compared with $\mathbf{b}_M$. For a uniform ionosphere it follows from (17b) and (20a)

$$\frac{|(\mathbf{b}_F)_\perp|}{|\mathbf{b}_M|}=\frac{\Sigma_H}{2\Sigma_P}. \tag{25}$$

The downward continuation

Equation (16a) can be used to get some more insight into the properties of the wave field below the ionosphere. As may easily be verified from (16a) $\mathbf{b}_A$ is a curl free vector field. This can be represented as the gradient of a scalar potential W and one gets

$$\Delta W=\frac{\mu_0}{2}(\nabla\times\mathbf{I}_{sf})_Z \tag{26}$$

at $z=-h$. Further down in the atmosphere we have Laplace's equation $\Delta W=0$ because no currents are flowing there.

Assuming a horizontal variation of W with $\exp(ik_x x + ik_y y)$ one finds from Laplace equation $k_z=\sqrt{k_x^2+k_y^2}$ where k_z is the vertical wave number and it has been assumed that W vanishes at $z=+\infty$. Therefore, each spectral component of the magnetic potential W or of $\mathbf{b}_A$ will be damped according to the factor $\exp(-k_z(z+h))$ when doing the downward continuation of the wave field away from the ionosphere. This, of course, is a well known fact from potential theory and is widely used in the reverse way i.e. upward continuation of ground-based magnetic fields towards the ionospheric sources (cf. Siebert and Kertz, 1957; Weaver, 1964; Mersmann et al., 1979).

In the case of a uniform conductivity distribution or if $(\nabla\Sigma_{P,H}\times\mathbf{E}_I)_Z=\mathbf{0}$ $\mathbf{I}_{sf}$ and $\mathbf{I}_{irr}$ are identical to the Hall and Pedersen currents, respectively, and one has just above and below the ionosphere

$$\mathbf{b}_M=-\mu_0\,\Sigma_P\,\mathbf{E}\times\mathbf{e}_B \tag{27a}$$

$$\mathbf{b}_A=\frac{\mu_0}{2}\,\Sigma_H\,\mathbf{E}. \tag{27b}$$

From this and using the above described downward continuation a simple relation between the magnetospheric magnetic field $\mathbf{b}_M$ and the magnetic field $\mathbf{b}_G$ measured at the ground is found

$$\frac{b_G^K}{b_M^K}=\frac{1}{2}\frac{\Sigma_H}{\Sigma_P}\exp(-k_z h) \tag{28}$$

where K denotes the spectral component with horizontal wave numbers k_x and k_y. The above relation is in accord with an expression derived earlier by Hughes and Southwood (1976). The exponential decay of each spectral component follows as a natural consequence of the magnetic field below the ionosphere being solely due to the source free part of the ionospheric currents. In general the factor $\Sigma_H/2\Sigma_P$ must be replaced by the ratio $|\mathbf{I}_{sf}|/|2\cdot\mathbf{I}_{irr}|$ as given by Eqs. (18)

Model calculations

The Reflection at the Ionosphere

In all the following model calculations the electric and magnetic fields as well as the corresponding currents vary with time $\sim\exp(i\omega t)$ and only the real part is regarded. Furthermore all model situations regarded are either shown for $t=0$ or $t=T/4$ and thus represent snapshots of the physical situation assumed.

Let us first assume model situations where $(\nabla\Sigma_{P,H}\times\mathbf{E}_I)_Z$ vanishes everywhere and thus Eq. (13) holds. As has been shown by Hughes (1974) or Newton et al. (1978) the ionospheric reflection coefficient is typically very large and chosen for our calculations to be ~ 0.98 which gives one with a typical value $\Sigma_P=4$ S a wave conductivity $\Sigma_W=5\cdot 10^{-2}$ S. Furthermore a ratio $\Sigma_H/\Sigma_P=2$ is assumed. With these values we find from (13), because of $\Sigma_W\ll\Sigma_P$ a simple relation for the total ionospheric electric field and the magnetic field just below the ionosphere:

$$\mathbf{E}\cong\frac{2\Sigma_W}{\Sigma_P}\,\mathbf{E}_I \tag{29a}$$

$$\mathbf{b}_A\cong 2\mu_0\,\Sigma_W\,\mathbf{E}_I. \tag{29b}$$

The magnetic field below the ionosphere thus depends mainly on $\mathbf{E}_I$ and best represents the structure of the incident electric field while the total ionospheric electric field depends on both the Pedersen conductivity and the electric field of the incident wave. This is of some importance when comparing ground-magnetic observations with measurements of the ionospheric electric field.

As examples, two different model situations are considered (Fig. 2 and Fig. 3). Following results by Walker et al. (1979) who found that the ionospheric electric field of pc 5 pulsations is much larger in the N–S component than in the E–W component and is confined in latitudinal direction to a very narrow region, in each case considered, the model electric field of an incident wave is assumed to have only a N–S component which varies in the N–S direction like a Gaussian function and is constant in the E–W direction. For both model situations the assumed conductivity distribution is of parabolic shape and also varies only in the N–S direction.

For the first situation assumed (Fig. 2), outside the region of high conductivity, the constant value 0.5 S for the Pedersen conductivity is chosen. Using Eqs. (13) and (27) the total ionospheric electric field and the magnetic field just below the ionosphere has been computed. Note that from Eqs. (27) it can be seen that the N–S component of the magnetic field just below the ionosphere is the same as the E–W component of the magnetic field above the ionosphere due to our choice $\Sigma_H/\Sigma_P=2$.

As one expects from (29b) the magnetic field reflects the structure of the incident electric field. It is also easily understandable from (29a) that the maximum of the total ionospheric electric field is significantly shifted with respect to that of the incident electric field and therefore also with respect to that of the magnetic field. The reason for this is the shift of the maximum of the conductivity distribution against that of the incident electric field. Observational evidence for the situation as displayed in Fig. 2 has recently been given by Poulter et al. (1982) reporting about coordinated magnetic and electric field measurements with the TRIAD satellite and the STARE radar, respectively. Glaßmeier et al. (1981) also reported about shifts of the spatial maxima of ground-magnetic and ionospheric electric field observations.

The conductivity distribution for the second model (Fig. 3) is much broader and has the constant value 0.1 S outside the high conductivity region. As a result of Eq. (13), the distribution of the total electric field shows a clear double-peak distribution. The appearance of the secondary maximum depends critically on the electric field and the conductivity distribution chosen, especially where both $\mathbf{E}_I$ and Σ_P are very small. Thus double-peak distributions are not expected to be a regular feature observable in the ionospheric electric field of ULF-pulsations. However, Nielsen and Allan (1983)

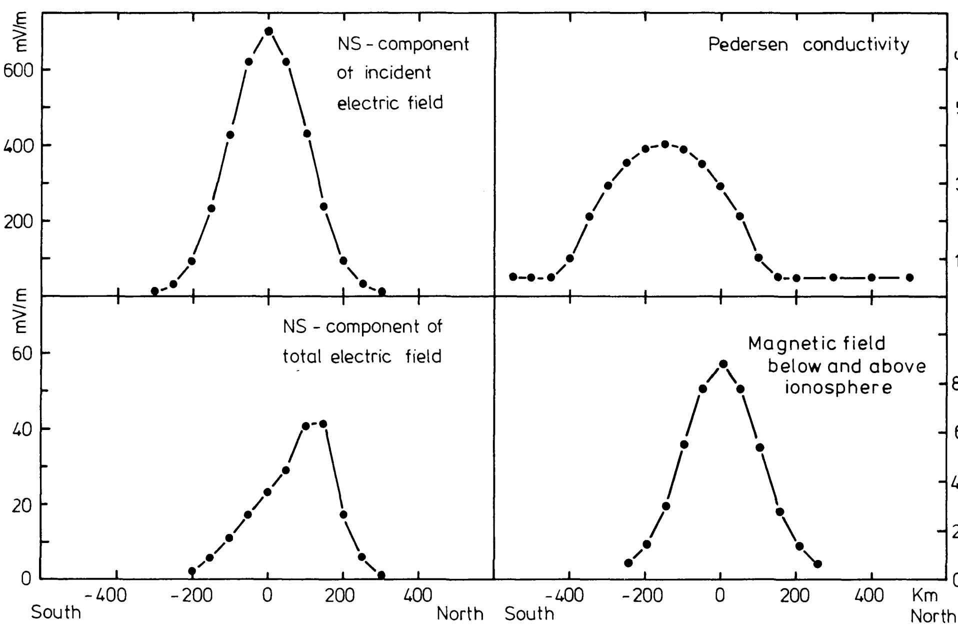

Fig. 2. N-S profiles of the N-S component of the electric field of the incident wave, the adopted model Pedersen conductivity, the resulting N-S component of the total ionospheric electric field and the associated N-S component of the magnetic field just below the ionosphere which is identical to the E-W component of the magnetic field above the ionosphere because $\Sigma_H/\Sigma_P = 2$ is assumed

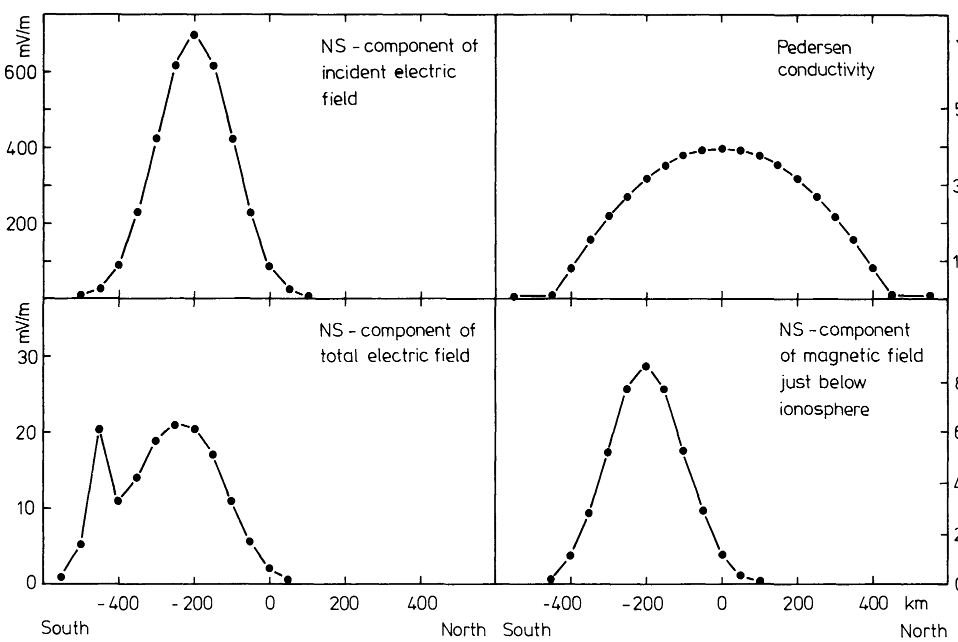

Fig. 3. As Fig. 2 but for a broader conductivity distribution and $\Sigma_P = 0.1$ S outside the high conductivity region

report a rare double-resonance pulsation event and discuss the observed latitude variation of the ionospheric electric field in terms of a special magnetospheric plasma density distribution. Figure 3 shows that an alternative explanation is possible when analysing the effect of the ionosphere on ULF-waves in more detail.

To consider the effect of conductivity gradients perpendicular to the incident electric field, Eq. (14) must be solved numerically. A successive overrelaxation procedure (cf. Mitchell and Griffiths, 1980) has been used on a 80×40 points grid with 80 points in the E-W direction and a spacing of 25 km between adjacent points. The electric field of the incident wave used has only a N-S component, is constant in the E-W direction and varies in the N-S direction in accord with the fieldline resonance theory (cf. Tamao, 1965; Southwood, 1974). The amplitude variation and the spatial phase are displayed in the top panel of Figure 4 and the situation discussed is a snapshot at the time $t = 0$. The conductivity distributions regarded in the following examples change in E-W direction and are constant in N-S direction. As boundary values of Eq. (14) solutions of Eq. (13) have been used.

For different ratios of Σ_H/Σ_P the N-S component of the reflected and total electric field along an E-W profile north of the maximum of the incident electric

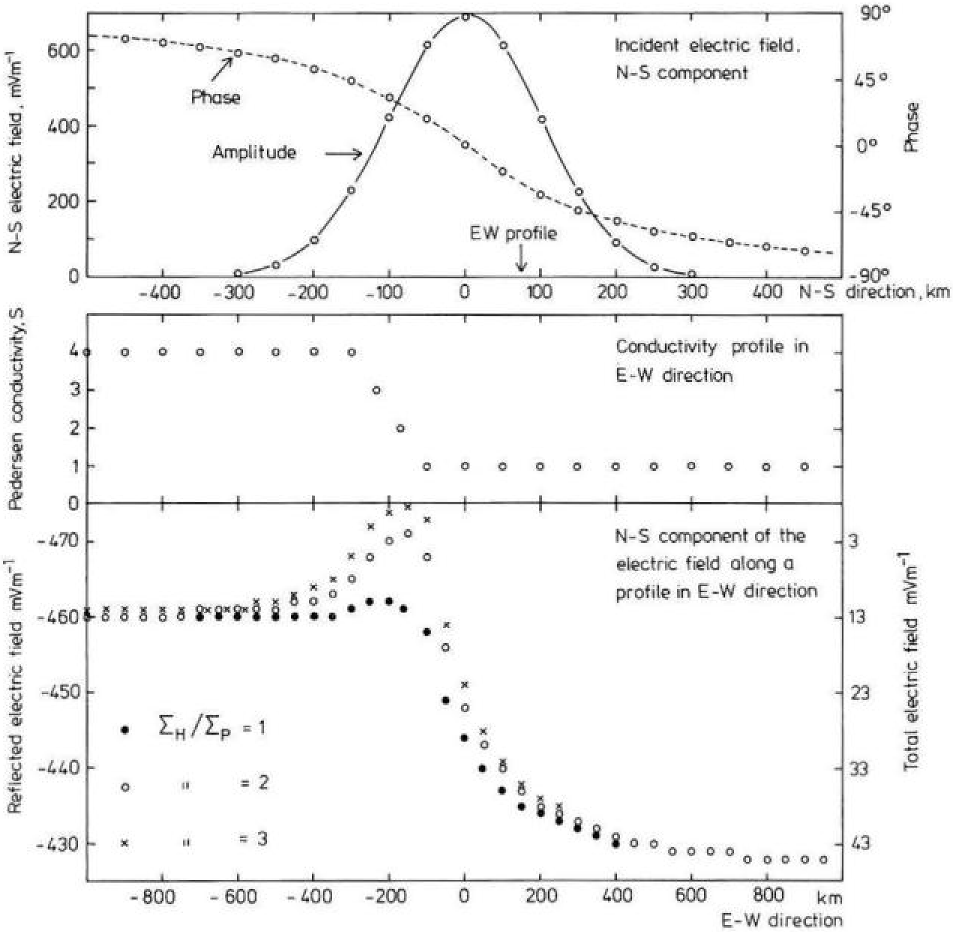

Fig. 4. An N–S profile of the amplitude and spatial phase of the incident electric field (*top panel*), and E–W profile of the ionospheric Pedersen conductivity variation (*center panel*) and an E–W profile of the N–S component of the resulting reflected and total electric field (*bottom panel*) for different ratios of Σ_H/Σ_P. A time variation of the incident electric field $\exp(i\omega t)$ is assumed and only the real part is regarded for $t=0$. The arrow in the top panel indicates the location of the E–W profile in the bottom panel north of the maximum of the incident field

field (see the arrow in the top panel) is displayed in Fig. 4. Far away from the gradient zone the reflected electric field is as in the uniform case and is given by Eq. (13). Thus our apriori assumption for the boundary values is justified by this result and will be used for all further model calculations. Approaching the gradient zone the reflected electric field increases, with the increase depending on the Σ_H/Σ_P ratio. Outside the gradient zone the electric field decreases rapidly to values given by Eq. (13).

To understand this behaviour of the electric field a more detailed discussion of the physics of the reflection process is necessary. If the incident Alfvén-wave approaches a uniform ionosphere an ionospheric current system with $\nabla\cdot\mathbf{I}_I=\Sigma_P\nabla\cdot\mathbf{E}_I$ builds up and field-aligned currents $j_{\parallel}^P=\Sigma_P\nabla\cdot\mathbf{E}_I$ are necessary to balance the system. However, these necessary field-aligned currents $j_{\parallel}^P$ cannot be carried by the wave-field whose field-aligned currents are given by $j_{\parallel}=\Sigma_W\nabla\cdot\mathbf{E}_I$. Thus polarisation charges build up which produce the electric field of the reflected wave, $\mathbf{E}_R$. This electric field also is associated with necessary field-aligned currents having the reverse sign as $j_{\parallel}^P$ and a size suitable to balance the field-aligned current situation at the ionosphere: the sum of the necessary field-aligned currents, $\Sigma_P\cdot(\nabla\cdot\mathbf{E}_I+\nabla\cdot\mathbf{E}_R)$, must equal those carried by the wave, $\Sigma_W\cdot(\nabla\cdot\mathbf{E}_I-\nabla\cdot\mathbf{E}_R)$ (see also Eq. (10)).

Now, if there are additional necessary field-aligned currents $j_{\parallel}^H$, associated with the incident wave due to conductivity gradients (see Fig. 5) the reflected electric field distribution necessary to balance the field-aligned currents at the ionosphere must change with respect to the uniform case. For the situation presented schematically in Fig. 5 and whose numerical solution is given in Fig. 4 the gradient of the Hall conductivity increases the field-aligned currents north of the maximum of the incident electric field and thus the reflected electric field must also be increased. South of the maximum the gradient associated necessary field-aligned currents $j_{\parallel}^H$ lower those associated with the divergence of the incident electric field and the reflected electric field is expected to be decreased. This discussion of the problem holds qualitatively only because we have not included the effect of the E–W component which appears

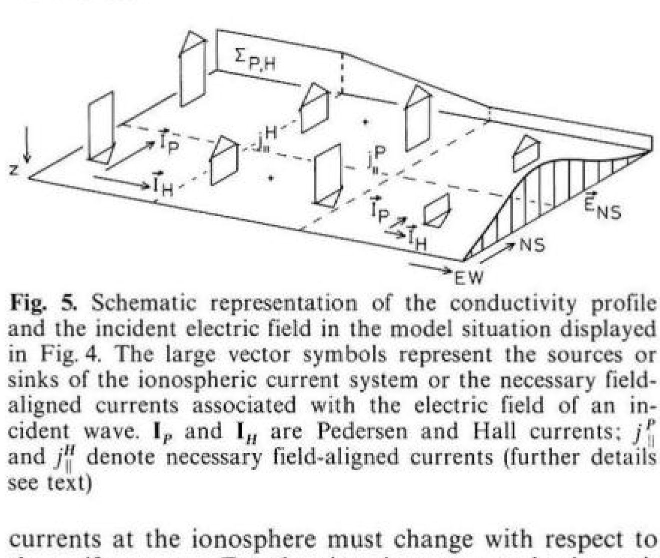

Fig. 5. Schematic representation of the conductivity profile and the incident electric field in the model situation displayed in Fig. 4. The large vector symbols represent the sources or sinks of the ionospheric current system or the necessary field-aligned currents associated with the electric field of an incident wave. $\mathbf{I}_P$ and $\mathbf{I}_H$ are Pedersen and Hall currents; $j_{\parallel}^P$ and $j_{\parallel}^H$ denote necessary field-aligned currents (further details see text)

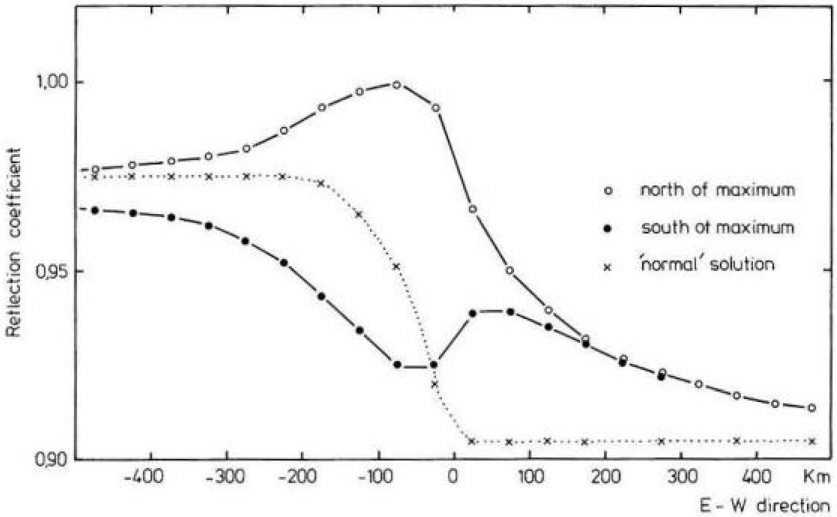

Fig. 6. E–W profiles of the reflection coefficient north and south of the maximum of the incident electric field (see arrow in the top panel of Fig. 4; the southern location is symmetric to the northern one) together with the 'normal' solution for the reflection coefficient as given by Eq. (13). The reflection coefficient is given for $\Sigma_H/\Sigma_P=2$

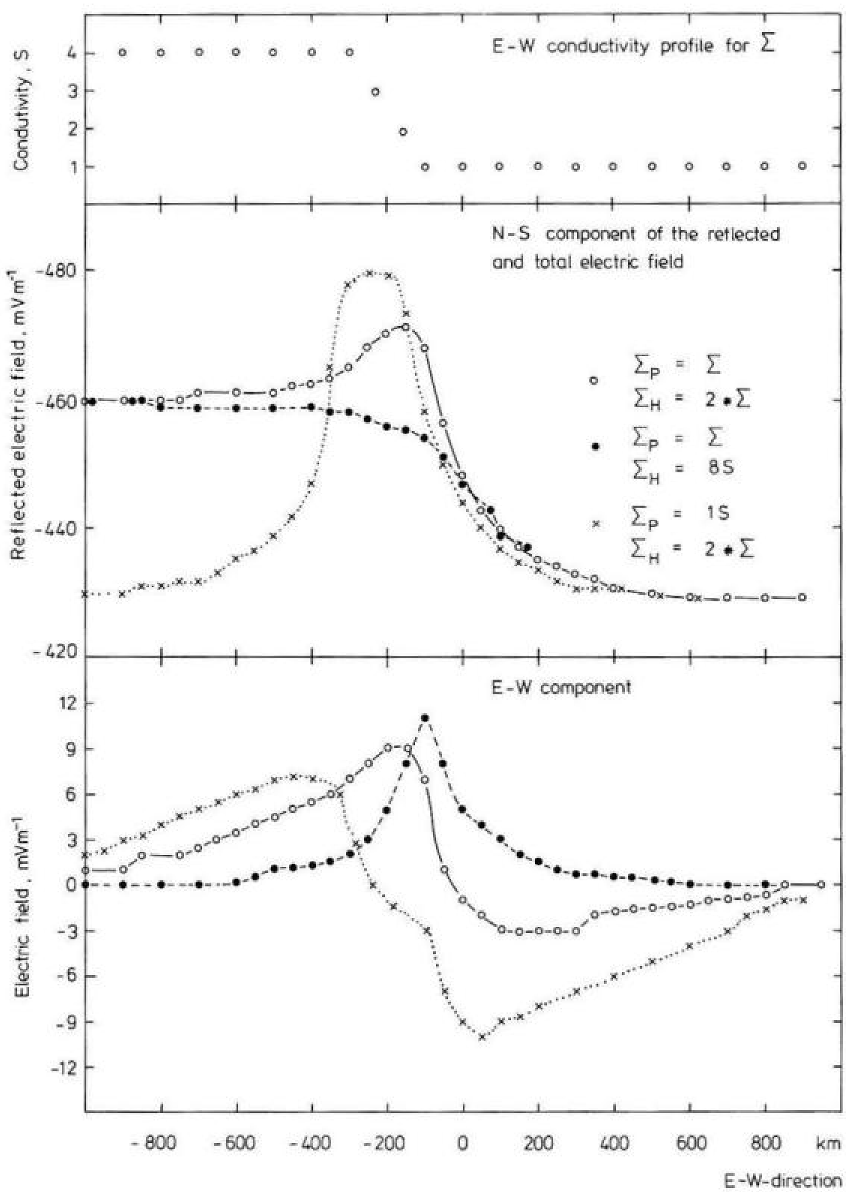

Fig. 7. E–W profiles of the N–S component and E–W component of the reflected electric field for a model situation with non-constant Σ_H/Σ_P ratio. The conductivity distributions used are based on the profile shown in the top panel. The incident electric field is as displayed in Fig. 4

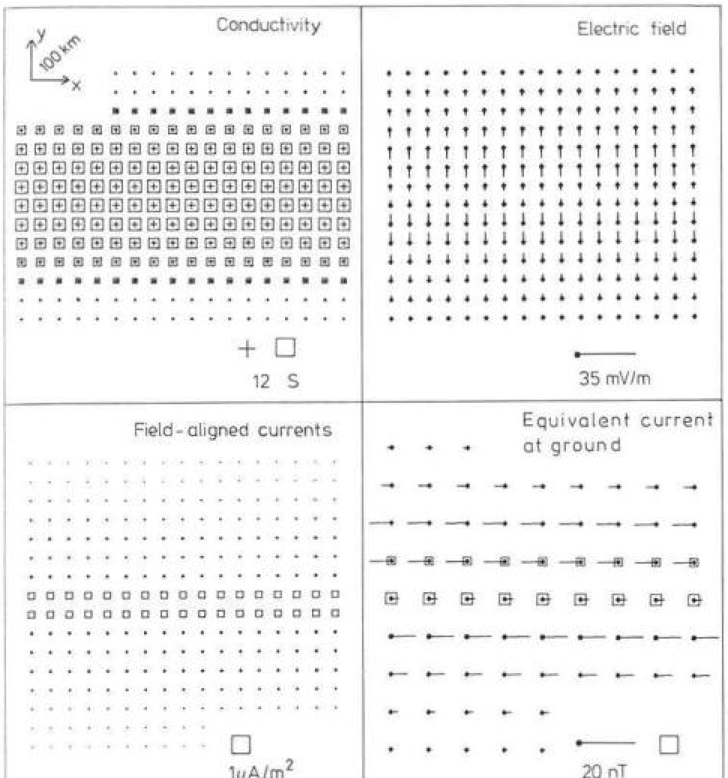

Fig. 8. Conductivity and total ionospheric electric field distribution as well as field-aligned currents and the equivalent current pattern (i.e. magnetic disturbance vectors rotated clockwise by 90° if viewed from above) at the ground for a model situation where $(\nabla\Sigma_{P,H} \times \mathbf{E})_Z$ vanishes. The incident electric field is as adopted for the model situation displayed in Fig. 4 but for the time instant $t = T/4$. Squares and crosses in the top left panel denote Hall and Pedersen conductivity, respectively. Squares in the lower right panel denote a negative magnetic Z-component at ground level

in the reflected electric field due to the fact that the distribution of the polarisation charges changes also in E-W direction in the gradient zone. However, this does not significantly alter the results of our qualitative discussion, as Fig. 6 shows, where the effective reflection coefficient is defined as the ratio of the total reflected electric field amplitude and the amplitude of the incident electric field. In Fig. 6 the reflection coefficient given by Eq. (13), irrespective of any gradients is denoted as the 'normal solution'. Inspection of Fig. 6 shows significant variations between the effective and the 'normal' reflection coefficient with the effective coefficient being decreased (increased) south (north) of the maximum of the incident electric field in accord with our qualitative discussion. Figure 6 also shows the basic asymmetry of the model with respect to the maximum of the incident electric field. This will be of some importance later on when discussing the influence of non-uniform conductivity distribution on ground-based observations of ULF-waves.

While in the model discussed in Fig. 4 the ratio Σ_H/Σ_P was everywhere constant Fig. 7 shows results of a numerical solution of (14) for Σ_H/Σ_P changing in the E-W direction. Considering the case where Σ_H is constant and Σ_P changes in the E-W direction, is of some interest because the incident electric field causes no additional necessary field-aligned currents in the gradient zone. Therefore, one could expect Eq. (13) to be a solution of (14) but due to the change of the polarisation charges across the gradient zone the reflected electric field also changes in the E-W direction and there is also an E-W component in the reflected electric field (see bottom panel of Fig. 7). This component gives rise to additional necessary field-aligned currents which may be too large to get a balance of the field-aligned current situation. This balance is then accomplished by the E-W component increasing in the gradient zone towards east (north of the maximum of the incident field, see Fig. 7). The numerical solution of (14) for the situation considered in Fig. 7 furthermore shows the N-S component of the reflected electric field to be symmetric and the E-W component to be antisymmetric with respect to the maximum of the incident field. Thus similar arguments to those above hold south of the maximum.

Another case of interest is when the Pedersen conductivity is constant throughout and the Hall conductivity changes in the E-W direction (Fig. 7). For this case, the numerical solution with the same electric field distribution of the incident wave as used in Fig. 4 shows a sharply increasing N-S component which maximizes in the middle of the gradient zone and falls off outside of it to values given by Eq. (13). Where the N-S component has its maximum the E-W component changes sign from being positive in the west to negative in the east. The N-S component of the reflected electric field reaches values as large as those of the incident field but of opposite sign. Thus the effect of the reflected electric field in this case is to reduce the divergence of the Hall currents in the gradient zone which was noted earlier by Ellis and Southwood (1983).

The influence of a non-uniform ionosphere on ground magnetic observations

Knowing both the ionospheric conductivity and the ionospheric electric field (derived as described above) gives one the possibility to construct the total iono-

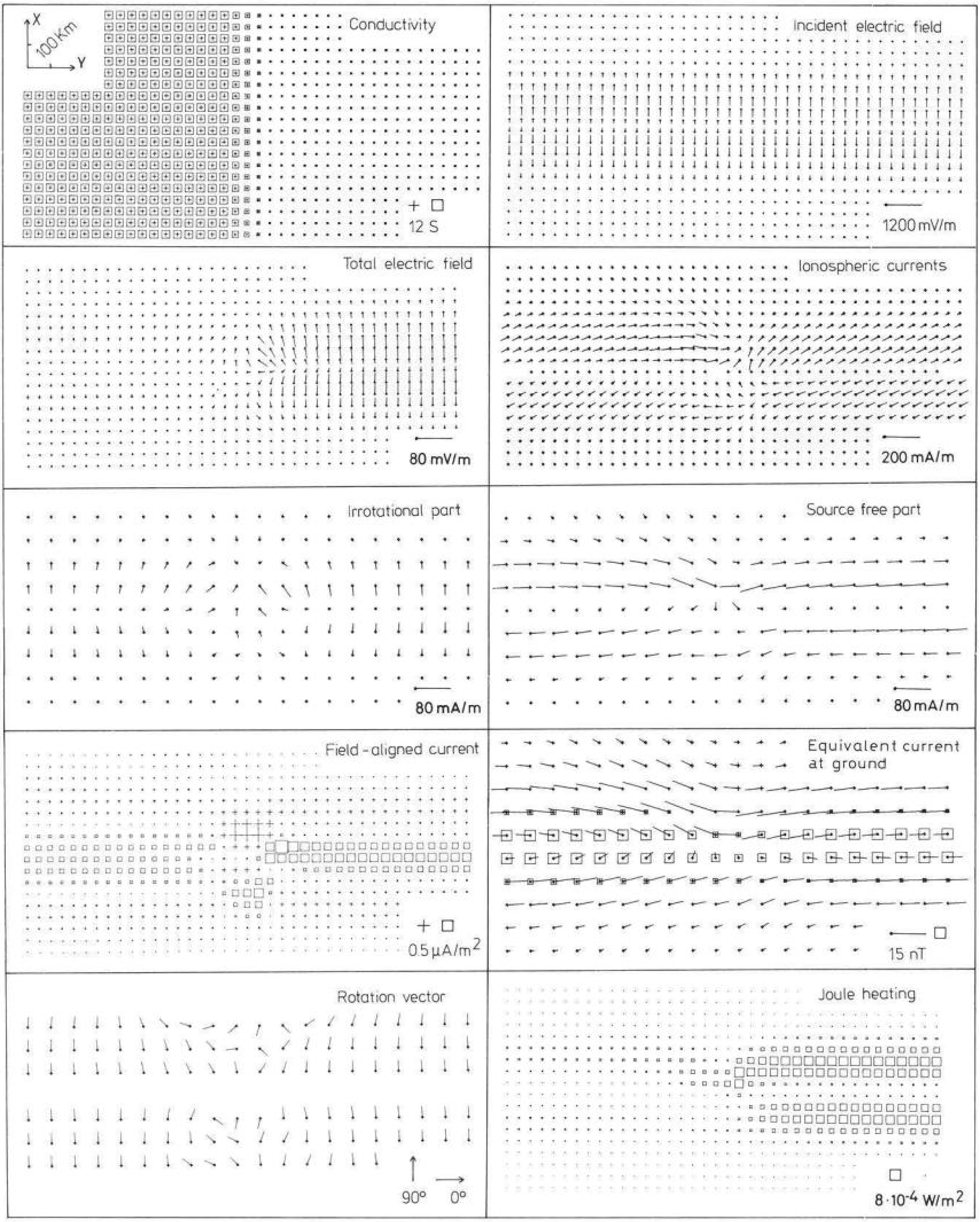

Fig. 9. Conductivity and incident electric field distribution for a model situation to demonstrate the ground magnetic effects of a non-uniform ionosphere. The incident electric is as adopted for the model situation displayed in Fig. 4 but for the time instant $t = T/4$. The conductivity variation in E-W direction is as displayed in the middle panel of Fig. 4. Squares and crosses in the upper left panel denote Hall and Pedersen conductivity, respectively. In the panel showing the distribution of the field-aligned currents squares and crosses denote upward and downward field-aligned currents, respectively. For the numerical calculations a grid spacing of 25 km was used while in the figure only solutions at each second point are shown. For the irrotational part, the source free part, the equivalent current at ground and the rotation vector only every fourth point of the grid is displayed. The angle the rotation vector makes with the y-axis gives the angle by which the magnetic field just below the ionosphere is rotated with respect to that far away above the ionosphere if viewed from above. For all panels the left and bottom rows correspond to each other

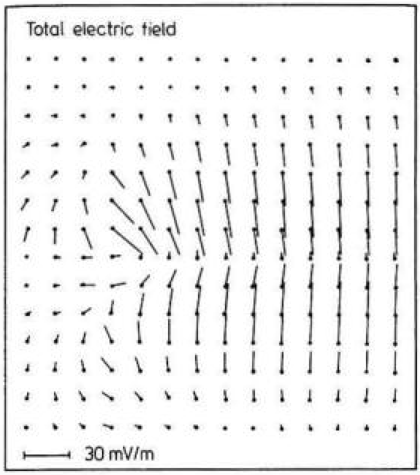

Fig. 10. A more detailed representation of the total electric field distribution for the model situation displayed in Fig. 9

spheric current system and, by using the Biot-Savart law, the magnetic disturbance at the ground (cf. Baumjohann et al., 1981).

To demonstrate this procedure Fig. 8 shows a model situation with an incident electric field similar to that displayed in the top panel of Fig. 4 and the situation discussed is a snapshot for $t=T/4$ where T is the period of the wave. The conductivity distribution is of parabolic shape in the N–S direction with a maximum value for Σ_P of 4 S, and is constant in the E–W direction (upper left panel of Fig. 8). A ratio $\Sigma_H/\Sigma_P=2$ is assumed. Equation (13) holds and, with $\Sigma_W=5.0 \cdot 10^{-2}$ S, a total electric field as displayed in the upper right panel of Fig. 8 results. From the total ionospheric current distribution the field-aligned currents can be derived and a band of enhanced upward directed field-aligned current flows where **E** changes sign (cf. Greenwald and Walker, 1980). The ground magnetic disturbances are represented in terms of equivalent currents i.e. the magnetic disturbance vector is rotated clockwise by 90° if viewed from above. Due to the electric field and the conductivity being constant in the E–W direction, we see purely westward currents flowing in the North and eastwards currents flowing in the South.

The situation shown in Fig. 8 may serve as reference for the following model calculations (Fig. 9) where the same incident electric field is used as before, but a conductivity distribution as shown in the middle panel of Fig. 4 is used, i.e. there is a change of the conductivity in the E–W direction only. With $\Sigma_W=5 \cdot 10^{-2}$ S and $\Sigma_H/\Sigma_P=2$, Eq. (14) has been solved numerically to yield the total ionospheric electric field (see upper right panel in Fig. 9). The section (Fig. 10) shows clear deviations of the electric field as compared with the uniform conductivity case. The total electric field vector is significantly rotated in the gradient zone due to the polarisation charges changing in E–W direction. Both the electric field and the conductivity give the total ionospheric current system which can be split into source free and irrotational parts (see middle panels of Fig. 9). While in the uniform case the source free part is equivalent to the Hall current system, part of the Pedersen currents in the non-uniform case contribute to the source free part. On the other hand part of the Hall current system contributes to the irrotational part of the ionospheric current system. This and the E–W component of the total electric field which appears due to the conductivity gradients give rise to N–S components in the source free part.

The gradient zone also acts to redistribute the field-aligned currents as compared with the uniform conductivity case (see Figs. 8 and 9) with strong downward directed currents in the northern part of the gradient zone and upward directed currents flowing in the south. Remembering that the field-aligned currents and the irrotational part of the ionospheric current system together with the polarisation currents form a poloidal current system whose magnetic effect is not detectable at the ground, gives one the equivalent current system of the ground-magnetic disturbance. Comparison of the corresponding panels in Fig. 9 shows the close relationship between the source free and the equivalent current system of the ground-magnetic disturbance. This later current system has a clear vortex-like structure due to the N–S current components in the source free part. A N–S component in the equivalent current system means a magnetic disturbance in the D-component and thus, when crossing the gradient zone from west to east, one detects an enhanced D/H ratio as has been observed by Saka et al. (1982) for the local time region around sunrise, where gradients of the ionospheric conductivity are expected.

Using radar measurements of the ionospheric electric field during times of pulsation activity, Walker et al. (1979) tried to predict the corresponding ground magnetic disturbance using a uniform ionosphere. For the H component they found good agreement between the predicted and actually measured field values but failed to predict the D component. The discussion of the model situation displayed in Fig. 9 shows that this failure to predict the D component is probably due to choosing a uniform ionosphere rather than one with a suitable non-uniform conductivity.

Thus observational results by Saka et al. (1982) and Walker et al. (1979) are understandable taking into account the influence of a non-uniform ionosphere as described in the present work. Further work is under way to fit in more detail STARE electric field measurements reported about by Walker et al. (1979) and simultaneous observations made by the Scandinavian Magnetometer Array (Küppers et al., 1979) using the theoretical model and the numerical code outlined above.

Hughes (1974) was the first who stated clearly that the effect of a uniform ionosphere is to rotate the magnetic disturbance vector below the ionosphere by 90° counterclockwise with respect to the disturbance vector above. However, our model calculations (see lower left panel of Fig. 9) show prominent deviations from this 90° rotation in the gradient zone. The angle the rotation vector (see bottom left panel of Fig. 9) makes with the x-axis is the rotation angle as derived from a numerical evaluation of the integral in Eq. (18a) and using (18b). At the northern and southern boundary as well as in the middle part, where the magnetic fields are rather small, the derived rotation angle is not reliable and has been neglected in Fig. 9. The deviations of the rotation angle from 90° are due to the break-down of the simple separation of the ionospheric current system into Hall and Pedersen current system as source free and irrotational parts, respectively, in regions where conductivity gradients occur. Thus any detailed comparison of satellite and ground-based magnetic observations of ULF-pulsations needs details about the ionospheric conductivity distribution on which the ionospheric rotation angle depends strongly.

As a matter of completeness the ionospheric Joule heating due to the wave electric field has been computed but no remarkable new features appear and early work by Greenwald and Walker (1980) still holds.

Summary and conclusions

A theory has been outlined to describe the influence of the ionosphere on ULF-pulsations for arbitrary distributions of the height-integrated ionospheric conductivity.

Using the concept of splitting a vector field into source free and irrotational parts the elliptical Eq. (14) for the potential of the reflected electric field is derived by matching the irrotational part, $\mathbf{I}_{irr}$, of the ionospheric height-integrated sheet current system with the polarisation currents of the wave flowing in the magnetosphere.

Both, $\mathbf{I}_{irr}$ and $\mathbf{j}_P$, together with the field-aligned currents carried by the wave-field, form a poloidal current system which has no magnetic effect below the ionosphere where the magnetic field is due solely to the source free part of the ionospheric sheet current system.

The concept of splitting a vector field into source free and irrotational parts has been used much earlier in a similar way by Dungey (1963) to study the effect of the ionosphere on hydromagnetic waves. In his attempt to tackle the problem Dungey (1963) has separated the atmospheric magnetic field into two parts and showed that the one with $(\nabla \times \mathbf{b})_Z \neq 0$ i.e. the part associated with a vertical current is effectively screened from the ground. The remaining part of the ground magnetic disturbance is thus curl-free and, when represented by a scalar potential, can easily be continued upwards towards the current carrying ionosphere and is seen to be due to the source free part of the ionospheric current system. Thus our model is quite in accord with Dungey's (1963) early work. For a uniform ionosphere the magnetic field observable at the ground is found to be due to the Hall currents alone which means a counterclockwise rotation by 90° of the magnetic field below the ionosphere with respect to that in the magnetosphere. This agrees with earlier work by for example, Nishida (1964) and Hughes (1974).

If the magnetic field below the ionosphere is due to the source free part of the ionospheric current system a scalar potential for the magnetic field can be introduced which satisfies Laplace's equation. It is easily shown that the horizontal Fourier components of the wave-field below the ionosphere are damped by a factor $\exp(-k_z h)$ (cf. Siebert and Kertz, 1957; Hughes and Southwood, 1976; note that the ionosphere is assumed at a height $z=-h$). The source free part of the ionospheric currents gives rise to a magnetic field not only below but also above the ionosphere where the magnetic field is that of a fast mode wave whose amplitude decreases exponentially away from the ionosphere i.e. it is a surface wave.

For specific distributions of the electric field of the incident wave and the height-integrated ionospheric conductivity, the reflected electric field has been determined using Eq. (13) or a numerical solution of Eq. (14). It is shown that double-peaked total ionospheric electric field distributions, shifts of the ionospheric field maximum against that of the magnetic field below and above the ionosphere, as well as strong electric field anomalies which are confined to the conductivity gradient zones, may occur. By electric field anomaly we understand deviations from electric field distributions expected for a uniform ionosphere. Due to conductivity gradients the simple separation of the ionospheric current system into Hall- and Pedersen-currents as source free and irrotational parts, respectively, breaks down and the distribution of polarisation charges giving rise to the reflected wave builds up, depending on the conductivity distribution, and any electric field anomaly produced by this is confined to the gradient zones and falls off rapidly outside.

Double-peak electric field distributions as mentioned above are not expected to be a regular feature and we suppose them to be associated with rare double-resonance pulsations as reported about recently by Nielsen and Allan (1983). However, though we are able to explain observed latitude profiles of the electric field amplitude, nothing can be said about phase variations within the limits of our model. Further work is necessary and under consideration to solve the full wave equation with non-uniform ionospheres as boundaries of the magnetospheric cavity.

Our numerical calculations (cf. Fig. 9) furthermore show that, due to the above mentioned break-down of the separation of the ionospheric sheet currents into Hall- and Pedersen current system, in the case of a non-uniform ionosphere the Pedersen currents also contribute to the magnetic field at the ground. Thus in the gradient zone of our model we find an enhanced D/H ratio as has been reported by Saka et al. (1982) in the dawn sector.

We have also been able to show that the 90°-rotation (Hughes, 1974) does not hold in general and the rotation of the disturbance vector below and above the ionosphere depends strongly on the conductivity distribution (see Fig. 9).

As a matter of comparison it should be noted that the physical ideas used in the present paper are also used in other branches of magnetospheric physics. One example is the motion of a conducting body in a magnetized plasma (cf. Drell et al., 1965; Goertz, 1980, Neubauer, 1980), and the model used by Luzemann (1982) to describe the generation of Alfvén-waves due to the motion of Io in the Jovian magnetosphere yields the same Eq. (14) used in this paper to describe the wave field launched from the ionosphere. Equation (14) can also be used to analyse the wave field generated in the ionosphere by periodic modulation of the auroral electrojet as it has been reported by Stubbe and Kopka (1977) (see also Fejer and Krenzien, 1983).

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