J Geophys (1983) 52:140–142

Journal of Geophysics

Short Communication

Ocean Tides and Periodic Variations of the Earth's Rotation

H.-R. Baader, P. Brosche, and W. Hövel

Sternwarte der Universität Bonn, Auf dem Hügel 71, 5300 Bonn 1, Federal Republic of Germany

Abstract. We consider the periodic acceleration of the Earth's rotation rate caused by the oceanic M_2-tide and the corresponding cumulative effect in universal time UT1. The necessary information stems from a hydrodynamical model of the world's oceans. The time variations of the contributions to the relative angular momentum balance and of the solid Earth's center-of-mass are provided. As for the theoretical aspects, two hypotheses are examined: the 'quasi-isolated-Earth'-hypothesis, which is basic for the derivation of the $\Delta\omega$- and ΔUT1-effects, is acceptable at least as a first-order approximation, whereas the idea of 'locked oceans' is not. The main result is a ΔUT1-contribution with a total range of 0.05 ms, thus nearly detectable by means of modern observational techniques: this effect essentially originates from the tidal currents.

Key words: Earth's rotation – M_2-tide – Angular momentum balance of the oceans

Introduction

The "classical" effects of precession and nutation have been understood for a long time. They are a result of the reaction of a non-spherical rigid Earth exposed to the action of tidal torques from the Moon and the Sun. Later on, elastic deformations of the solid Earth were also taken into account. Recently, Yoder et al. (1981) re-studied the periodic tidal variations of the Earth's rotation rate and thereby, for the first time, estimated "the effect of fluid core and ocean tides on the dynamical motion of the Earth". They argued on the basis of two assumptions:

- Oceans act as if locked to the mantle;
- External torques are negligible, i.e., the Earth is an isolated system in a tidal period.

Because of the second assumption the whole Earth's angular momentum $P_z = C\omega_z$ is conserved, and hence by means of the resulting relation:

$$\frac{\Delta\omega_z}{\omega_z} = -\frac{\Delta C}{C} \tag{1}$$

changes in ω and in universal time UT are derivable from variations of C (C: polar moment of inertia, ω_z: angular velocity of the whole Earth, the rotation vector is fixed to the z-coordinate – as a permanent simplifying assumption hereafter).

The purpose of this note is to examine the above assumptions and to provide a more realistic view plus the corresponding estimates for the oceanic M_2-tide. Some aspects of the topic were already mentioned briefly by Brosche (1982).

Our main interest is dedicated to the secular influence of the oceanic tides on the Earth's rotation. Therefore, we regard the solid Earth only in a most rough and simple way, i.e. as a *rigid* sphere, except for its equilibrium tide response reducing the tangential forces of the ocean model by the Love factor $\gamma = 1 + k - h = 0.69$. The world's oceans in contrast are considered in detail (e.g. including shelf seas): our work is based on a modified version of Zahel's (1970) nonlinear $4° \times 4°$-HN-model (see also Brosche and Hoevel, 1982), mainly applied to the M_2-tide. The model output allows *direct* access to all angular momentum and moment of inertia quantities in their dependence on the tidal phase $\tau = \sigma t$ (σ: angular velocity of the M_2-tide). Of course, any theoretical model of a partial oceanic tide provides for this information, at least in principle, but it seems unusual to publish the velocities too.

Comments on Zahel's Model

First we would like to comment on the apparently outstanding Zahel-model values for the M_2-tide given in a recent comparative compilation of certain characteristic parameters of diverse ocean models (Yoder et al., 1981). They seem unrepresentative to the authors of this paper, who are working with original and modified versions of the Zahel-model. One reason is that the M_2-effect on UT1 quoted in Table 4 of Yoder et al. (1981) is approximately twenty-six times greater than our corresponding ΔUT1-amplitude of ~ 0.008 ms (see Eq. 6). Also their harmonic coefficients h_{22} (Table 3) differ considerably from the values listed in Table 6.4 of Lambeck (1980), which show fair agreement with our findings. Our main argument against the reliability of that spherical harmonic analysis of Zahel's model is the very poor mass conservation (evident in the huge values of h_{00} in Table 3), whereas we find an excellent fulfillment of this essential constraint: the relative error in the water elevation is $\frac{\Delta\zeta}{\zeta} \sim 10^{-13}$, i.e. the defects of the mass balance would induce modifications of this order of magnitude. Furthermore, the reported "coordinate offsets of the

Offprint requests to. P. Brosche

Table 1. Amplitudes of the M_2-variations of the solid Earth's center-of-mass ($\delta\mathbf{S} = \mathbf{A}\cos\sigma t + \mathbf{B}\sin\sigma t$) relative to its time-average position (or also to the center-of-mass of the whole Earth) in 10^{-3} m. From the origin the z-axis parallels the rotation vector, the x- and y-axes point towards the Greenwich meridian and 90° E, respectively

Component of vector $\delta\mathbf{S}$	**A**	**B**
δx	−1.5	3.6
δy	2.8	3.9
δz	−3.7	1.5

Table 2. Typical contributions (amplitudes) to the relative polar angular momentum balance of the world's oceans during an M_2-period (units: 10^{21} kg m^2 s^{-2}). The storage term results from the net effect of the other torques quoted. (Incidentally the Coriolis torque will counteract the changes in the moment of inertia in later context; the contribution of the pressure gradient force dominates the ocean-solid Earth interaction)

Torque due to	Order of magnitude
Tidal potential force (= oceanic-lunar interchange)	0.3
(Its period average = secular value	4.4×10^{-5})
Pressure gradient force	2.0
Coriolis force	0.8
Lateral eddy viscosity	10^{-2}
Bottom friction	2×10^{-3}
Storage (= time variation of angular momentum)	3.0

center-of-mass of mantle" (i.e. of mantle-plus-core relative to the position of the whole Earth's barycenter, according to their calculation; see Tables 3 and 5) are also too high compared to our 10^{-3} m-order results for the modified Zahel-model in Table 1.

The Mechanical Isolation Hypothesis

As is shown by Table 2, with its estimates of the contributions to the oceanic angular momentum balance, the neglect of the angular momentum transfer to the Moon via the tidal potential forces (listed in the first line) is a first-order approximation during an M_2-period (Baader, 1982). In particular, the first two figures are remarkable because of the fact that we cannot infer a negligible role of the potential force term during the period (mainly due to the 'undisturbed' ocean depth) from its very small time average (effect of the 'disturbed' water elevation).

On the other hand, this periodic transfer of angular momentum between the water and the Moon is only a part of the total interchange between the Earth and its satellite. From the fact that the geoid has tesseral undulations which are one order of magnitude smaller than the depth of the oceans (Gaposchkin, 1974) we now conclude that the total periodic change of the Earth's angular momentum is correspondingly smaller than the value in the first line of Table 2. We therefore accept the quasi-isolation hypothesis of the Earth at least in the sense indicated above.

In principle we should have listed a term depending on the total time derivative of ω in Table 2. During an M_2-period $\dot{\omega}$ is of the order 10^{-17} s^{-2}; its contribution to the oceanic angular momentum balance consequently is of the order 10^{17}–10^{18} kg m^2 s^{-2} and thus of relative order 10^{-3}–10^{-4} (as can easily be estimated by means of the isolation hypothesis via Eq. (5) or by means of the solid Earth's angular momentum balance). Its effect is therefore neglected, as usually.

The Influence of the Tidal Currents

From the different mechanisms changing the oceanic angular momentum we shall now go on to discuss the manifestations of its contents. We then have to consider the time variability of only two kinds of oceanic quantities:

- the variation of the inertia tensor $\boldsymbol{\theta}$ due to changing water elevations,
- the relative water motion (in the terrestrial frame) corresponding to a relative angular momentum P_r.

Therefore the total oceanic angular momentum is no longer

$$P_z = \theta_{zz}\omega \quad (2)$$

(z-component only; θ_{zz} is the oceanic part of C)

but $$P_z = \theta_{zz}\omega + P_{rz} = P_{\theta z} + P_{rz}, \quad (3)$$

(Lambeck, 1980; Munk and MacDonald, 1960).

The idea of 'locked oceans' means that the $P_{\theta z}$ contribution should dominate. If for the moment we regard only the time dependent parts and out of these, the overwhelming harmonic ones (see also Baader, 1982), we obtain

$$P_{\theta z}(\tau) = 6.93\sin(\tau + 41.4°) \quad 10^{24}\ \text{kg m}^2\ \text{s}^{-1}$$
$$P_{rz}(\tau) = 24.6\ \sin(\tau - 189.5°) \quad 10^{24}\ \text{kg m}^2\ \text{s}^{-1}$$

hence

$$P_z(\tau) = 20.93\sin(\tau - 204.39°) \quad 10^{24}\ \text{kg m}^2\ \text{s}^{-1}.$$

It can be seen that the amplitude of P_{rz} is larger than that of $P_{\theta z}$ by a factor 3 and that the phase angles are completely different. Therefore we conclude that the hypothesis of 'locked' oceans is not justified and should be discarded – at least for the M_2-tide, as Yoder et al. (1981) were already inclined to do. Nevertheless it is possible that loading and self-attraction lead to a slightly stronger coupling to the solid Earth.

Our P_{rz} corresponds to a typical zonal relative velocity of the oceans $u = 4 \times 10^{-3}$ m s^{-1}, (which may be converted to a differential rotation of the order $\omega_{rOC} = u/R \sim 6.3 \times 10^{-10}$ s^{-1}; subscript OC for ocean, R. Earth's radius).

The Effects in ω and UT

In what follows our line of reasoning very much resembles that of previous treatments of the atmospheric aspect of the topic (Hide et al., 1980). If we assume a rigid solid Earth (SE), the change of its angular momentum will be

$$\Delta P_{SE} = -\Delta P_{OC}, \quad (4)$$

because of the conservation in the whole $SE-OC$ system (i.e. neglecting the atmosphere). Consequently

$$\Delta\omega_{SE} = \Delta P_{SE}/\theta_{SE} = -\Delta P_{OC}/\theta_{SE}. \quad (5)$$

By means of (5) we relate the changes of the oceanic quantities $P_{\theta z}$, P_{rz}, P_z to the changes in ω (dropping the subscript

SE because ω is the same for the oceans in our definition of relative motions). We use $\theta_{SE}=0.804\times10^{38}$ kg m^2 (Lambeck, 1980). If one considers a decoupled mantle only, then $\theta_{\text{mantle}}=0.71\times10^{38}$ kg m^2, and the following values should be raised by a factor of 1.13.

$\Delta\omega = 8.62 \sin(\tau-138.6°)$ 10^{-14} s^{-1} due to the moment of inertia only,
$= 30.60 \sin(\tau- 9.5°)$ 10^{-14} s^{-1} due to relative motions only,
$= 26.03 \sin(\tau- 24.39°)$ 10^{-14} s^{-1} due to the combined effect.

A time integration leads to the change in UT1:

$$\Delta \mathrm{UT1} = \int \frac{86400}{2\pi}\, \mathrm{s}\Delta\omega\, dt \tag{6}$$

ΔUT1 $= 0.008$ ms $\sin(\tau+131.4°)$ due to the moment of inertia, only,
$= 0.030$ ms $\sin(\tau- 99.5°)$ due to relative motions only,
$= 0.025$ ms $\sin(\tau+245.61°)$ totally.

It should be noted that the total range of 0.05 ms runs close to the measurement accuracy of Very Long Baseline Interferometry and Lunar Laser Ranging. In the case of fortnightly tides the ΔUT1-effect is presumably even more important.

References

Baader, H.-R.. Balance problems in tidal computations. In: Tidal Friction and the Earth's Rotation, P. Brosche, J. Sündermann, eds.. pp. 122–164. Berlin, Heidelberg, New York: Springer, 1982

Brosche, P.: Oceanic tides and the rotation of the Earth. In: Sun and Planetary System, W. Fricke, G. Teleki, eds.: pp. 179–184. Dordrecht: D. Reidel Publ. Comp. 1982

Brosche, P., Hövel, W.: Tidal friction for times around the presence. In. Tidal Friction and the Earth's Rotation, P. Brosche, J. Sündermann, eds.: pp 175–182. Berlin: Springer, 1982

Gaposchkin, E.H.: Earth's gravity field to the eighteenth degree and geocentric coordinates for 104 stations from satellite and terrestrial data. J. Geophys. Res. **79**, 5377–5411, 1974

Hide, R., Birch, N.T., Morrison, L.V., Shea, D.J., White, A.A.. Atmospheric angular momentum fluctuations and changes in the length of the day. Nature **286**, 114–117, 1980

Lambeck, K.: The Earth's variable rotation: geophysical causes and consequences. London: Cambridge University Press 1980

Munk, W.H., MacDonald, G.J.F. The rotation of the Earth, a geophysical discussion. London: Cambridge University Press 1960

Yoder, C.F., Williams, J.G., Parke, M.E.: Tidal variations of Earth rotation. J. Geophys. Res. **86**, 881–891, 1981

Zahel, W.: Die Reproduktion gezeitenbedingter Bewegungsvorgänge im Weltozean mittels des hydrodynamisch-numerischen Verfahrens. Mitt. Institut für Meereskunde, 17, University of Hamburg 1970

Received September 1, 1982; Revised version November 16, 1982
Accepted November 26, 1982