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Monte Carlo Simulation of Collisionless Shocks Showing Preferential Acceleration of High A/Z Particles*

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Abstract. A collisionless, quasi-parallel shock is simulated using Monte Carlo techniques. In this kinetic approach, scattering of all velocity particles (from thermal to high energy) is assumed to occur such that the mean free path is directly proportional to velocity times the mass to charge ratio and inversely proportional to the plasma density. Within the constraints of this assumption, the shock profile and velocity spectra are obtained showing preferential acceleration of high A/Z particles relative to protons.

Key Words: Cosmic rays – Shock acceleration

Introduction

Analytic treatments of the acceleration of charged particles by collisionless shocks have been successful in describing some of the basic properties of this mechanism. The work of Krymsky (1977), Axford et al. (1977), Bell (1978), and Blandford and Ostriker (1978) shows that steady state shocks can yield high energy power law spectra similar to that observed in many astrophysical environments. Eyponential spectra, as observed within the solar system, can also be derived when adiabatic losses (Fisk and Lee 1980) or finite shock size (Scholer et al. 1980; Terasawa 1981; Eichler 1981) are considered. Also, Axford et al. (1977) and Drury and Volk (1981) consider cosmic ray mediation and derive solutions in which sufficient cosmic ray pressure prevents a shock, with the bulk kinetic energy going into cosmic rays rather than heat.

In the above treatments, the number flux of cosmic rays is generally conserved through the shock; that is, the cosmic rays are treated as a second fluid component, with the question of their origin left open. Bell (1978) proposes that the shock picks up a few superthermal particles from the shocked fluid, but the rate of this process remains a free parameter in his discussion. Our own work, which models production of cosmic rays at the shock directly from the thermal plasma, is motivated by the following considerations. Observations suggest that shocks routinely put a significant fraction of their energy into cosmic rays in addition to shock heating the thermal fluid, and this requires that the number of "injected" particles, if it is a free parameter, be chosen very carefully. If there are too few, their energy content subsequent to acceleration remains small. If there are too many, the thermal shock is washed out, and the thermal fluid is not shock heated, contrary to observations of supernova remnants etc. Other arguments against pre-injection, in the context of galactic cosmic rays, have been made on observational grounds by Eichler (1980), Cowsik (1980), and Fransson and Epstein (1980).

The physical details of injection, that is, if and how a shock creates new cosmic rays from the thermal plasma, depends on the shock structure. A model of cosmic ray production at shock fronts was proposed by Eichler (1979). The "injection rate" of thermal particles into the cosmic ray pool is fixed by the assumptions of the model, i.e. that thermal particles are themselves compressed by converging scattering centers, (e.g. as in the shock model of Parker (1961)), and that their acceleration is limited only by the presence of higher energy particles. The resulting injection rate is just what is needed to maintain a dynamically significant level of cosmic rays.

The premise that any particle in the upstream thermal plasma is about as likely as any other particle of the same ion species to end up as a cosmic ray suggests that the relative composition of cosmic rays can be calculated, in principle, by solving for the shock structure. Conversely, measurements of cosmic ray composition can (again, in principle) provide observational tests of shock models that feature cosmic ray production.

In treating first-order Fermi acceleration of cosmic rays (see, in particular, Axford et al. 1977 and Blandford and Ostriker 1978), the procedure generally followed is to solve a diffusion equation namely,

$$\frac{\partial}{\partial x}\left(uf-k\,\frac{\partial f}{\partial x}\right)=\frac{1}{3}\left(\frac{\partial u}{\partial x}\right)\frac{\partial}{\partial p}(pf) \tag{1}$$

where u is the plasma flow velocity, $f(x,|\vec{p}|)$ is the cosmic ray distribution function, x is the position measured in the shock frame, p is the momentum and k is the diffusion coefficient. This diffusion approximation holds only when the thermal velocity of the particles is large compared to the flow velocity. In addition, one or more of the following simplifying assumptions are made: (1) treating the shock as a discontinuity (that is, neglecting cosmic ray back pressure on the incoming plasma), (2) ignoring or simplifying energy dependence of the diffusion coefficient, and (3) assuming an infinite

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scale to the accelerating system. The difficulties in achieving an analytic solution and the inability of the diffusion approximation to describe the behavior of low energy particles have led us to attempt a Monte Carlo simulation of this problem. This kinetic approach can describe the behavior of thermal particles *if* the proper scattering law is known and, therefore, can model how high energy particles might be drawn from the thermal pool.

Description of Monte Carlo Method

In this simulation of a parallel shock we follow the trajectories of particles originating far upstream in the thermal plasma, allowing them to scatter *isotropically* and *elastically* in the frame of the massive scattering centers. The particles gain energy by scattering off converging flows and are observed far downstream as they leave the system. We assume that the scattering of particles of all velocities (including the thermal particles responsible for creating the collisionless shock) can be represented by a mean free path, λ, given by

$$(1/3)\,\lambda v = k_0 (v/c)\, R^{\alpha} \tag{2}$$

where $R = \text{rigidity} = pc/(Ze)$, v is the particle velocity measured in the plasma frame, α is a constant (Jones 1978), c is the velocity of light, e is the electronic charge, Z is the charge number and k_0 is assumed to be inversely proportional to the plasma density, ρ. Equation (2) can be written (for non-relativistic particles) as

$$\lambda = \lambda_0 (A/Z)^{\alpha} (v/u_2)^{\alpha} (\rho_2/\rho) \tag{3}$$

where A/Z is the mass to charge ratio, u_2 is the downstream flow velocity, and ρ_2 is the downstream plasma density. This assumption determines the shock structure as the bulk motion is randomized by isotropic scattering.

Our calculations are non-relativistic and for this preliminary work we consider only cases where $\alpha = 1$. This choice simplifies computation since it implies a constant collision time and has some observational justification in that a diffusion coefficient of approximately this form has been deduced from observations of diffuse ions upstream of the earth's bow shock (see Scholer et al. 1980; Forman 1981; Ellison 1981).

The size of the system is defined by a free escape boundary located a distance d upstream from the shock (defined as $x = 0$).

Results

Discontinuous Shock

As an illustration we present results where the shock is taken to be discontinuous. This is done for two injection velocities for a strong shock ($u_1 = 4u_2$, see Fig. 1) where the flow velocity, $u(x)$, is given by

$$u(x) = u_1, \quad x < 0$$
$$u(x) = u_2, \quad x \geq 0.$$

In this first case, the velocity of the injected particles, v_i, is large compared to u_2. Figure 2 shows the integral spectra obtained for $v_i = 10\,u_2$ (the free escape distances are measured in λ_0, the mean free path of a particle with $A/Z = 1$, $v = u_2$, and $\rho = \rho_2$.

The expected slope for an integral, non-relativistic velocity spectrum is given by (Axford et al. 1977 and Bell 1978)

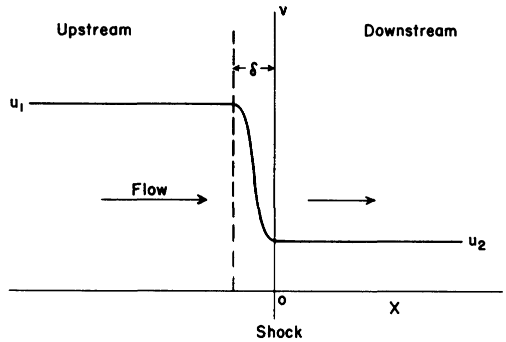

Fig. 1. Schematic velocity profile of a strong collisionless shock. In discontinuous shocks, δ is small compared to the mean free path of the accelerated particles.

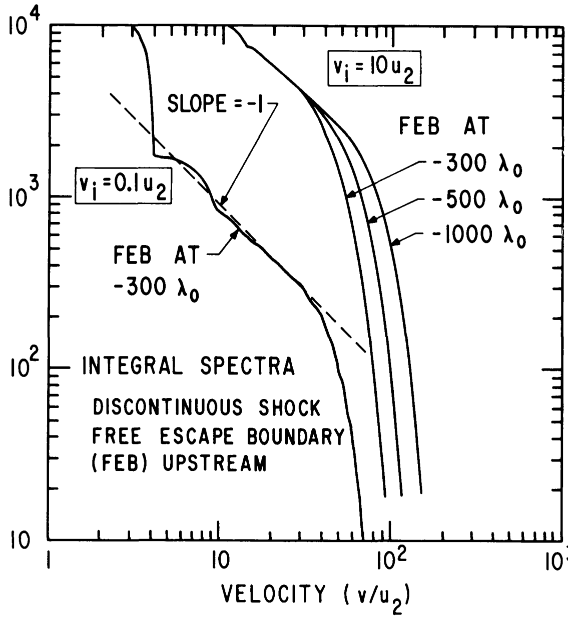

Fig. 2. Integral velocity spectra for a discontinuous shock with $v_i = 0.1$ and $10\,u_2$. The *dashed line* shows the expected slope obtained from the diffusion approximation. The position of the free escape boundary is measured in units of λ_0, the mean free path of a particle with $A/Z = 1$, $v = u_2$, and $\rho = \rho_2$.

$$\text{slope} = -\frac{3u_2}{u_1 - u_2} \tag{4}$$
$$= -1 \quad \text{for} \quad u_1/u_2 = 4.$$

The effect of the finite scale of the accelerating system is seen in the cutoff that occurs when

$$\frac{u_2 d}{k} \sim 1. \tag{5}$$

Significant differences from the predicted power law occur when the injected velocity is of the order or smaller than the flow velocities. Figure 2 shows the result for $v_i = 0.1\,u_2$. Above about $10\,u_2$, the expected power law and cutoff occur but for low velocities we see the "heating" effect that results when all particles make their first crossing of the shock. After one crossing, the injected particles will have a velocity measured in the downstream plasma frame given by

$$v = \sqrt{[v_i \cos\theta + (u_1 - u_2)]^2 + (v_i \sin\theta)^2} \tag{6}$$

where θ is the angle the particle makes with the shock normal as it crosses the shock. It is seen that even if v_i is zero, particles will acquire a velocity boost of $u_1 - u_2$, consistent with the Rankin-Hugoniot relations for an initially cold plasma. This means there will always be particles downstream with velocities greater than u_2 and, therefore, a definite proportion will diffuse back upstream across the shock and participate in the first order acceleration. The high energy particles in the power law spectrum are drawn naturally from the thermal population.

Particle Smoothed Shock

The Rankin-Hugoniot relations for particle, momentum, and energy flux are respectively,

$$\rho(x)\,u(x) = \text{constant} = A$$

$$P(x) + \rho(x)\,u(x)^2 = \text{constant} = B$$

$$\rho(x)\,u(x)\left[\tfrac{1}{2}u(x)^2 + \frac{\gamma P(x)}{\rho(x)(\gamma-1)}\right] = \text{constant} \tag{7}$$

where P is the x-component of pressure measured in the plasma frame, u the plasma flow velocity, and γ the ratio of specific heats (taken in what follows to be 5/3). The first two of these equations can be combined to give

$$u(x) = [B - P(x)]/A. \tag{8}$$

In our simulation we choose an initial velocity profile $u(x)$ and then calculate A, B and $P(x)$. A new velocity profile is then calculated using Eq. (8) and the process repeated until the particle and momentum fluxes are approximately constant across the shock.

For any real shock acceleration process, the predicted power law slope (Eq. 4) will not be obtained at very high energies. That this must be the case is seen in the fact that Eq. 4, if it describes the spectrum to unlimited energies, places an infinite amount of energy in the cosmic ray tail. To avoid this, particles will either escape from the system by geometric effects from a finite shock or from a lack of scattering at some sufficiently high energy, the time of acceleration will become longer than the lifetime of the shock, or the shock will be smoothed to the point that it no longer produces a slope of -1 at the highest energies. If particles escape, this loss of energy will in effect soften the equation of state of the gas and allow a compression ratio greater than 4. Equations (7), when modified for particle escape give the compression ratio r,

$$r = \frac{5 + \sqrt{25 - 16[1 - (2q/\rho_1 u_1^3)]}}{2[1 - (2q/\rho_1 u_1^3)]} \tag{9}$$

where q is the rate of energy loss from the high energy particles leaving the system and the subscripts indicate values far upstream. We have assumed a high Mach number shock and that the flux of particles leaving the system is small. In these preliminary calculations we include a velocity cutoff above which particles escape freely from the system but neglect any possible effect on the compression ratio.

Figure 3 shows the resultant velocity profile for $r = 4$, an injection velocity of $v_i = 0.07\,u_2$, $A/Z = 1$, an infinite scale, and a velocity cutoff of $50\,u_2$. As can be seen from the figure, the transition takes place on essentially two length scales. There is an abrupt transition caused by the large number of low velocity particles on a scale of order λ_0, and a large scale

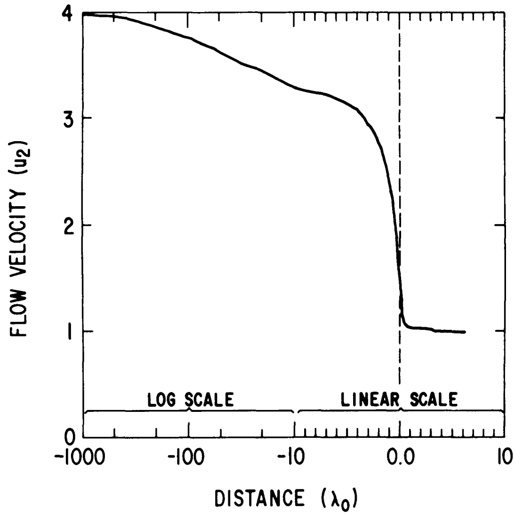

Fig. 3. Upstream velocity profile obtained by demanding that the particle and momentum fluxes be conserved (within 20% of the far upstream values). The free escape boundary is at infinity and a velocity cutoff at $v = 50\,u_2$ is imposed.

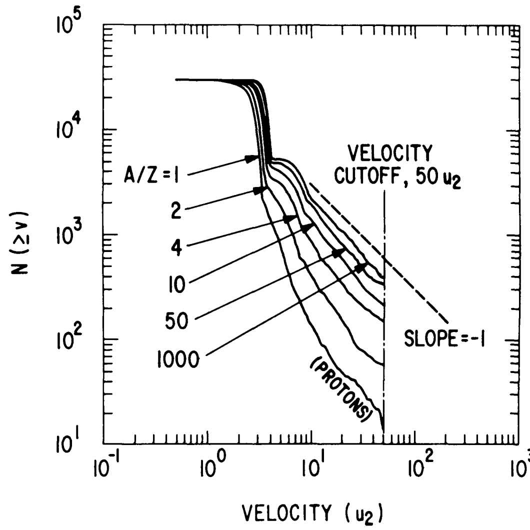

Fig. 4. Integral velocity spectra for different A/Z particles with $v_i = 0.07\,u_2$ using the velocity profile shown in Fig. 3. The free escape boundary is at infinity and a velocity cutoff at $v = 50\,u_2$ is imposed.

smoothing caused by the few high velocity particles that stream far out in front of the shock. The relative importance of this high velocity tail depends on the proportion of thermal particles that are accelerated to high energies which in turn depends on the shape and position of the shoulder in Fig. 2. It also depends on the velocity cutoff and indeed, the profile will be smoothed farther upstream as the cutoff is raised and higher energy particles are accelerated.

These results must be considered suggestive rather than quantitative, but it is clear that if the shock profile is smoothed as described above and the diffusion coefficient increases with energy, the spectrum will not be a strict power law but will be flatter at high energies than at low.

Enhancement of High A/Z Particles

The combination of a finite scale velocity transition and a mean free path dependent on rigidity naturally produces an enhancement of high A/Z particles as suggested by Eichler (1979). To see this we assume the shock profile is determined by protons ($A/Z=1$) and the we have a small admixture of high A/Z particles. The high A/Z species will have a longer mean free path, see a sharper shock, and therefore be accelerated more than the protons. This will result in an over abundance of high A/Z particles relative to protons at high velocities.

Figure 4 shows spectra for different A/Z using the velocity profile of Fig. 3.

Again it must be emphasized that the relative importance of this effect depends on the shape and height of the low velocity shoulder in Figs. 2 and 4; that is, on the details of how the thermal particles interact with the collisionless shock which is addressed in this calculation only through the very simple scattering law of Eq. (3).

The presence of this effect depends on two things. First, the shock must be smoothed by the back pressure from high velocity particles and second, the injection velocity must be low enough so that the particles do not see the shock as a discontinuity. In other words, a seed particle description with large injection velocities will not show an enhacement effect.

Conclusions

The kinetic simulation of shock acceleration using the Monte Carlo method offers an alternative with several advantages to solving the diffusion equation. It becomes simple to introduce a velocity dependent mean free path for scattering, a finite scale either by a velocity cutoff or a free escape boundary, and no restriction $v \gg u$ need be made. This allows a self-consistent calculation of the shock wherein the pressure of the accelerated particles produces a velocity profile that conserves the momentum flux (within 20%) of these particles across the shock. We obtain the following results:

a) For discontinuous shocks, a power law with an exponential cutoff at approximately $u_2 d/k=1$ results when $v_i \gg u_2$.

b) For $v_i < u_2$, within the restriction of the simple scattering law that we use, the proportion of high energy particles drawn from the thermal population is obtained.

c) The inclusion of the back pressure of the scattering particles on the inflowing plasma produces a smoothing of the shock profile. This implies that the spectra are steeper than for a discontinuous shock. This effect may be balanced to some degree by the increase in the compression ratio due to the energy loss from escaping particles.

d) The combination of a finite scale shock profile and a mean free path dependent on A/Z implies that high A/Z particles will be enhanced at high velocities over protons. (It has been shown by Cesarsky (1981) that for certain source temperatures the enhancement of galactic cosmic rays over local galactic abundances may show a positive correlation with A/Z). This effect saturates as A/Z increases and, for the parameters used here, produces an enhancement in the helium to proton ratio by about a factor of 3.

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