Plasma-gas interactions in planetary atmospheres and their relevance for the terrestrial hydrogen budget

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Abstract. The relevance of charge exchange processes for the evaporation of neutral gases out of planetary gravitational fields has been known for quite a long time. For the planetary escape, however, collisions of the escaping constituent with both neutral and ionized atmospheric species operate in building up the escape flux. For the earth, hydrogen collisions with O-atoms and O^+- and H^+-ions lead to about comparable contributions to the hydrogen escape at heights below 800 km, whereas at larger heights only charge exchange collisions need to be considered. In the present work, the lower region is therefore described by particle and energy flux conservation concepts, whereas in the upper region free-flight kinetic concepts are used. We give solutions for the height profiles of the hydrogen density, temperature and escape flux. The inclusion of charge exchange collisions increases the scale height, and consequently a smaller decrease of the hydrogen density is given. The hydrogen temperature, however, is reduced by including this effect. The charge exchange processes contribute about 70% to the total hydrogen escape which is of the order of a few 10^7 atoms/cm^2 s at the plasmapause.

Key words: Plasma-gas interactions – Charge exchange collisions – Escape

Introductory remarks

It has been known for quite a long time that thermal loss mechanisms alone cannot account for the total hydrogen escape from the earth's atmosphere. Rather, it has been suspected that non-thermal, i.e. ion-induced, processes due to charge exchange with ionospheric O^+- and H^+-ions, also contribute to hydrogen escape. The extraordinary importance of such charge exchange processes also for non-terrestrial exospheres has just recently been stressed in papers by Cloutier et al. (1978), Eviatar et al. (1979), Ip and Axford (1980), Goertz (1980), Brown and Schneider (1981), Goertz and Ip (1982) and Ip (1982). An ion pick-up from Io's ionosphere via charge exchange collisions may contribute to its atmospheric losses. The underlying idea in all these papers is that due to charge exchange processes, newly created neutrals, most probably H- or O-atoms, are injected into the "escape branch" of the velocity distri-

bution function at a rate in excess of the counteracting thermalization rate. The population density in this branch can thus be enhanced compared to the thermal one, resulting in enhanced escape rates. In this context it may also be mentioned that charge exchange interaction of precipitating terrestrial keV O^+-ions with atmospheric oxygen under geomagnetically disturbed conditions represents a source of escaping O-atoms (Torr et al., 1974, Torr and Torr, 1979). The insufficiency of the thermal escape concept to describe the terrestrial hydrogen geocorona has been pointed out by Bertaux (1975), Vidal-Madjar (1978) and Vidal-Madjar and Thomas (1978). There it was shown that the Lyman-Alpha inferred variation of the exobasic hydrogen density with thermospheric temperature is incompatible with the assumption of a Jeans-type escape of hydrogen (Jeans, 1925). Thus a charge-exchange-induced contribution to the terrestrial hydrogen escape has been proposed to solve this problem. Both charge exchange contributions and rarefied-gas-dynamical deviations from the Jeans escape, as discussed in Gross (1974), Fahr and Weidner (1977) and Fahr and Nass (1978), tend to weaken the temperature dependence of the net escape rate. Nevertheless, a quantitative value for the fractional escape contribution due to charge exchange processes has not yet been given in a rigorous treatment. In order to give as realistic an answer as possible, several different approaches towards this problem have been attempted, most of these being concerned with the specific problem of the terrestrial hydrogen escape where charge exchange collisions of H-atoms with protonospheric O^+- and H^+-ions play the dominant role (Maher and Tinsley, 1977; Chamberlain, 1977; Hodges et al., 1981; Shizgal and Lindenfeld, 1982). An extensive review of this work was recently given by Fahr and Shizgal (1983). In most of the earlier works, with the exception of Shizgal and Lindenfeld (1982), charge exchange escape contributions had been considered exclusively. However, as the latter authors could show, the resulting escape rates are substantially reduced if, in addition to charge exchange collisions, elastic collisions of H-atoms with O-atoms are also included. To arrive at a more conclusive answer concerning the fractional contributions by thermal and "non-thermal" processes we have reinvestigated this problem with special emphasis on the coupling of both processes.

Theoretical concept for the terrestrial case

In order to calculate the hydrogen loss we subdivide the upper atmosphere into two zones: (1) a transition region between 300 and 800 km where hydrogen atoms suffer collisions with both the neutral component of the background gas, i.e. with oxygen atoms, and with the ionized component, i.e. oxygen ions and protons; and (2) the region between 800 km and the plasmapause where nearly no elastic collisions and only a few charge exchange collisions between hydrogen atoms and fast protonospheric protons occur. Due to the presence of elastic collisions there exists a strong coupling in the lower region between the non-escaping and the escaping branch of the hydrogen velocity distribution function. To overcome the mathematical difficulties connected with this coupling we use the concepts of mass flow and energy flow continuity separately in this region, for these two categories of hydrogen particles. The build-up of the hydrogen escape in this region will be described by a parametrized quasi-Maxwellian representation of the hydrogen velocity distribution f_H with density, temperature and degree of anisotropy as unknown parameters. The determination of these parameters as functions of height is reached by integration of the continuity equations.

Due to the ineffectiveness of elastic collisions in the upper region in producing escaping particles we can use a simpler concept here and consider only local charge-exchange-induced production of escaping particles multiplied by their escape probabilities to space as a representation of the change of the escape flux with height. Deviating, however, from the practice of Shizgal and Lindenfeld (1982), we also consider the spherical geometry of the problem and thus take into account the effect of charge-exchange-induced escaping particles first moving to lower levels but with perigees greater than 800 km and therefore potentially contributing to the escaping branch of f_H. First we want to discuss the physics in the transition region and start with the description of the hydrogen velocity distribution function f_H.

Distribution function

To represent this distribution function we use here the following expression, given by Fahr and Nass (1978):

$$f_H(r, v') = f_{ne}^{\pm} + f_e^+ + f_e^-$$
$$= \frac{1}{G_1 + G_2 + G_3} \cdot \left\{ M(r, v') h(v_{esc} - v') \right.$$
$$+ M(r, v') h(v' - v_{esc}) h\left(\frac{\pi}{2} - \theta\right)$$
$$\left. + \alpha(r) M(r, v') h(v' - v_{esc}) h\left(\theta - \frac{\pi}{2}\right) \right\} \quad (1)$$

where $v' = |\hat{v} - \hat{v}_{Diff}|$ with $\hat{v}_{Diff}$ being that particular diffusion velocity at the 300 km level ensuring flux continuity of the H-atoms over the transition region. This diffusion velocity is small in comparison with thermal velocities, and thus we can identify v' with v. h is the step function, $M(r, v)$ is the Maxwellian distribution function according to a locally defined hydrogen tem-

perature $T_H(r)$, the subscripts "ne" and "e" stand for "non-escaping" and "escaping", and superscripts $+$, $-$ for upward and downward distributions, respectively. The quantities G_1, G_2 and G_3 are normalization factors, given explicitly by:

$$G_1 = \int_0^{v_{esc}} \int_0^{\pi} \int_0^{2\pi} f_{ne}^{\pm} d^3 v = erf(\bar{v}_H) - \frac{2}{\sqrt{\pi}} \exp(-\bar{v}_H^2), \quad (2)$$

$$G_2 = \int_{v_{esc}}^{\infty} \int_0^{\pi/2} \int_0^{2\pi} f_e^+ d^3 v = \tfrac{1}{2}(1 - G_1), \quad (3)$$

$$G_3 = \int_{v_{esc}}^{\infty} \int_{\pi/2}^{\pi} \int_0^{2\pi} f_e^- d^3 v = \alpha \cdot G_2, \quad (4)$$

where

$erf(x) =$ error function,

$$v_{esc} = \sqrt{\frac{2\gamma M}{r}},$$

$\gamma =$ gravitational constant,
$M =$ mass of Earth,
$r =$ distance from centre of Earth,

$$\bar{v}_H = \sqrt{\frac{\gamma M m_H}{k T_H r}},$$

$m_H =$ mass of hydrogen atom,
$k =$ Boltzmann constant.

The factor $\alpha(r)$ occurring in the distribution function (1) takes into account the fact that the distribution function $f_e^{\pm}$ for escaping particles is increasingly anisotropic with r. This results from the continuous decrease of collision rates between hydrogen atoms and the background gas in higher atmospheric regions responsible for the production of the downward-flying particles with an energy greater than the escape energy. Obviously, below a certain height r_c, $\alpha(r)$ has to be equal to one, meaning that an isotropic Maxwellian prevails. Going upwards from this level α will decrease and at large heights r the value α will asymptotically tend to zero, meaning that there will be no downward-flying particles with escape energy. Hence one expects a specific functional relation between α and the collision probability which we formulate in the following manner

$$\alpha(r) = 1 - (1 - \beta) w_e(r, \infty), \quad (5)$$

where $w_e(r, \infty)$ is the angle averaged probability that a particle with an averaged escape velocity representative of the whole escape category can leave the spherical atmosphere without suffering any collisions above r:

$$w_e(r, \infty) = \int_0^{\pi/2} \exp\left\{ -\sum_i q_i \bar{N}_{i, e}(r, \theta) \right\} \sin\theta \, d\theta. \quad (6)$$

Downward-flying particles with an energy greater than the escape energy are produced not only by collisions between upward-flying escaping particles and the background gas but also by charge-exchange collisions between fast protons and hydrogen or oxygen atoms. This means that under the additional effect of these collisions $\alpha(r)$ will not decrease towards zero as rapidly as in the case of elastic collisions alone. To account for this we introduce the quantity β in Eq. (5). This quan-

tity β has to be zero if there are no ionized hydrogen atoms in upper atmospheric regions. On the other hand, we can calculate β from the requirement that there must be a continuous transition between the transition region (300–800 km) and the higher atmospheric regions (800 km – plasmapause).

The quantity $\bar{N}_{i,e}(r, \theta)$ is the mean column density of the i'th background gas component seen by an averaged escaping hydrogen atom at r flying upwards with an angle θ against the radius vector, i.e.:

$$\bar{N}_{i,e}(r, \theta) = \int_r^\infty n_i(r') \frac{\bar{v}_{i,e}^{rel}}{\bar{v}_e \cos \theta'} \, dr'. \qquad (7)$$

Here $\bar{v}_e'$ is the mean velocity of all escaping hydrogen atoms at r'.

$\bar{v}_{i,e}^{rel}$ is the mean relative velocity of a hydrogen atom moving with velocity $\bar{v}_e'$ with respect to the i'th background gas component of temperature $T_i(r')$, particle mass m_i and Maxwellian velocity distribution $M[T_i(r')]$:

$$\bar{v}_{i,e}^{rel} = \sqrt{\frac{2kT_i}{\pi m_i}} \exp\left(-\frac{m_i \bar{v}_e'^2}{2kT_i}\right)$$
$$+ \bar{v}_e' \cdot \left(1 + \frac{kT_i}{m_i \bar{v}_e'^2}\right) erf\left(\sqrt{\frac{m_i \bar{v}_e'^2}{2kT_i}}\right). \qquad (8)$$

According to the conservation of angular momentum, the angle θ' of Eq. (7) is given by

$$\cos \theta' = \sqrt{1 - \left(\frac{r \bar{v}_e}{r' \bar{v}_e'}\right)^2 \sin^2 \theta}. \qquad (9)$$

By introducing the averaged escape velocity in Eq. (6) we give only an approximation to the escape probability. In principle we have to integrate this probability over the whole velocity distribution, but it can be shown that our approximation differs only slightly from the rigorous expression.

Energy flux continuity in the transition region

Because of the disparate masses of the collision partners, the energy coupling due to mutual elastic collisions between the light escaping gas constituent (hydrogen) and the relatively heavy neutral background gas (oxygen) is small. Thus, due to productions of escaping atoms, the energy that is locally extracted from the escaping gas constituent cannot be locally replaced by elastic collisions with oxygen. Therefore, there is no local balance between energy losses and gains. This unbalanced situation consequently leads to a height gradient of the temperature of the escaping constituent (T_H) in the transition region, whereas under geomagnetic quiet conditions the oxygen temperature T_O can be expected to remain at its asymptotic thermospheric temperature value T_O. This is due to its high thermal conductivity and the non-existence of energy losses by escape. As was shown by Torr et al. (1974), precipitating O^+-ions at higher latitudes under disturbed conditions can represent an energy source at exobasic levels. However, this has not been considered here since its influence on the averaged global escape is estimated

to be of minor importance. The hydrogen temperature T_H, however, may drop below T_O in order to establish a local thermal energy source for the escaping constituent.

In a previous paper, Fahr and Nass (1978) also examined an additional energy loss for hydrogen embedded in oxygen. As is known, inelastic collisions between hydrogen and oxygen can give rise to the excitation of the $^3P_1$ state of oxygen followed immediately by an emission of a $63\,\mu$ photon. This process in principle should be able to cool down hydrogen even further because oxygen, due to its high thermal conductivity, can be taken as an infinite heat reservoir with temperature T_O. A more accurate examination of this specific process by Durrance and Thomas (1979), who took into account the characteristics of the optically thick radiation field of the $63\,\mu$ emission line, shows, however, that if some simplifying assumptions with regard to the relevant inelastic cross-sections can be made, this specific temperature reduction can be neglected. Therefore, the energy situation in the transition region can be described by the following heat conduction equation:

$$\frac{d}{dr}\left(\kappa(r)\frac{d}{dr}T_H(r)\right)$$
$$= n_H n_O q_{H,O}^{el} \bar{v}_{H,O}^{rel} \varepsilon \tfrac{3}{2} k [T_O - T_H]$$
$$+ n_P n_H q_{P,H}^{ex} \bar{v}_{P,H}^{rel} \tfrac{3}{2} k [T_I - T_H]$$
$$+ n_P n_O q_{P,O}^{ex} \bar{v}_{P,O}^{rel} \tfrac{3}{2} k [T_I - T_H]$$
$$- n_H n_{O^+} q_{H,O^+}^{ex} \bar{v}_{H,O^+}^{rel} \tfrac{3}{2} k [T_I - T_H]$$
$$- \frac{d}{dr} \phi_e^{eff}(r). \qquad (10)$$

The terms on the right-hand side stand for the following processes:

1) energy coupling with oxygen,
2) energy coupling with protons,
3) energy gain due to collisions between protons and oxygen,
4) energy loss due to collisions between hydrogen and oxygen ions,
5) energy loss due to local escape contributions,

where

$$\kappa(r) = \bar{\kappa} n_H \frac{\sqrt{T_H(r)}}{n_O q_{H,O}^{el} + n_P q_{P,H}^{ex} + n_{O^+} q_{H,O^+}^{ex}},$$
$$\bar{\kappa} = 1.23 \times 10^{-12} \; [\text{g cm}^3 \, \text{grad}^{-3/2} \, \text{s}^{-3}],$$

$n_i(r) =$ density of the i'th background gas component,

$\quad \varepsilon = 2 m_H m_O / (m_H + m_O)^2$,

$T_O =$ oxygen temperature,

$T_I =$ ion temperature (taken to be common for O^+- and H^+-ions).

The cross-section for elastic collisions between H- and O-atoms can be adopted with (Liwshitz and Singer, 1966):

$$q_{H,O}^{el} = 3 \times 10^{-15} \; \text{cm}^2, \qquad (11)$$

whereas the charge-exchange cross-sections are slightly velocity dependent and given by (Storm, 1970; Rapp, 1963)

$$q_{P,H}^{ex} = (1.64 \times 10^{-7} - 1.6 \times 10^{-8} \log(v_{P,H}^{rel}))^2 \qquad (12)$$

with

$$10^5 \leq v_{P,H}^{rel} \leq 5 \times 10^7 \text{ cm/s}$$

and

$$q_{P,O}^{ex} = \tfrac{1}{3}(1.64 \times 10^{-7} - 1.6 \times 10^{-8} \log(v_{P,O}^{rel}))^2 \qquad (13)$$

with

$$3 \times 10^5 \leq v_{P,O}^{rel} \leq 5 \times 10^7 \text{ cm/s.}$$

We use identical cross-sections for $H^+ - O$ collisions and for $H - O^+$ collisions, since in thermal equilibrium the ratio of the reaction rate $[HO^+]/[H^+O]$ is indicated to be $9:8$ (Chamberlain, 1956).

The required mean relative velocities can be calculated by the following formula:

$$\bar{v}_{i,j}^{rel} = \int_0^\infty \int_0^\pi \int_0^{2\pi} f_i \, d^3 v_i$$
$$\cdot \int_0^\infty \int_0^\pi \int_0^{2\pi} f_j \sqrt{v_i^2 + v_j^2 - 2 v_i v_j \cos \theta_j} \, d^3 v_j. \qquad (14)$$

If the velocity distribution functions of the background gas components are Maxwellians, which is a very reasonable assumption, then the integrations can be carried out analytically:

$$\bar{v}_{i,j}^{rel} = \sqrt{\frac{8k}{\pi}\left(\frac{T_i}{m_i} + \frac{T_j}{m_j}\right)} \qquad (i, j \neq H),$$

$$\bar{v}_{H,j}^{rel} = \frac{1+\alpha}{2(G_1 + G_2 + G_3)} \sqrt{\frac{8k}{\pi}\left(\frac{T_H}{m_H} + \frac{T_j}{m_j}\right)} \qquad (15)$$
$$+ \frac{1-\alpha}{2(G_1 + G_2 + G_3)} I_9(H, j).$$

(The factor $I_9(H, j)$ is explained in the Appendix.) The local energy flux associated with the escape flux at some height level r is given by

$$\phi_e^{eff}(r) = n_H(r) J_e^E(r) w_e(r, \infty) \qquad (16)$$

with

$$J_e^E = \int_{v_{esc}}^\infty \int_0^{\pi/2} \int_0^{2\pi} (\tfrac{1}{2} m_H v^2) v \cos\theta f_e^+ \, d^3 v$$
$$= \frac{1}{G_1 + G_2 + G_3} \frac{m_H}{2\sqrt{\pi}} \left(\frac{2k T_H}{m_H}\right)^{3/2}$$
$$\cdot \exp(-\bar{v}_H^2)\{\bar{v}_H^4 + \tfrac{1}{2}\bar{v}_H^2 + \tfrac{1}{2}\}. \qquad (17)$$

Gain and loss processes

The particle flux continuity equation can be written in the following form:

$$\text{div } J = \text{div } J_{ne}^+ + \text{div } J_{ne}^- + \text{div } J_e^+ + \text{div } J_e^-, \qquad (18)$$

where the subscripts "ne", "e" stand for "non-escaping" and "escaping" particles and superscripts $+$, $-$ for upward- and downward-directed flux, respectively.

This means that the total flux is built up by all particles crossing the height level r in any direction. Each contribution to the total flux J is derived by considering the relevant gain and loss processes responsible for the individual fluxes, i.e.

$$\text{div } J_{ne}^+ = P_{ne}^{N,+} + P_{ne}^{I,+} + L_G,$$
$$\text{div } J_e^+ = P_e^{N,+} + P_e^{I,+}, \qquad (19)$$

where the indices N, I stand for gain or loss processes due to collisions of hydrogen with the neutral and ionized background gas components, respectively. The term L_G is a gravitationally caused loss term and will be explained later.

The flux of hydrogen atoms at each height level r is composed of upward- or downward-flying particles starting at lower or higher levels r' and reaching r without suffering any collisions with the background gas. Thus contributions from all these levels r' have to be integrated, yielding

$$J_{ne}^+(r) = \frac{\displaystyle\int_{r_c}^r \left(\frac{r'}{r}\right)^2 n_H(r') v_{ne}^{eff,+}(r', r) w_{ne}(r', r) \, dr'}{\displaystyle\int_{r_c}^r w_{ne}(r', r) \, dr'}. \qquad (20)$$

An analogous equation describes the upward-directed flux of escaping particles. Here $w_{ne}(r', r)$ is the probability that the particles will fly without collisions from r' to r and is given by an expression similar to Eq. (6).

The quantity $v_{ne}^{eff,+}(r', r)$ is the mean radial effusion velocity of non-escaping hydrogen atoms ascending from r' to r. We have to take into account that the flux of non-escaping particles at a height level r is built up only of those particles starting at a lower level r' and in fact arriving at the upper level r. This means that at r' these particles must have a velocity greater than a certain minimum velocity. Therefore, $v_{ne}^{eff,+}(r', r)$ is given by:

$$v_{ne}^{eff,+}(r', r) = \int_{v_0}^{v_{min}} \int_0^{\theta_{max}} \int_0^{2\pi} f_{ne}^\pm v \cos\theta \, d^3 v$$
$$+ \int_{v_{min}}^{v_{esc}} \int_0^{\pi/2} \int_0^{2\pi} f_{ne}^\pm v \cos\theta \, d^3 v. \qquad (21)$$

Here v_0 is that particular minimum velocity belonging to an angle $\theta = 0$ between the velocity vector of the particles and the radius vector. On the other hand, if the angle θ lies between 0 and θ_{max}, where

$$\theta_{max} = \arcsin\left(\frac{r}{r' v}\sqrt{v^2 - 2\gamma M\left(\frac{1}{r'} - \frac{1}{r}\right)}\right), \qquad (22)$$

then these particles must have a minimum velocity which is between v_0 and v_{min}.

By introducing the following two quantities

$$\bar{v}_{min} = \sqrt{\frac{m_H v_{min}^2}{2k T_H}} \qquad (23)$$

and

$$\bar{v}_0 = \sqrt{\frac{m_H v_0^2}{2k T_H}}, \qquad (24)$$

the mean radial effusion velocity of non-escaping particles can be given analytically by

$$
v_{ne}^{eff,\,+}(r',r) = \frac{0.5}{G_1 + G_2 + G_3} \sqrt{\frac{2kT_H}{\pi m_H}} \left(\frac{r}{r'}\right)^2
$$
$$
\cdot \{e^{-\bar{v}_0^2} - (1 + \bar{v}_{min}^2 - \bar{v}_0^2)\, e^{-\bar{v}_{min}^2}\}
$$
$$
+ \frac{0.5}{G_1 + G_2 + G_3} \sqrt{\frac{2kT_H}{\pi m_H}}
$$
$$
\cdot \{(1 + \bar{v}_{min}^2)\, e^{-\bar{v}_{min}^2} - (1 + \bar{v}_H^2)\, e^{-\bar{v}_H^2}\}. \tag{25}
$$

An expression for the mean radial effusion velocity of escaping particles is obtained more easily because particles having a velocity greater than the escape velocity at r' can always reach the upper level r, i.e.

$$
v_e^{eff,\,+}(r') = \int_{v_{esc}}^{\infty} \int_0^{\pi/2} \int_0^{2\pi} f_e^+\, v \cos\theta\, d^3 v
$$
$$
= \frac{0.5}{G_1 + G_2 + G_3} \sqrt{\frac{2kT_H}{\pi m_H}} (1 + \bar{v}_H^2)\, e^{-\bar{v}_H^2}. \tag{26}
$$

The quantity $v_e^{eff,\,-}(r',r)$ which enters the equation for the downward-directed flux of escaping particles J_e^- is given by

$$
v_e^{eff,\,-}(r',r) = \int_{v_{esc}}^{\infty} \int_{\pi - \theta_g}^{\pi} \int_0^{2\pi} f_e^-\, v \cos\theta\, d^3 v
$$
$$
= \alpha(r) \frac{0.5}{G_1 + G_2 + G_3} \sqrt{\frac{2kT_H}{\pi m_H}}
$$
$$
\cdot \left\{ \left(\frac{r}{r'}\right)^2 (1 + \bar{v}_H^2) + \bar{v}_0^2 \right\} e^{-\bar{v}_H^2}, \tag{27}
$$

where

$$
v_0 = \sqrt{2\gamma M \left(\frac{1}{r} - \frac{1}{r'}\right)}. \tag{28}
$$

Since a particle starting at an upper level r' must reach the lower level r in order to contribute to the flux considered there, its velocity vector can only attain angles with the radius vector within a certain range. The limiting angle θ_g is found by angular momentum considerations.

The flux considered is then given by

$$
J_e^-(r) = \frac{\displaystyle\int_r^{r_G} \left(\frac{r'}{r}\right)^2 n_H(r')\, v_e^{eff,\,-}(r',r)\, w_e(r',r)\, dr'}{\displaystyle\int_r^{r_G} w_e(r',r)\, dr'}. \tag{29}
$$

Here r_G is the limiting height of the transition region, i.e. $r_G = 800$ km.

To obtain an equation for the downward-directed flux of non-escaping particles we have to find an expression for the quantity $v_{ne}^{eff,\,-}$. But this is not trivial because this quantity is built up in two parts, namely (1) all particles produced at heights r' above r and flying downward; and (2) all particles starting at heights below r, reaching their apogee above r and again flying downward. To overcome these difficulties we reconsider the hydrogen velocity distribution f_H given in Eq. (1). Since this distribution is taken to be purely isotropic for the non-escaping branch of the velocity space, the upward-directed flux of non-escaping particles must be consistently balanced by a corresponding downward-directed flux, i.e.

$$
J_{ne}^-(r) = J_{ne}^+(r). \tag{30}
$$

Let us now consider the gain and loss processes and start with those processes due to charge-exchange collisions. These collisions can be essentially characterized in the following simple way: the neutral and the ionized particle exchange an electron without changing their flight paths in doing so; this means there is no momentum transfer in such a collision [see for instance Mapleton (1972)]. Therefore, the production of upward-flying escaping hydrogen atoms resulting from collisions between protons and oxygen atoms is simply given by

$$
P^{I,\,+} = n_O n_P \int_{v_{esc}}^{\infty} \int_0^{\pi/2} \int_0^{2\pi} q_{P,O}^{ex}(v_{rel})\, f_P\, d^3 v_P \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} f_O v_{rel}\, d^3 v_O
$$
$$
= n_O n_P q_{P,O}^{ex}(\bar{v}_{rel}) \int_{v_{esc}}^{\infty} \int_0^{\pi/2} \int_0^{2\pi} f_P\, d^3 v_P \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} f_O v_{rel}\, d^3 v_O, \tag{31}
$$

where $\bar{v}_{rel}$ is the mean relative velocity between the two particles and can be calculated by evaluating the sixfold integral of the second part in Eq. (31)

$$
\bar{v}_{rel} = \tfrac{1}{2} \cdot \{I_7(P, O) - I_9(P, O)\}. \tag{32}
$$

(The integrals I_1 through I_9 are explained in the Appendix.) Adding all the gain and loss processes in the same way for both upward-flying non-escaping and escaping hydrogen atoms, we come to the following two expressions, giving net production rates of H-atoms of both categories.

$$
P_{ne}^{I,\,+} = n_H n_P \cdot \{q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot [I_9(P, H) - I_6(P, H)]
$$
$$
+ q_{P,H}^{ex} \cdot I_3(P, H) + \alpha \cdot q_{P,H}^{ex} \cdot I_4(P, H)\}
$$
$$
- n_H n_P \cdot \{q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot I_9(H, P)\}
$$
$$
+ n_O n_P \cdot \{q_{P,O}^{ex} \cdot \tfrac{1}{2} \cdot I_9(P, O)\}
$$
$$
+ n_H \cdot n_{O^+} \cdot \{q_{H,O^+}^{ex} \cdot \tfrac{1}{2} \cdot I_9(H, O^+)\}. \tag{33}
$$
$$
P_e^{I,\,+} = n_H n_P \cdot \{q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot I_6(H, P)
$$
$$
+ \alpha \cdot q_{P,H}^{ex} I_2(P, H) + q_{P,H}^{ex} \cdot I_1(P, H)\}
$$
$$
- n_H n_P \cdot \{q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(H, P) - I_9(H, P)]\}
$$
$$
+ n_O n_P \cdot \{q_{P,O}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(P, O) - I_9(P, O)]\}
$$
$$
- n_H n_{O^+} \cdot \{q_{H,O^+}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(H, O^+) - I_9(H, O^+)]\}. \tag{34}
$$

The quantities $P_{ne}^{I,\,-}$ and $P_e^{I,\,-}$ are similarly found and given by:

$$
P_{ne}^{I,\,-} = P_{ne}^{I,\,+} \tag{35}
$$
$$
P_e^{I,\,-} = n_H n_P \cdot \{q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot I_6(H, P)
$$
$$
+ \alpha \cdot q_{P,H}^{ex} \cdot I_2(P, H) + q_{P,H}^{ex} \cdot I_1(P, H)\}
$$
$$
- n_H n_P \cdot \{\alpha \cdot q_{P,H}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(H, P) - I_9(H, P)]\}
$$
$$
+ n_O n_P \cdot \{q_{P,O}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(P, O) - I_9(P, O)]\}
$$
$$
- n_H n_{O^+} \cdot \{\alpha \cdot q_{H,O^+}^{ex} \cdot \tfrac{1}{2} \cdot [I_7(H, O^+) - I_9(H, O^+)]\}. \tag{36}
$$

The expression Eq. (35) holds since the velocity distribution functions entering the calculations are isotropic Maxwellians, whereas $P_e^{I,\,-}$ is not equal to $P_e^{I,\,+}$ because of the anisotropy of the distribution functions.

The gain processes due to elastic collisions between hydrogen and oxygen can be formulated in the following way:

$$P_e^{N,\,\pm} = \frac{0.5}{t_{rel}}\,(n'_{H,\,e} - n_{H,\,e}). \tag{37}$$

Here $n'_{H,\,e}$ is the equilibrium density of escaping hydrogen atoms if the hydrogen temperature were equal to the thermospheric temperature T_O, and t_{rel} is the relaxation time necessary for hydrogen to attain the oxygen temperature if there were no escape. This time was defined by Fahr and Nass (1978) as

$$t_{rel} = \frac{1}{q_{H,\,O}^{el}\, n_O\, \varepsilon\, \sqrt{\dfrac{8\,k\,T_H}{\pi\,m_H}}}. \tag{38}$$

The actual density of escaping hydrogen atoms is denoted by $n_{H,\,e}$. The quantities $n_{H,\,e}$ and $n'_{H,\,e}$ are thus given by

$$n_{H,\,e} = n_H\,\frac{G_2 + G_3}{G_1 + G_2 + G_3}, \tag{39}$$

$$n'_{H,\,e} = n_H \int_{v_{esc}}^{\infty} \int_0^{\pi} \int_0^{2\pi} M(v,\,T_O)\,d^3 v$$

$$= n_H \cdot \left[(1 - \mathrm{erf}(\bar{v}'_H)) + \frac{2}{\sqrt{\pi}}\,\bar{v}'_H\,e^{-\bar{v}'^2_H} \right], \tag{40}$$

$$\bar{v}'_H = \sqrt{\frac{\gamma\,M\,m_H}{k\,T_O\,r}}. \tag{41}$$

The factor 0.5 in Eq. (37) indicates that we only deal with the production into the upper/lower half of the velocity space. A corresponding expression for non-escaping H-atoms is obtained in analogy to Eq. (37)

$$P_{ne}^{N,\,\pm} = \frac{0.5}{t_{rel}}\,(n'_{H,\,ne} - n_{H,\,ne}). \tag{42}$$

Finally, we return to the term L_G of Eq. (19). As already mentioned, this term is a gravitationally caused loss term taking into account the fact that not all non-escaping particles leaving a lower level r' will finally reach r. This means L_G is simply the gradient of the upward-directed flux of non-escaping particles weighted by the probability that these particles do not suffer any collision between their origin and the upper level r, i.e.

$$L_G = \frac{\int_{r_c}^r \frac{d}{dr}\left\{ \left(\frac{r'}{r}\right)^2 n_H(r')\,v_{ne}^{eff,\,+}(r',r) \right\} w_{ne}(r',r)\,dr'}{\int_{r_c}^r w_{ne}(r',r)\,dr'}. \tag{43}$$

It should be mentioned here that we have a term similar to L_G in the continuity equation for downward-flying escaping particles. This term, however, is mainly due to the spherical geometry of the problem and accounts for those particles with perigees $r_p > r$.

Simplified concept for great heights

Above the 800-km level, collisions between hydrogen and the background gas components become less and less important. This is reflected by the fact that the value of $\alpha(r)$, which is a certain measure of the collision probability above r, is less than 0.2 (see next section). Therefore, the height levels are energetically decoupled from each other. In order to obtain an expression for the escape flux one only has to sum up the local gains into the escape regime. Due to the rare collisions above 800 km, we can also assume that the velocity distribution function for non-escaping hydrogen atoms will stay isotropic. Taking advantage of the fact that above 800 km only collisions between hydrogen and protonospheric protons need be considered, the fluxes are given by

$$J_e^+(r) = \left(\frac{r_G}{r}\right)^2 J_e^+(r_G)\,w_e(r_G,r)$$
$$+ \int_{r_G}^r \left(\frac{r'}{r}\right)^2 n_{H,\,ne}\,n_P$$
$$\cdot \left\{ q_{P,\,H}^{ex} \cdot \tfrac{1}{2}\cdot I_6(H,P) \right\} w_e(r',r)\,dr'$$
$$+ \int_{r_G}^r \left(\frac{r'}{r}\right)^2 n_{H,\,ne}\,n_P$$
$$\cdot \left\{ q_{P,\,H}^{ex}\,\frac{1}{2} \sqrt{1 - \left(\frac{\bar{v}_e(r_G)\,r_G}{\bar{v}_e\,r'}\right)^2}\,I_6(H,P) \right\} \tilde{w}_e(r_p,r)\,dr', \tag{44}$$

$$J_e^-(r) = \int_r^{r_{pp}} \left(\frac{r'}{r}\right)^2 n_{H,\,ne}\,n_P$$
$$\cdot \left\{ q_{P,\,H}^{ex}\,\frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{\bar{v}_e(r)\,r}{\bar{v}_e\,r'}\right)^2}\right] I_6(H,P) \right\} w_e(r',r)\,dr', \tag{45}$$

where r_{pp} is the height of the plasmapause. The subscript "ne" indicates that we deal only with the non-escaping part of the total hydrogen density, whereas collisions of escaping hydrogen atoms with protons are considered as ineffective since they probably reproduce an escaping H-atom. The third term of Eq. (44) takes into account downward-flying hydrogen particles produced at r' with a perigee $r_p \geq r_G$, which contribute to the escape flux at r (see Fig. 1). The terms given in curly brackets represent the product between charge-exchange cross-section and mean relative velocity. Since the term $\tilde{w}_e(r_p,r)$ differs slightly from the aforementioned collision probabilities, it is given here explicitly

$$\tilde{w}_e(r_p,r) = \int_{\pi/2}^{\pi - \theta_g} \left\{ \exp\left[-2\int_{r_p}^{r'} q_{P,\,H}^{ex}\,n_P\,\bar{v}_e^{rel}\,dr'' \right/ \right.$$
$$\left(\bar{v}_e'' \sqrt{1 - \left(\frac{r_p\,v_p}{r''\,\bar{v}_e'}\right)^2} \right) \right]$$
$$\cdot \exp\left[-\int_{r'}^r q_{P,\,H}^{ex}\,n_P\,\bar{v}_e^{rel}\,dr'' \right/$$
$$\left. \left(\bar{v}_e'' \sqrt{1 - \left(\frac{r'\,\bar{v}_e}{r''\,\bar{v}_e'}\right)^2 \sin^2\theta}\right) \right] \right\} \sin\theta\,d\theta, \tag{46}$$

where $\bar{v}_e$ is the mean velocity of escaping hydrogen atoms at r' but refers to the ion temperature, and $\bar{v}_e''$ is
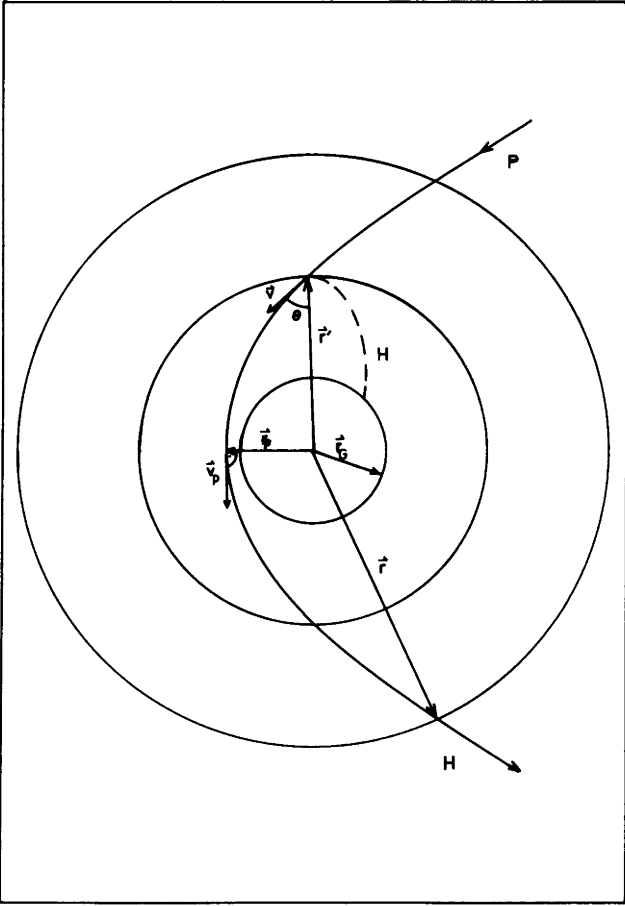
Fig. 1: A downward-flying hydrogen particle produced at r' with perigee $r_p \geqq r_G$ has a certain probability of contributing to the escape flux at r

Fig. 2: The density profiles of oxygen O, oxygen ions O^+ and protons H^+ in a 1,000 K exosphere

Fig. 3: Ion temperature distribution for an exospheric temperature of 1,000 K

the velocity at r''. r_p and v_p are the height of the perigee and the velocity at this height, respectively (see Fig. 1).

Results for the terrestrial case

Input parameters and numerical procedure

In order to calculate the density and temperature distribution of hydrogen we need height profiles for the ion densities and the ion temperature as well as for the density profile of atomic oxygen. The latter can be obtained from a barometric law with temperature T_O modified by a factor first given by Chamberlain (1963), taking into account the reduction of the populated velocity space volume with increasing geocentric distance. Thus one obtains

$$n_O(r) = n_O^B(r) \cdot \left\{ 1 - \sqrt{1 - \left(\frac{r_c}{r}\right)^2} \exp\left(-\frac{\gamma m_O M}{k T_O r} \frac{r_c}{r_c + r}\right)\right\},$$

(47)

where

$$n_O^B(r) = n_O(r_c) \exp\left[\frac{\gamma m_O M}{kT} \cdot \left(\frac{1}{r} - \frac{1}{r_c}\right)\right].$$

(48)

The density value at the lower boundary of the transition region $n_O(r_c)$ is taken from the CIRA model (Jacchia, 1972). The ion data are obtained from the
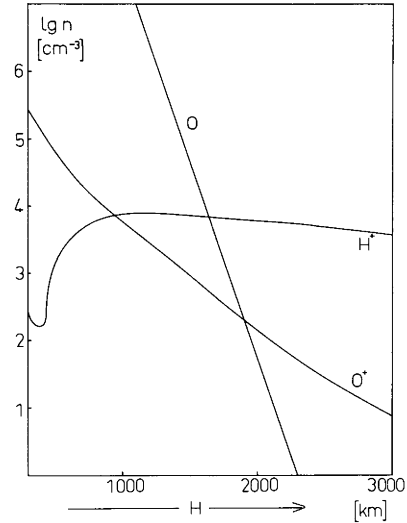
"International Reference Ionosphere" (Rawer et al., 1978) valid up to 1,000 km. For greater heights we use a data representation in accordance with Raitt et al. (1975). Thus we obtain the density profiles and ion temperature distribution for an exospheric temperature of 1,000 K, as shown in Figs. 2 and 3.
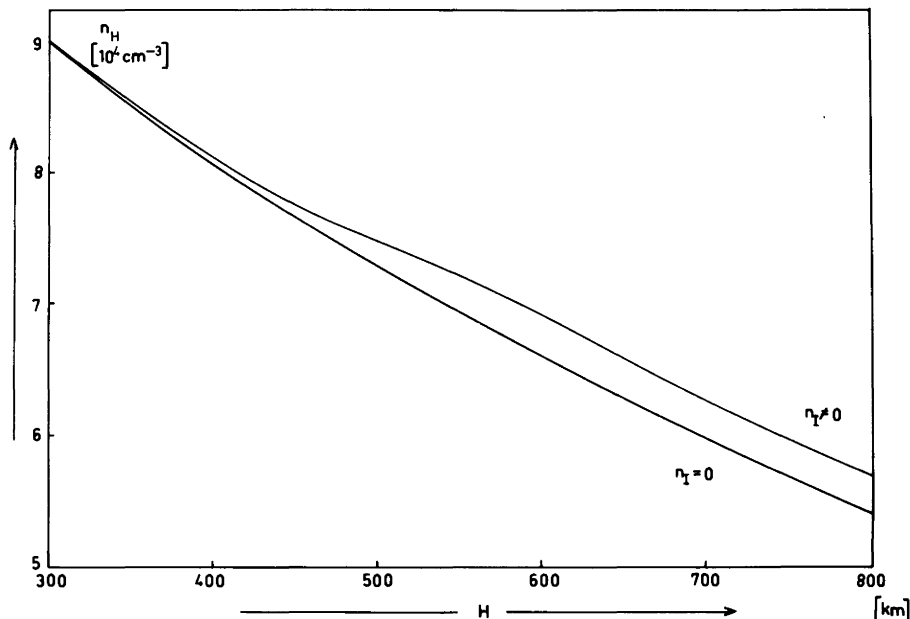
Fig. 4: The hydrogen density distribution. The *upper curve* ($n_I \neq 0$) is obtained by taking into account, in addition to elastic collisions, collisions between neutral and ionized particles. The *lower curve* is obtained by neglecting the latter collisions ($n_I = 0$)

To obtain the hydrogen temperature at each height level r we have to solve the heat conduction equation, Eq. (10). We solved this differential equation by using an extrapolation method (Bulirsch and Stoer, 1966), starting with the following boundary conditions:

$$T_H(r_c) = 1,000 \text{ K},$$

$$\left.\frac{dT_H}{dr}\right|_{r_c} = 0$$

and then integrating upwards step by step. The hydrogen density values entering these calculations can be obtained from the particle continuity equation, Eq. (18). In order to keep the integration error small, we choose a very narrow step width $\Delta r = r' - r$ of only 2 km. This small stepping also justifies our further expression, Eq. (25), for the mean radial effusion velocity of non-escaping particles. In this expression we obviously neglect all satellite particles with perigees between r and $r' = r - \Delta r$. But, of course, the number of these particles becomes negligible if we take this narrow step width. The solution of Eq. (18) at r requires a knowledge of downward-directed fluxes of escaping and non-escaping particles involving hydrogen densities and temperatures above r which are not yet available at this stage of calculation. In order to circumvent this difficulty, we start in a so-called zeroth iteration with a reduced particle continuity equation considering only upward-flying H-atoms:

$$\text{div } J = \text{div } J_{ne}^+ + \text{div } J_e^+$$
$$= P_e^{I,+} + P_{ne}^{I,+} + P_e^{N,+} + P_{ne}^{N,+} + L_G. \tag{49}$$

In this way we obtain a first-order solution for the hydrogen density and temperature distribution at each height level r up to the limiting height r_G of the transition layer. In a next iteration step – now beginning at r_G – we calculate n_H and T_H down to the height r_c, but now taking into account the whole particle continuity equation, Eq. (18). This iteration process is continued

until

$$\max_{r_c \leq r \leq r_G} \left| \frac{n_H^{i-1}(r) - n_H^i(r)}{n_H^i(r)} \right| = \text{const.}, \tag{50}$$

where $n_H^i(r)$ is the hydrogen density at height level r after the i'th iteration. Fortunately, we need only 4–5 iterations to fulfill the condition (50).

Density and temperature distribution

The hydrogen density values calculated with the above-mentioned method are shown in Fig. 4. The upper curve is obtained by taking into account, in addition to elastic collisions, collisions between neutral and ionized particles; the lower curve is obtained by neglecting the latter collisions ($n_I = 0$). The enhanced density values of the upper curve express the fact that collisions between protons and neutral oxygen atoms can produce neutral hydrogen atoms, whereas the counteracting process consisting of collisions between hydrogen atoms and oxygen ions is not very effective.

In order to illustrate the iteration process, the maximum relative differences of the calculated densities as a function of the iteration step are given in Table 1.

The relatively large differences after the second iteration step depends on the fact that, for the first time in both the upward- and downward-directed fluxes, hy-

Table 1

i	$\left\| \dfrac{n_H^{i-1} - n_H^i}{n_H^i} \right\|_{max}$	
	$n_I = 0$	$n_I \neq 0$
0	—	—
1	1.2×10^{-3}	2.11×10^{-3}
2	1.3×10^{-2}	1.50×10^{-2}
3	3.0×10^{-4}	1.50×10^{-3}
4	-5.0×10^{-4}	-1.00×10^{-3}
5	3.0×10^{-4}	1.50×10^{-3}

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drogen densities derived with the help of the whole particle continuity equation, Eq. (18), were used.

In Fig. 5 we show the resulting hydrogen temperature profiles, and, as expected, there is a temperature decrease with height. Due to the additional temperature coupling between hydrogen and the hot, ionized background gas components, this temperature decrease is less pronounced compared to that decrease caused by elastic collisions between hydrogen and oxygen atoms only.

The temperature reduction for the latter case ($n_I = 0$) obtained at the boundary of the transition region, i.e. $\Delta T_H(r_G) = 17$ K, can be compared with a value given previously by Fahr and Weidner (1977). They assume a linear temperature decrease over a certain height range Δr, so that the energy flux due to escape is balanced by a heat flux originating at lower heights.

$$\phi_e^{eff} = \kappa(r)\,\frac{T_O - T_H}{\Delta r}.$$

They obtained an estimate of the dimension Δr over which this temperature reduction takes place by calculating the average number of collisions needed to transfer the thermal energy difference $k \cdot \Delta T_H$ between oxygen and hydrogen to the hydrogen atoms. By using this simplified energy continuity equation they obtained a value of $\Delta T_H = 25$ K. Comparing this value to the one presented here, it can be deduced that their simple model overestimates the temperature reduction by a factor of 1.5, but that the expected effect was qualitatively well described.

In Fig. 6 we show the height dependence of the factor α needed for the escape branch of the hydrogen velocity distribution f_H. As mentioned above, a quantity β has to be calculated from the requirement of a continuous transition between the transition region and the higher atmospheric regions if charge-exchange collisions are considered ($n_I \neq 0$). This was done by comparing the downward-directed flux J_e^- calculated by Eq. (29) with the corresponding flux evaluated by Eq. (45). The two quantities become equal if

$$\beta = 0.15.$$

A value of two earth radii was taken for the height of the plasmapause in these calculations.

Above the 800 km level, $\alpha(r)$ can be calculated from the ratio J_e^-/J_e^+. In Fig. 6 these values are also shown. As expected, $\alpha(r)$ decreases with height monotonically.

Escape flux

The total escape flux, i.e.

$$J_e^{tot}(r) = J_e^+(r) - J_e^-(r), \tag{51}$$

is shown in Figs. 7 and 8. The fluxes are calculated with the help of Eqs. (29), (44) and (45). The dashed-dotted line shows the result if collisions between neutral and ionized particles are also taken into account ($n_I \neq 0$), while the solid line gives the result only for elastic collisions taken into consideration ($n_I = 0$). It can be seen that only above the 1,000 km level does the production of escaping hydrogen atoms by charge-ex-
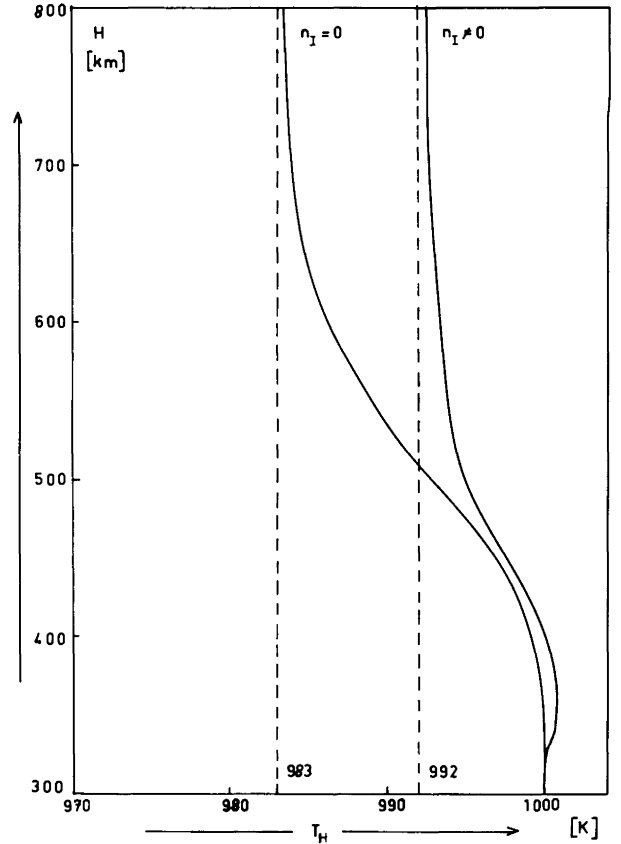

Fig. 5: The height profiles of the exospheric hydrogen temperature are shown. The labelling of the profiles has the same meaning as in Fig. 4
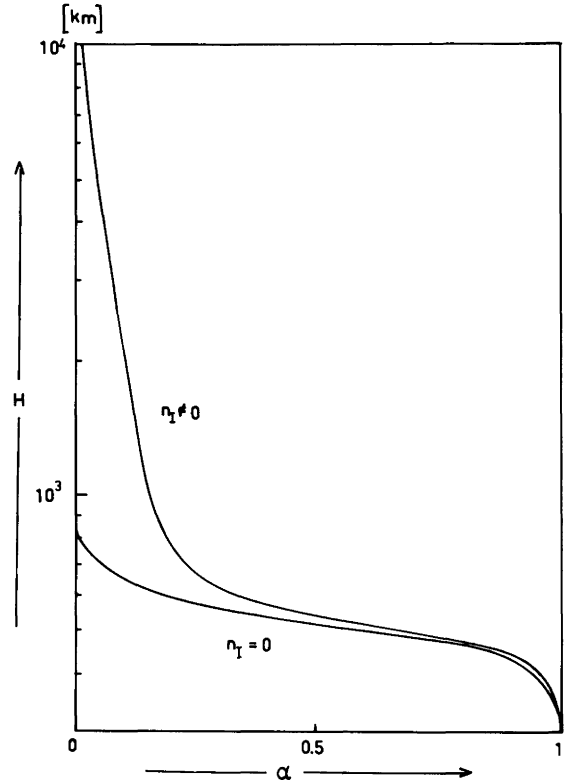

Fig. 6: Height profiles of the factor α needed for the escape branch of the hydrogen velocity distribution f_H [see Eq. (1)]. The labelling of the profiles has the same meaning as in Fig. 4
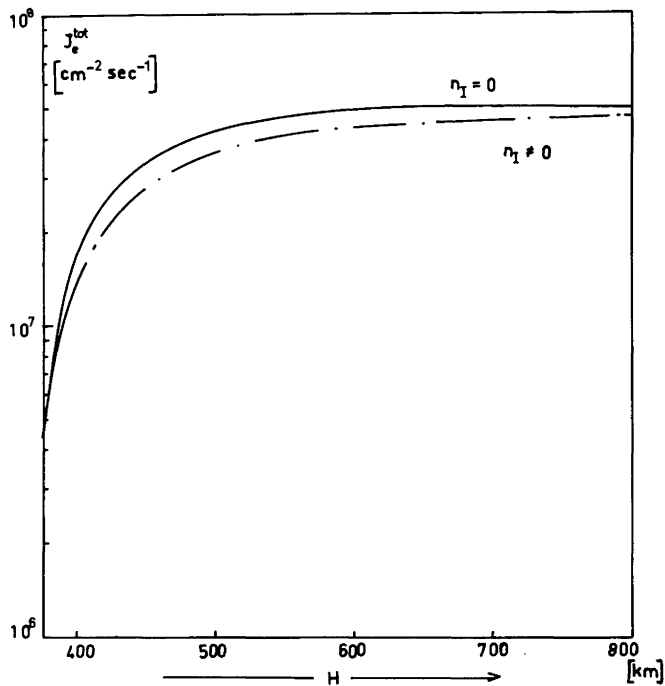
Fig. 7: The height dependence of the total escape flux is shown. The *dashed-dotted line* shows the result if collisions between neutral and ionized particles are taken into account ($n_I \neq 0$), while the *full line* gives the result only for elastic collisions taken into consideration ($n_I = 0$)

change processes lead to an enhancement of the total escape flux to values well above the pure thermal escape flux. Below this height the situation is somewhat reversed since charge-exchange collisions enhance both the upward-directed and the downward-directed escape flux. This means that the enhanced background gas density, due to the presence of the ions, partly impedes the escape flux of hydrogen atoms.

In Fig. 9 we show the height dependence of the upward-directed escape flux J_e^+ for both cases. Ob-

viously, the resulting flux given by the dashed-dotted line ($n_I \neq 0$) everywhere exceeds the flux given by the full line ($n_I = 0$). This is due to the fact that collisions between neutral and ionized particles lead to an enhancement of the local hydrogen density, especially the partial density in the escape branch, and therefore an enhanced escape flux J_e^+ results. At heights where no further production of escaping particles takes place, the decrease of J_e^+ must be proportional to $1/r^2$. This fact is indicated by the dashed line in Fig. 9.

An interesting result is found by comparing the pure thermal escape flux J_e^t with the total flux J_e^{tot} at the plasmapause

$$\frac{J_e^{tot}}{J_e^t} = 2.4. \tag{52}$$

This means that the ion-enhanced terrestrial escape flux is of the same order as the pure thermal one.

If the value of the total excape flux at the plasmapause, i.e. 1.7×10^7 cm^{-2} s^{-1}, were extrapolated downwards to a height of 500 km, one would arrive at

$$J_e^{tot}(500 \text{ km}) = 1.3 \times 10^8 \text{ cm}^{-2} \text{ s}^{-1}. \tag{53}$$

This value can be compared to 1.5×10^8 cm^{-2} s^{-1} given by Hunten and Strobel (1974) and Liu and Donahue (1974a; b; c), who calculated the upward diffusive hydrogen flux at the turbopause level produced by dissociation processes of H-bearing molecular constituents at lower heights. As is obvious, both values are in good agreement, showing that our theory of planetary escape is able to account for the continuity of the hydrogen flux.

Critical review of the results

In this section we want to compare our results concerning ion-induced flux contributions to the terrestrial H-escape with results of earlier publications in this field.
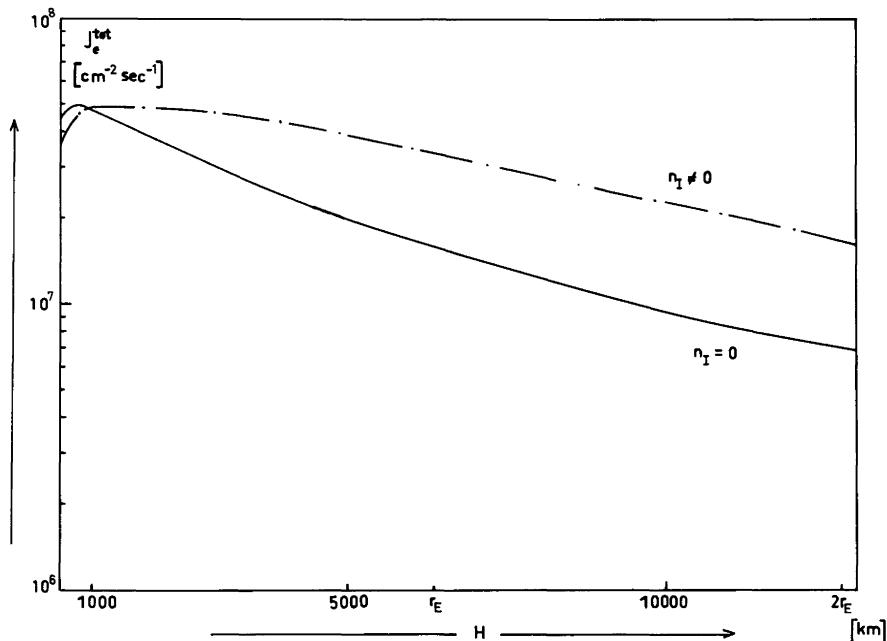

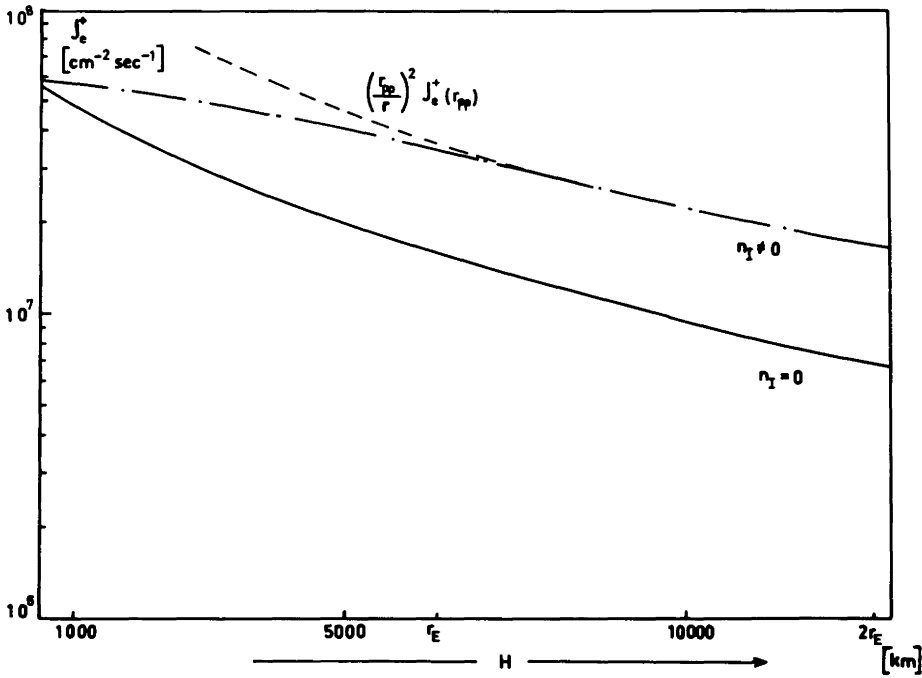**Fig. 8:** As Fig. 7, but the height dependence is shown up to the plasmapause

Fig. 9: The height dependence of the upward-directed escape flux is shown. The labelling of the profiles has the same meaning as in Fig. 7. The *dashed curve* is obtained if the value of the upward-directed escape flux were extrapolated downwards according to an inverse square law

As in all earlier contributions to this subject, it was demonstrated here that, due to the "charge-exchange contact" between neutral hydrogen and plasmaspheric ions ($T_I \approx 2 - 4 T_H$), the local H-density in the escape branch of f_H and hence the escape fluxes J_e are enhanced. However, as it turns out from our calculations the resulting enhancement is less pronounced than was claimed in earlier publications (Tinsley, 1973; Maher and Tinsley, 1977; Chamberlain, 1977; Maher, 1980; Maher and Tinsley, 1978; Hodges et al., 1981; Shizgal and Lindenfeld, 1982). A detailed analysis for this was given in a recent review by Fahr and Shizgal (1983).

Here we only want to mention the work of Shizgal and Lindenfeld (1982) which is the most suitable for a comparison with our results, with regard to ion-induced escape contributions to the terrestrial hydrogen escape. The latter authors use their former concept of local escape contributions (Lindenfeld and Shizgal, 1979) but expanded by inclusion of local charge-exchange collisions. Different height layers are considered as individually contributing to the total escape according to the local production rate of escaping H-atoms and the probability of an escape from this level to infinity is taken into account. After simplication of the collision process by the assumption of a constant charge-exchange cross-section and of Maxwell-Boltzmann distribution functions for the collision partner, the production term can be evaluated in a semi-analytic form.

The advantage of this approach clearly is its mathematical simplicity and clarity. This makes it very appropriate in studying the principal reaction of ion-induced escape fluxes to variations of the ion temperature T_I and of the neutral temperature T_O. However, it has to be kept in mind that this approach treats the ion-induced escape as de-coupled from the thermal one, i.e. no consistent treatment of local thermalization processes is included.

The main cause for their overestimation of the ion-induced escape fluxes is the assumption of constant ion temperature T_I and ion density n_I over the entire region above the critical height r_c up to the plasmapause. In particular, the very large escape flux enhancements that result for low temperatures T_O and high values of T_I/T_O should be drastically reduced if a variation of T_I with height, obeying $T_I = T_O$ at r_c, were considered. In view of this, enhancement factors (of 1–3 over thermal escape) seem to be indicated by the results of Shizgal and Lindenfeld (1982), which are entirely compatible with our results.

In our approach, we have carefully taken into account the local H-particle productions into the escape category due to both elastic and inelastic collisions using a consistently determined distribution function f_H for the calculations of collisional productions and losses, i.e. the depopulation of the escape branch has also been taken consistently into account. The thermalization or relaxation of the local function f_H towards the local background distribution function was also taken into account using energy and particle flow continuities in the transition layer above r_c. The quantity that is not consistently determined in this approach is the ion distribution function taken to be a Maxwellian with the local ion temperature $T_I = T_I(r)$. This is probably why even our results are still on the high side.

Appendix

In order to give expressions for the mean relative velocities, the following four integrals must be evaluated:

$$I_1(1,2) = \int_{v_{esc}}^{\infty} \int_{0}^{\pi/2} f_1 \, d^3 v_1 \int_{v_{esc}}^{\infty} \int_{0}^{\pi/2} f_2 \, v_{rel} \, d^3 v_2, \qquad (54)$$

$$I_2(1,2) = \int_{v_{esc}}^{\infty} \int_{0}^{\pi/2} f_1 \, d^3 v_1 \int_{v_{esc}}^{\infty} \int_{\pi/2}^{\pi} f_2 \, v_{rel} \, d^3 v_2, \qquad (55)$$

$$I_3(1,2) = \int_{0}^{v_{esc}} \int_{0}^{\pi/2} f_1 \, d^3 v_1 \int_{v_{esc}}^{\infty} \int_{0}^{\pi/2} f_2 \, v_{rel} \, d^3 v_2, \qquad (56)$$
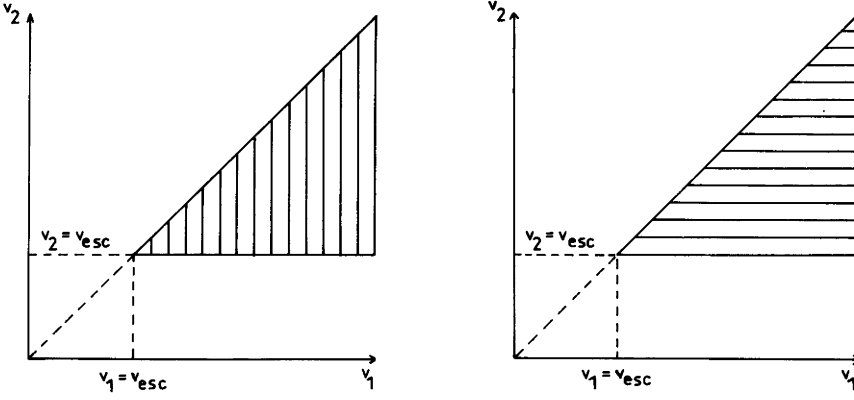
$$I_4(1,2) = \int\limits_0^{v_{esc}} \int\limits_0^{\pi/2} f_1 \, d^3v_1 \int\limits_{v_{esc}}^\infty \int\limits_{\pi/2}^\pi f_2 \, v_{rel} \, d^3v_2, \qquad (57)$$

where

$$d^3v_i = v_i^2 \sin\theta_i \, d\theta_i \, d\psi_i \, dv_i,$$

$$f_i = \left(\frac{m_i}{2\pi k T_i}\right)^{3/2} \exp\left(-\frac{m_i v_i^2}{2kT_i}\right) \qquad (i=1,2),$$

$$v_{rel} = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta_2}.$$

The integration over the angles can be carried out immediately by taking into account that

$$\int v_{rel} \sin\theta_2 \, d\theta_2 = \int \frac{v_{rel}^2}{v_1 \cdot v_2} \, dv_{rel}.$$

This yields for the integral (54):

$$\int\limits_{v_{esc}}^\infty \int\limits_0^{\pi/2} f_1 \, d^3v_1 \int\limits_{v_{esc}}^\infty \int\limits_0^{\pi/2} f_2 \, v_{rel} \, d^3v_2$$

$$= 4\pi^2 \left(\frac{m_1}{2\pi k T_1}\right)^{3/2} \left(\frac{m_2}{2\pi k T_2}\right)^{3/2} \int\limits_{v_{esc}}^\infty v_1^2 \exp\left(-\frac{m_1 v_1^2}{2kT_1}\right) dv_1$$

$$\cdot \left\{ \int\limits_{v_{esc}}^{v_1} \left[\frac{-2(v_1-v_2)^3}{6v_1 v_2} + \frac{2}{6v_1 v_2} (v_1^2+v_2^2)^{3/2}\right] v_2^2 \right.$$

$$\cdot \exp\left(-\frac{m_2 v_2^2}{2kT_2}\right) dv_2$$

$$+ \int\limits_{v_1}^\infty \left[\frac{-2(v_2-v_1)^3}{6v_1 v_2} + \frac{2}{6v_1 v_2} (v_1^2+v_2^2)^{3/2}\right] v_2^2$$

$$\left. \cdot \exp\left(-\frac{m_2 v_2^2}{2kT_2}\right) dv_2 \right\}.$$

The first double integral can be rewritten after consideration of the region of integration. Here we first have to integrate over a vertical strip from $v_2 = v_{esc}$ to $v_2 = v_1$ and then sum up all these strips (see Fig. 10). The same region of integration is covered by integrating first over a horizontal strip from $v_1 = v_2$ to $v_1 = \infty$ and then sum up all these strips, i.e.

$$\int\limits_{v_{esc}}^\infty \int\limits_{v_{esc}}^{v_1} \dots dv_2 \, dv_1 = \int\limits_{v_{esc}}^\infty \int\limits_{v_2}^\infty \dots dv_1 \, dv_2.$$

If we now introduce new variables of integration $(v_1 \to v_2, \; v_2 \to v_1)$ in the above-mentioned integral, we come to the following expression for integral (54)

$$\int\limits_{v_{esc}}^\infty \int\limits_0^{\pi/2} f_1 \, d^3v_1 \int\limits_{v_{esc}}^\infty \int\limits_0^{\pi/2} f_2 \, v_{rel} \, d^3v_2$$

$$= 4\pi^2 \left(\frac{m_1}{2\pi k T_1}\right)^{3/2} \left(\frac{m_2}{2\pi k T_2}\right)^{3/2}$$

$$\cdot \left\{ \int\limits_{v_{esc}}^\infty v_1^2 \exp\left(\frac{m_2 v_1^2}{2kT_2}\right) dv_1 \int\limits_{v_1}^\infty \left[\frac{-2(v_2-v_1)^3}{6v_1 v_2}\right. \right.$$

$$+ \frac{2}{6v_1 v_2} (v_1^2+v_2^2)^{3/2}\Big] v_2^2 \exp\left(-\frac{m_1 v_2^2}{2kT_1}\right) dv_2$$

$$+ \int\limits_{v_{esc}}^\infty v_1^2 \exp\left(-\frac{m_1 v_1^2}{2kT_1}\right) dv_1 \int\limits_{v_1}^\infty \left[\frac{-2(v_2-v_1)^3}{6v_1 v_2}\right.$$

$$\left. \left. + \frac{2}{6v_1 v_2} (v_1^2+v_2^2)^{3/2}\right] v_2^2 \exp\left(-\frac{m_2 v_2^2}{2kT_2}\right) dv_2 \right\}.$$

As can be seen by inspection of this expression, now only one double integral has to be evaluated. The result for the second integral is then gained from the result of the first one by changing m_1 into m_2 and T_1 into T_2. The integration of the given double integral is straightforward and has not been given in detail. Furthermore, it can be shown that:

$$I_5(1,2) = \int\limits_{v_{esc}}^\infty \int\limits_0^\pi f_1 \, d^3v_1 \int\limits_{v_{esc}}^\infty \int\limits_0^\pi f_2 \, v_{rel} \, d^3v_2$$

$$= 2 \cdot \{I_1(1,2) + I_2(1,2)\} \qquad (58)$$

and

$$I_6(1,2) = \int\limits_0^{v_{esc}} \int\limits_0^\pi f_1 \, d^3v_1 \int\limits_{v_{esc}}^\infty \int\limits_0^\pi f_2 \, v_{rel} \, d^3v_2$$

$$= 2 \cdot \{I_3(1,2) + I_4(1,2)\}. \qquad (59)$$

In addition, Eq. (58) yields the well-known result $(v_{esc} \to 0)$

$$I_7(1,2) = \int\limits_0^\infty \int\limits_0^\pi f_1 \, d^3v_1 \int\limits_0^\infty \int\limits_0^\pi f_2 \, v_{rel} \, d^3v_2$$

$$= \sqrt{\frac{8k}{\pi} \left(\frac{T_1}{m_1} + \frac{T_2}{m_2}\right)} \qquad (60)$$

and with the help of Eqs. (58), (59) and (60) we yield

$$I_8(1,2) = \int_0^{v_{esc}} \int_0^{\pi} f_1 \, d^3 v_1 \int_0^{v_{esc}} \int_0^{\pi} f_2 \, v_{rel} \, d^3 v_2$$

$$= \{I_7(1,2) - (I_5(1,2) + I_6(1,2))\} - I_6(2,1) \qquad (61)$$

and

$$I_9(1,2) = \int_0^{v_{esc}} \int_0^{\pi} f_1 \, d^3 v_1 \int_0^{\infty} \int_0^{\pi} f_2 \, v_{rel} \, d^3 v_2$$

$$= I_8(1,2) + I_6(1,2). \qquad (62)$$

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