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Some problems with *S, SKS* and *ScS* observations and implications for the structure of the base of the mantle and the outer core

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Abstract. Complicated radially symmetric models of the seismic velocity structure at the base of the mantle (Bullen's D″ region) and the uppermost outer core have been inferred from analyses of the waveforms and relative amplitudes of *S, SKS* and *ScS* phases. Using radially symmetric structure, it has been difficult to construct physically realizable models of the rheology of D″ that simultaneously satisfy *P* and *S* amplitudes and slownesses in the core shadow. These data are reviewed in the light of an increasing body of evidence that the structure of D″ is characterized by heterogeneities having a broad spectrum of scale lengths.

Depending on the region and range interval of D″ sampled, *S* waveforms can be found that support either a radially simple or complex model of D″. The complex models have one or more first-order discontinuities in velocity. The particle motion measured by three-component recordings of some *S*+*ScS* waveforms is consistent with a discontinuous increase in *S* velocity 250–300 km above the core-mantle boundary. The observed particle motion in these examples cannot readily or alternatively be explained by either general anisotropy or by strong lateral velocity gradients in D″. Sufficient variability in *S* waveforms and travel times exists, however, that any radially symmetric model having a strong degree of complexity should be accepted with caution until all of the competing effects of lateral heterogeneity and possible anisotropy in D″ are fully investigated. The distribution and scale lengths of heterogeneities in D″ may account for regional differences in the properties of D″ inferred from waveform data, including features that mimic intrinsic attenuation and anisotropy

Key words: Lower mantle – *S* waves – Earth structure

Introduction

Almost every conceivable variation of *P* and *S* velocities has been proposed for the lowermost 200 km of the mantle [Bullen's (1950) D″ region]. The data fit by the various models have included the amplitudes, waveforms, travel times and apparent slownesses of core-diffracted *P* and *S* waves (e.g. Doornbos and Mondt, 1979; Ruff and Helmberger, 1982) and *S, SKS* and *ScS* waves (e.g. Mitchell and Helmberger, 1973; Lay and Helmberger, 1983a, b). The only common feature among the many different studies is a recognition that the behaviour of body waves sampling the base of the mantle cannot be explained by simple extrapolation of the *P* and *S* velocity profile in the region immediately above D″.

This extrapolation is performed using the theory of finite strain and the assumptions of homogeneity of composition and phase and an adiabatic gradient in temperature. The departure of a velocity profile in D″ from this extrapolation can be explained by relaxing any single one or combination of these assumptions. Independent geophysical evidence suggests that the assumption to be relaxed is adiabaticity. Estimates of the geotherm of the lower mantle and the heat flux from the core point to the existence of a thermal boundary layer at the base of the mantle (Jeanloz and Richter, 1979). Hence, many interpretations of the velocity profiles of D″ have attempted to determine a plausible thermal structure to account for specific models. Velocity models having either a simple reduction or reversal in gradient with depth can usually be shown to be consistent with compositional homogeneity and the effects of a single thermal boundary layer at the core-mantle boundary (Cleary, 1974; Jones, 1977). More complicated models having first-order discontinuities or zones of rapid velocity increase or decrease have been taken to indicate either compositional changes (Anderson and Hanks, 1972; Ruff and Anderson, 1980) or the existence of multiple thermal boundary layers (Ruff and Helmberger, 1982).

Many of these D″ models and their interpretations have ignored a growing body of evidence that D″ is characterized by a heterogeneous velocity structure. This evidence includes studies of the short-period precursors to the *PKP-DF* branch (Haddon and Cleary, 1974; Husebye et al., 1976; Haddon, 1982) and studies that have inverted large catalogues of *P* travel times for properties of the lowermost mantle (Comer and Clayton, 1984; Dziewonski, 1984). A significant result of both types of studies is that D″ seems to possess 2%–3% variations in velocity over a broad spectrum of characteristic scale lengths, from a few tens of kilometres to over 1000 km. This paper will review both the radially symmetric and laterally heterogeneous models that have been proposed for D″ and consider how the distribution of heterogeneities in D″ might affect observations of long-period body waves, specifically *S* waves. The example data and synthetic calculations will concentrate on the results of Lay and Helmberger (1983a, b). The intent of the paper, however, is to emphasize that the search for a D″ model must include all of the possible effects of velocity heterogeneity on body waves sampling D″.

P, *S* and *K* velocity profiles

Simple radially symmetric

Figure 1 summarizes two *P* and *S* velocity profiles proposed for the base of the mantle. The models are chosen to be representative of simple forms. The PREM profiles (Dziewonski and Anderson, 1981) were constructed to satisfy a large data set of normal mode eigenfrequencies and *P* and *S* travel times. The feature that defines D″ in PREM is a second-order discontinuity in the *P* and *S* velocity profiles 150 km above the core-mantle boundary. Dziewonski and Anderson (1981) note that this feature is primarily dictated by a sudden change in slope of $\frac{dT}{d\Delta}$ of *P* waves at 90°. The scatter in *S* travel times precluded an inversion for such fine scale features and the gradient change shown in Fig. 1 followed from the assumption that second-order discontinuities in the *S*-velocity profile exist at the same depths as in the *P*-velocity model.

Doornbos and Mondt (1979) prefer negative rather than reduced positive velocity gradients in D″. They investigated perturbations of PEM (Dziewonski et al., 1975) needed to obtain agreement the spectral decay and apparent $\frac{dT}{d\Delta}$,s of core-diffracted *P* and *S* waves. Model PEM-L01 from this study has reversals of both *P*- and *S*-velocity profiles starting 75 km above the core-mantle boundary. The gradients with depth for *P* and *S* velocity are both equal to $-0.0019\ s^{-1}$. This gradient, which is positive with respect to radius *r*, is subcritical for *P* waves and close but less than the critical gradient for *S* waves $\left(\frac{d\beta}{dr}=0.002\ s^{-1}\right)$ in D″. Thus, turning rays can exist for both *P* and *S* waves in D″. Stacey and Loper (1983) have noted that a model having negative velocity gradients in D″ such as these would require a compositional variation to be consistent with a plausible thermal structure. They also criticized the equivalence of *P* and *S* gradients as being inconsistent with second-order elasticity theory, which requires an increase in Poisson's ratio with depth. This does not, however, allow for the effects of temperature.

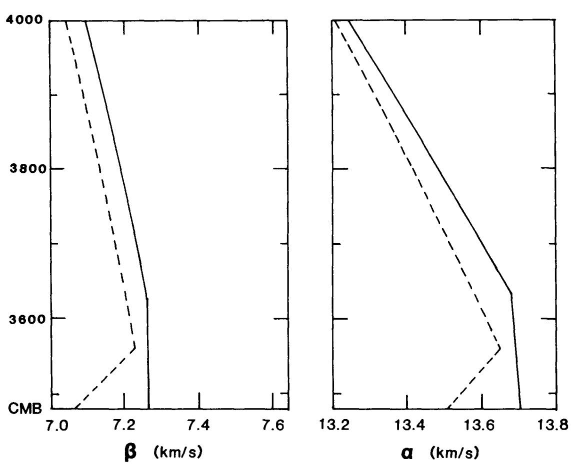

Fig. 1. The simplest models of D″ have reduced or reversed velocity gradients in the lowermost 50–200 km of the mantle. *Solid lines* are the isotropic PREM velocities at 1 Hz (Dziewonski and Anderson, 1981). *Dashed lines* are model PEM-L01 (Doornbos and Mondt, 1979). The lower velocities of PEM relative to PREM above D″ are due to the velocity dispersion of intrinsic anelasticity. PEM is referenced to 0.005 Hz rather than to 1 Hz

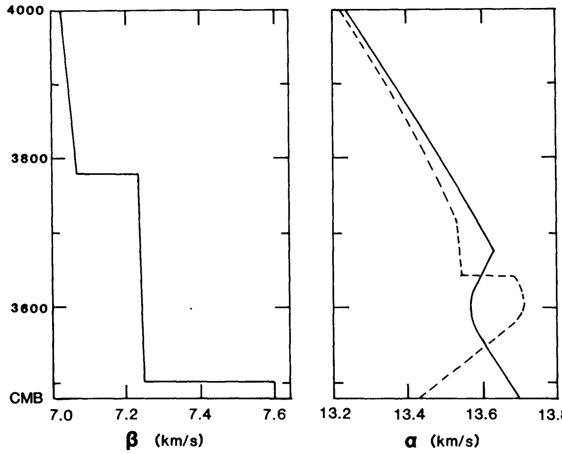

Fig. 2. Radially complex models have also been proposed for D″. The *S* model is by Lay and Helmberger (1983a). The thin (20 km thick) high *S* velocity layer at the base of the mantle was originally proposed by Mitchell and Helmberger (1973). The *P*-velocity profiles shown are (*solid line*) by Ruff and Helmberger (1982) and (*dashed line*) by Wright and Lyons (1981)

Slightly positive *P* and *S* velocity gradients in D″ are favoured by Mula and Müller (1980) and Mula (1981). Their analysis of diffracted *P*- and *S*-wave amplitudes and slownesses could not exclude a small negative gradient in *P* velocity of the size and width favoured by Doornbos and Mondt. They concluded, however, that the most reliable *S* data best agreed with a small positive rather than negative gradient in *S* velocity.

Complex radially symmetric

Another class of D″ models have velocity discontinuities or narrow zones in which *P* or *S* velocity rapidly increases or decreases (Fig. 2). Ruff and Helmberger (1982) fit small scale features of the amplitude curve of short-period *P*+*PcP* and diffracted *P* with a model in which *P* velocity rapidly decreases in a narrow zone 160 km above the core-mantle boundary and then smoothly increases. Although this study used a normalization and averaging procedure to reduce the amplitude fluctuations due to near receiver structure, some fluctuations may remain due to variations in source locations and near-source structure.

Wright and Lyons (1981) and Wright et al. (1985), using a treatment of array data to resolve interfering multiples in short-period *P* waveforms, favour a *P* structure having a rapid or discontinuous increase in *P* velocity 160 km above the core-mantle boundary. This is followed by a smooth decrease in *P* velocity toward the core-mantle boundary. A feature similar to this is found in Lay and Helmberger's (1983a, b) SLHO models of *S* velocity. A discontinuity is introduced 250–300 km above the core-mantle boundary to produce a triplication between *SH* and *ScSH* in the distance range 70°–85°. Without introducing lateral heterogeneity, however, this model cannot fit the *S* waveforms of specific data sets in the longer distance range beyond 100° (Schlittenhardt et al., 1985). A similarly shaped *P* profile cannot fit *P* waveforms unless the jump in *P* velocity is smaller by one-half or less in per cent than

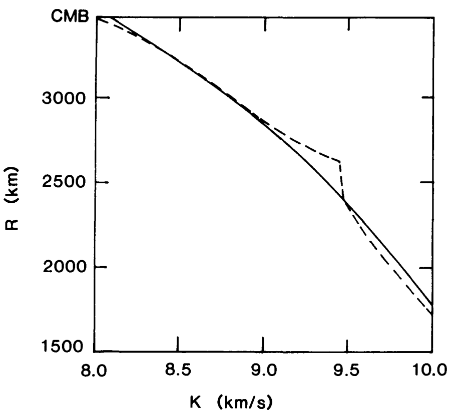

Fig. 3. Kind and Müller proposed a region of rapid velocity increase in the outer core to fit *SKS/SKKS* amplitude ratios (*dashed line*). The PREM velocity profile in the outer core is shown by the *solid line*

that in S velocity (Schlittenhardt et al., 1985; Lay and Young, submitted for publication). This P-velocity jump is closer in size to the one originally proposed by Wright and Lyons (1981) than the one most recently proposed by Wright et al. (1985).

Figure 3 compares the K-velocity profile of PREM with one determined by Kind and Müller (1977). Discussion of this model is included here because it is an another example of the distinctive behaviour of S waves interacting with the D″ region. Kind and Müller found that standard Earth models having a smooth profile of K velocity in the outer core predict *SKS/SKKS* ratios that are too large compared with those observed in the 100°–110° range. Such models also predict differential travel times $T_{SKS}-T_{SKKS}$ that are too small at ranges greater than 105°. In order to fit travel times and amplitudes, Kind and Müller introduce the structure shown in Fig. 3, which generates a short triplication of the SKS travel-time curve and destructive interference of phases in the *SKS* waveform. Schweitzer and Müller (personal communication) have now tested whether the inner core phases, which were neglected in the earlier study, can account for the observed amplitudes and travel times. They have concluded that inner core phases do not contaminate the *SKS* and *SKKS* waveforms.

Radially asymmetric

Lateral variations in the seismic velocities of the lower mantle have been found to be significant and resolvable in inversions of large catalogues of P travel times reported by the ISC (Dziewonski, 1984; Comer and Clayton, 1984). These inversions are aimed at retrieving radially asymmetric perturbations to a best-fitting spherically symmetric Earth structure. The strongest heterogeneities obtained in these studies are concentrated near 5700 km and the lowermost mantle. The velocity perturbations are as large as 0.1–0.2 km/s in D″, which may represent a smoothing-over of a more intense variation concentrated in either a narrower depth range or over smaller lateral scale lengths. Disregarding this possibility, these variations are nearly as large as those proposed in some of the complex, radially symmetric models of D″.

Anomalies in S travel time have generally been found to correlate with those in P travel time (Hales and Roberts, 1970) and this appears to also be true in the lower mantle. Schweitzer (1984) has compared the travel times of *SKS, SKKS* and *P4KP* predicted by the core model of Kind and Müller with observed travel times sorted into different regions. He found that a radially symmetric model cannot simultaneously satisfy observations in different regions and concluded that lateral heterogeneity must exist in either the outer core and/or the lower mantle. If the heterogeneity is placed in the lower mantle, the sense of the perturbations in S velocity correlate with the perturbations in P velocity found in the Dziewonski (1984) study. Schweitzer also reported regional variations in the *SKS/SKKS* ratio. Together with the variations in $T_{SKS}-T_{SKKS}$ times, these amplitude ratios are consistent with the sense of those expected for focussing and defocussing by lateral velocity variations; i.e. slow travel times correlate with high amplitude and fast travel times correlate with low amplitude.

Comparison of *S* data and *S* synthetics

Features of SH and SV waveforms predicted by PREM

Synthetic S waves have been computed using the isotropic PREM velocities and the techniques described in Choy (1977) and Cormier and Richards (1977). Profiles of the radial, transverse and an intermediate component of horizontal ground motion (Figs. 4 and 5) were calculated for the depth and focal mechanism of the 1970 September 5, Sea of Okhotsk event included in the studies of Mitchell and Helmberger (1973) and Lay and Helmberger (1983a, b). All components were calculated for a profile of stations along an azimuth at the epicentre representative of North American stations. The synthesis included the infinite set of phases $SKS+SKKS+\ldots$, which form an interference head wave along the underside of the core-mantle boundary.

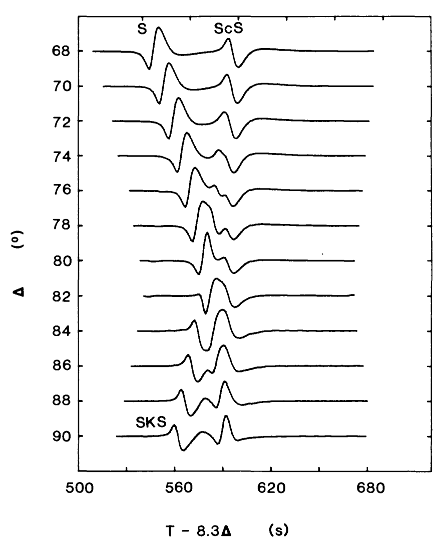

Fig. 4. Synthetic *SV* waveforms calculated in the isotropic 1 Hz PREM model for a point double couple source at 540 km depth and a LP-WWSSN instrument response. The focal mechanism was taken to be that determined by Strelitz (1975) for the September 5, 1970 Sea of Okhotsk earthquake. The takeoff azimuth was chosen to be 40°, which is representative of North American stations from the Sea of Okhotsk

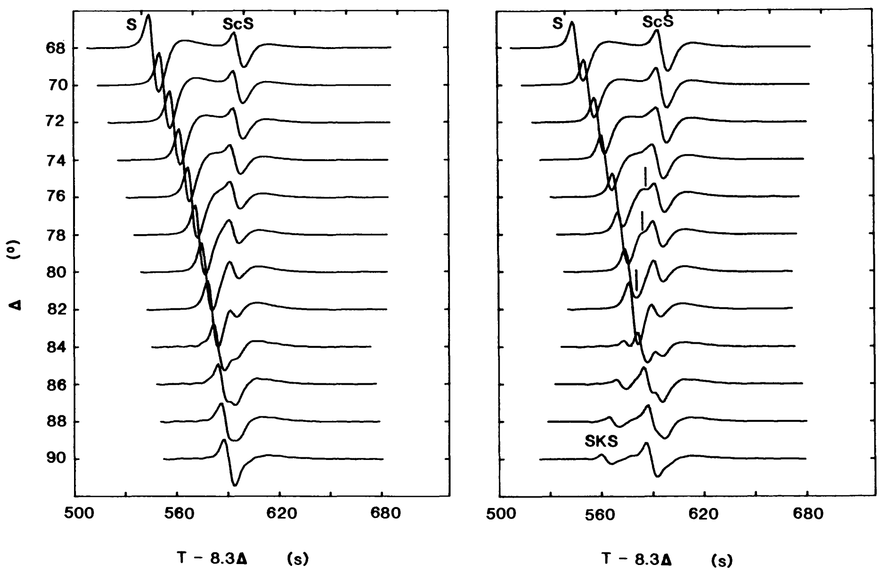

Fig. 5. Synthetic *SH* waveforms (*left*) and synthetic *SH* waveforms contaminated by *SV* energy (*right*) by rotating into an azimuth −20° from the true azimuth of arrival. The source is the September 5, 1970 Sea of Okhotsk earthquake described in Fig. 4. Note the perturbation in the waveforms at the *right* due to *SKS* starting at about 76°

Note that the *SKS* phase arrives between the *S* and *ScS* phases in the distance range 66°–80° and crosses over the S phase at 82°. Also note (Fig. 5) that for this focal mechanism at least, the *SKS* phase will be visible on a horizontal component even within 10°–15° of the true transverse component. It will be difficult to diagnose the perturbation marked in Fig. 5 as *SKS* on the basis of its measured slowness. This is because its small amplitude will make it hard to accurately correlate a peak or trough using a sparse long-period array. These features motivated a series of experiments, which are described in the following subsection, to determine whether the phase observed between transverse *S* and *ScS* by Lay and Helmberger (1983a, b) could be the *SKS* phase rather than a branch of a triplication due to a discontinuity near to core-mantle boundary.

SKS contamination of the transverse component

Laterally heterogenous structure along the ray path can modify both the orientation of the *S* polarization vector and the azimuth of the ray arrival (Cormier, 1984). When this occurs, two mechanisms exist by which *SKS* energy can be observed on the transverse component. First, deviations in ray path can cause the rotation of horizontal components using the great circle azimuth to contaminate the theoretical transverse component with *SKS* energy. Second, even for an *S* wave that has left the source as a pure *SH* wave, deviations in the *S* polarization vector can introduce an effective *SV* component of particle motion at the point of incidence of the *S* wave on the core-mantle boundary. This mechanism can produce *SKS* energy at azimuths that are nodal for *SV* radiation. Whether or not either mechanism operates, the *SKS* phase can be minimized by resolving the NS and EW components into radial and transverse components using an apparent rather than a great circle azimuth. The apparent azimuth can be determined either from an array measurement of vector slowness or from the orientation of the ellipsoid of particle motion.

As an experiment to see how the rotation of horizontal components can affect on *SH* or *SV* waveform, several events studied by Lay and Helmberger were digitized and rotated using a variable azimuth. Figure 6 shows the results

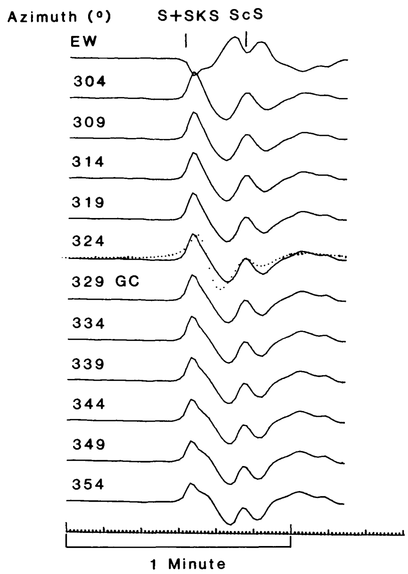

Fig. 6. *S* waveforms at station ATL recorded from the September 5, 1970 Sea of Okhotsk earthquake. ATL is 81.4° from this event. *S* has been retated into *SH* assuming a back azimuth that varies about the true great circle back azimuth of 329°. A synthetic *SH* (*dotted*) calculated in PREM fits most of the traces equally well until a back azimuth of 334° and greater, where an inflection due to the arrival of *SKS* begins to be amplified. At 81.4° ATL is close to the cross over distance of *S* and *SKS*. The time difference between *S* and *SKS* agrees well with the predictions of PREM

of one such experment. The *S* waveforms recorded by station ATL from the September 5, 1970 event were rotated using a sequence of different back azimuths. Note that beginning at the great circle azimuth of 329° and increasing towards rotations using larger back azimuths, a double inflection in the first downswing of the pulse is seen. This is generated by the interference of the *SKS* phase with the *S* phase and may be mistakenly identified as a branch of a new triplication. In this example, the waveforms and relative arrival times of all phases agree with those predicted by PREM, including the $S-SKS$ time difference.

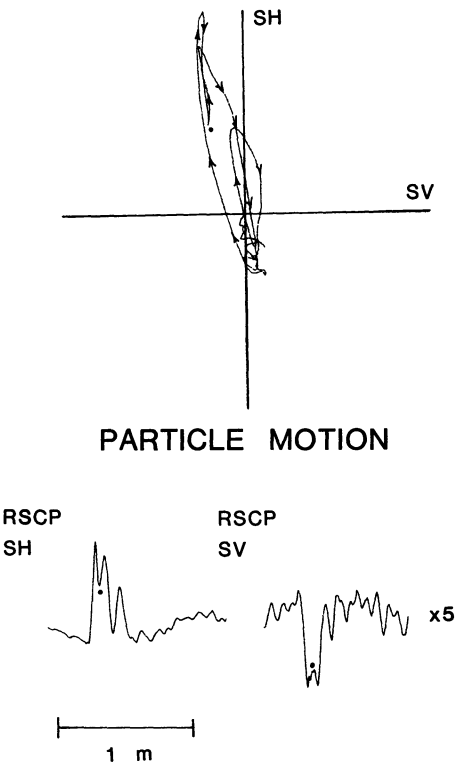

Fig. 7. Particle motion and S waveforms observed at station RSCP from a Sea of Okhotsk deep focus earthquake (April 20, 1984; 06 31 10.6 GMT; 50.12N, 148.74E; 582 km deep). The feature marked with the *dot* is not predicted by models of D″ such as PREM, which has a smooth profile in S velocity in the lower mantle. The particle motion remains nearly pure SH across a time window that includes the anomalous feature in the waveforms

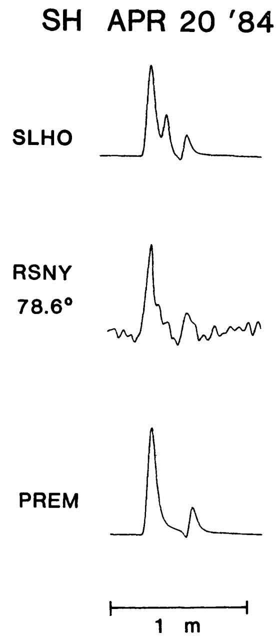

Fig. 8. A comparison of an $S+ScS$ waveform observed on the transverse component of station RSNY from the deep focus earthquake described in Fig. 7 with synthetic seismograms calculated in the PREM and SLHO models

Similar, more extensive tests (Lay and Young, submitted for publication), however, are not always successful in explaining the $S+ScS$ waveform observed on the transverse component. In many cases, good matches between observed and synthetic waveforms can be obtained using the PREM model only if so much SKS contamination is introduced as to be grossly inconsistent with both the focal mechanisms of the events and any likely lateral heterogeneity.

More precise tests of SKS contamination are possible with digitally recorded waveforms. The RSTN digital network, which is one of the few digital networks to have horizontal components in the broad and short-period bands, offers an opportunity to examine S particle motion. Broadband displacements from a deep focus earthquake beneath the Sea of Okhotsk, which occurred on April 20, 1984, were constructed from mid- and short-period recordings using the procedure described in Harvey and Choy (1982). From the analysis of short-period and broadband P waves, the source time function of this event is estimated to have a duration of approximately 2 s. Thus, at the time scale shown in Figs. 7–9, any complexity seen in the $S+ScS$ waveforms is due to the effects of earth structure rather than the source. The particle motion observed at station RSCP (82°) is shown in Fig. 7. The anomalous feature predicted by the D″ triplication of Lay and Helmberger is marked by a dot in the SH and SV waveforms and on the particle motion plot. The focal mechanism of this event and the azimuth of RSCP is such that SV radiation is nodal, with the SV waveform being about a factor of five smaller than the SH waveform. The particle motion remains nearly pure SH in the vicinity of the anomalous feature between the S and ScS waves. It does not suggest any SKS contamination, which would have signified itself in the plot of particle motion by an abrupt change to a more SV-like particle motion at the arrival of the anomalous feature.

In summary, some $S+ScS$ waveforms can be found in the 75°–82° range that are consistent with PREM, allowing for small amounts of SKS contamination. In many other cases, however, the waveforms are more consistent with the prediction of the SLHO model, having particle motions that cannot be explained by SKS contamination.

Comparision of data with PREM and SLHO synthetics

Using a source time function and focal mechanism of the April 20, 1984 event (Choy, personal communication), S waveforms were synthesized in the PREM and SLHO models and compared with those observed at the RSTN stations. All of the synthetics include an attenuation operator with $t^*_\beta = 2$, constant across the frequency band. In a model of the SLHO type, the interference of multiple branches of the travel-time curve can cause a broadening in the observed pulse. The relatively broad base of the direct S pulse at RSNY (78.6°) in Fig. 8 contains a suggestion of the intermediate phase predicted by the SLHO model. The size and sharpness of the velocity jump above the core-mantle boundary can be easily adjusted in an SLHO type model to obtain a better match to the RSNY waveform than that shown in Fig. 8. A good fit to the RSNY waveform can also be obtained with the PREM model by modifying the attenuation operator. In order to match the steep rise time of the waveform and to be consistent with the source duration inferred from P waveforms in the short-period band, the required attenuation operator must be strongly frequency dependent. At long periods, however, such an oper-

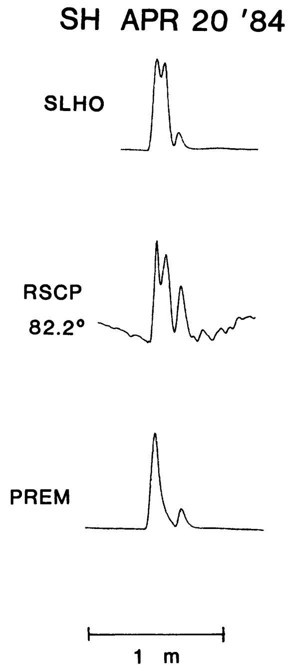

Fig. 9. A comparison of an $S+ScS$ waveform observed on the transverse component of station RSCP from the deep focus earthquake described in Fig. 7 with synthetic seismograms calculated in the PREM and SLHO models

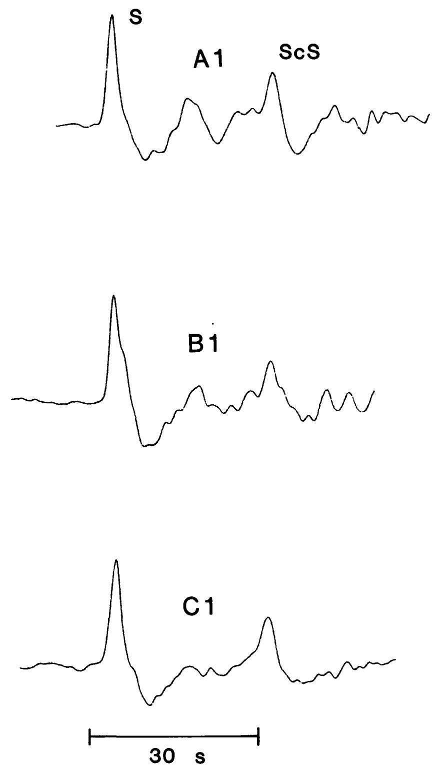

Fig. 10. Transverse $S+ScS$ waveforms observed at stations A1, B1 and C1 of the Graefenberg array from a deep focus earthquake beneath the Sea of Okhotsk (April 23, 1984; 21 40 35.5; 47.45N, 146.69E; 414 km deep). The data have been processed to simulate the response of a Kirnos broadband seismograph. The pulse between S and ScS is largest at A1, smaller at B1 and unidentifiable at C1

ator predicts an S-wave attenuation that is significantly larger ($t^*_\beta > 6$) than the S attenuation typically measured from a shallow focus earthquake (e.g. Brudick, 1978). A body wave from a deep-focus earthquake, such as the one studied here, is instead typically observed to be attenuated only by one half as much as one from a shallow-focus earthquake (e.g. Der et al., 1982). In this sense, an SLHO type model provides a better fit to the broad base of the waveform at RSNY than the PREM model coupled with an unreasonable attenuation model.

At station RSCP (82°), an SLHO type model provides unequivocally a better fit to the waveform than does PREM (Fig. 9). Again some slight adjustments in the size and depth of the D″ discontinuity are all that is necessary to obtain a good match.

A troubling feature remains in the waveform comparisons shown in Fig. 8: the observed ScS waveform at RSNY also appears to be broadened relative to the prediction of either the SLHO or PREM models. The broadening of the ScS phase might be dismissed as a consequence of its low signal-to-noise ratio. Similar broadening, however, is observed not only in the direct S and ScS at this station, but also at other RSTN stations at closer ranges. In these ranges, the pulse broadening cannot be easily explained by either the interference of triplicated branches or by an attenuation operator. Two possibilities that remain to be investigated are: (1) multi-pathing due to lateral rather than vertical heterogeneity and (2) shear-wave splitting due to anisotropy. The Discussion section outlines these possibilities in greater detail.

Waveform stability across an array

In other tests of the SLHO model, the stability and coherence of $S+ScS$ waveforms were observed across the Graefenberg array from another deep focus event beneath the Sea of Okhotsk. The centre elements, A1, B1 and C1, of three sub-arrays are approximately 50 km apart and the epicentral distance to any of the stations differs by no more than several tenths of a degree from 75°. Note that among the variations in the waveforms (Fig. 10) the phase between S and ScS is quite large on A1, smaller on B1 and practically absent on C1. The waveforms at B1 and C1 are generally more complex with a higher coda level than A1, suggesting that the crust may introduce more scattering and multi-pathing along the paths to A1 and B1 than the path to C1. In this example, the identification of the phase between S and ScS as a branch from a lower mantle triplication would be somewhat suspect unless one could demonstrate that the crustal effects are either unimportant along the paths to A1 and B1 or that they operate in such a way as to selectively defocus the intermediate phase along the path to C1.

Discussion

The scale lengths of heterogeneities and S polarization

The preceding examples show that SKS generally does not substantially contaminate the tranverse component of long-period records. In the short-period band, however, Murtha (1985) has found many examples of SKS on the transverse component. The variation of SKS contamination with frequency suggests a causal relation with the scale lengths and intensities of velocity heterogeneities of the earth. The possible scale lengths and intensities responsible for any *SKS* contamination are limited. This is because some distributions of heterogeneity are too intense to be consistent with the agreement in the long-period band of *S*-wave polarizations with focal mechanism solutions. Ray tracing experiments with *S* waves indicate that in order to strongly affect *S* polarization, heterogeneity in D″ must be significantly stronger than the 1% velocity fluctuations determined from inversions of large catalogues of *P* travel times. The ray tracing equations for *S*-wave trajectory and polarization have been integrated (Cormier, 1984) through both Dziewonski's heterogeneous model and models of descending slabs. These tests indicate that a large degree (greater than 10% velocity fluctuations) of coherent heterogeneity in D″ having long scale lengths (1000 km and greater) are required to produce observations of *SKS* on the transverse component. The only distribution consistent with any significant observations of *SKS* on the transverse component is a concentration of the heterogeneity in D″. Such a distribution would not affect *S* waveforms and polarizations measured in the long-period band at distances less than the *SKS* crossover (82°).

Ray tracing of the directly transmitted *S* wave, however, neglects the effects of multiple scattering and mode conversions by smaller scale heterogeneities. In the crust and lithosphere (Aki, 1980) and in the D″ region (Haddon and Cleary, 1974), the characteristic scale lengths of these smaller scale heterogeneities are estimated to be of the order of 10 km and the associated velocity fluctuations are estimated to be 10% or less. In order to explain the degree of *SKS* contamination observed on short-period records (Murtha, 1985) one must appeal either to the effects of scattering, in which multiple *SH* to *P* and *SV* mode conversions occur at sharp impedance contrasts, or to the effects of anisotropy.

General anisotropy from D″ heterogeneities

Any consideration of the distribution of heterogeneities in D″ should also include the constraints provided by analyses of *PKP-DF* precursors. The behaviour of the complex, short-period bundle of energy, which arrives before the *PKIKP* (*PKP-DF*) phase in the distance range (125°–143°) has been explained by seismic scattering of the *PKP-AB* and *-BC* phases by structural heterogeneity in D″ or by bumps on the core-mantle boundary (e.g. Haddon and Cleary, 1974; Husebye et al., 1976; Doornbos, 1978). This theory accounts for the detailed behaviour of the travel times, energy and frequency content and the time-distribution of apparent azimuth and slowness of the precursor wavetrain. The studies of the *PKP-DF* precursors have established the statistical properties of heterogeneities in D″ or bumps on the core-mantle boundary required to generate the amount of precursor energy observed. For heterogeneity in D″, these are a 2%–3% velocity fluctuation having scale lengths of several tens of kilometres. For bumps, these are heights of several hundreds of metres and having similar scale lengths. Significantly, an analysis and review by Haddon (1982) has demonstrated that more than one concentration of scale length of heterogeneity is needed to explain some of the precursor properties. Haddon finds that one type of precursor has an instantaneous arrival direction that systematically migrates with time along the precursor wavetrain. He suggests that this implies distributions of heterogeneities that are strongly directional in character, distributed along bands or lines having scale lengths up to 500 km.

Given the existence of this type of distribution of heterogeneities, it may be important to consider whether it may exhibit long-wavelength anisotropy. This is the anisotropy when a body wave averages over the elastic properties of a heterogeneous medium (Backus, 1962). Some examples discussed by Crampin (1984) include "periodic thin layering" and "checkerboard anisotropy". The directionality in the distribution of heterogeneities in D″ inferred from short-period *PKP – DF* precursors may effectively appear to be an anisotropic layer to long-period *S* waves.

This anisotropy may not only account for the apparent difference in arrival times on radial and transverse components observed for some *ScS* waves sampling D″ (Mitchell and Helmberger, 1973; Lay and Helmberger, 1983a; Fukao, 1984), but it may also account for apparent polarization and particle motion anomalies and fluctuations in amplitude and pulse width. The mechanism that produces fluctuations in amplitude and pulse width is described by Crampin (1981). Upon entering an anisotropic region, an *S* wave is split into two pulses. This splitting or birefringence is described by two fixed, orthogonal polarizations. These polarizations are determined from an eigenvector problem, whose solution depends on the elastic constants of the anisotropic region. For a region possessing general anisotropy, the fixed polarizations that travel at different velocities are not, in general, parallel to either the *SH* or *SV* components of particle motion. Thus both *SH* and *SV* waves will split in the anisotropic region and will be, thereafter, preserved along the remainder of the ray path. For weak anisotropy, in which the split pulses cannot be resolved by the instrument response, the split pulses either destructively interfere or can appear as a broader single pulse of diminished amplitude. The broadening may be mistakenly interpreted as the effect of a zone of increased intrinsic attenuation. Lateral variations in shear-wave splitting may then be an alternative mechanism that explains the low *ScSH*/*SH* ratios of Mitchell and Helmberger (1973), which they model with the effects of a thin-high-velocity layer at the core-mantle boundary, and the variations in the *SKS*/*SKKS* ratios of Kind and Müller (1977) and Schweitzer (1984).

Bolt and Niazi (1984) found a suggestion of azimuthal anisotropy in the slowness of diffracted *S* waves. Such observations, together with the differing arrival times of radial and transverse *ScS* reported in several studies, indicate that the possibility of general anisotropy in the D″ should not be dismissed until further study. The existence of general anisotropy can be diagnosed from the behaviour of the particle motion of *S* waves. In the example investigated in this paper (Fig. 10), the particle motion appeared to be nearly linear, having little evidence of either the elliptic or cruciform pattern characteristic of general anisotropy. Before any firm conclusions can be reached, however, more particle motions should be examined and compared with those expected for a radially homogeneous model of D″.

Another important model of D″ to investigate is one having transverse isotropy, in which *SH* and *SV* velocities are separate functions of ray parameter. This is a less radical generalization than a generally anisotropic model. It may be used to test the idea of Stacey and Loper (1983) of return convective flow being dominantly concentrated in a thin (order of 10 km) radial shell in the lowermost mantle.

Conclusions

Inversions of the travel times of *P* waves sampling D″ and studies of the detailed behaviour of short period precursors to *PKP*−*DF* indicate that the heterogeneous velocity structure of D″ is characterized by a broad spectrum of scale lengths, from several tens of kilometres to 1000 km. Precursor studies suggest that the distribution of the longer scale lengths are highly directional. One possibility, suggested by Haddon (1982), is that the heterogeneities may be either bands of corrugations on the core-mantle boundary or convective rolls having a short 10–20 km scale length and a longer 500–1000 km scale length concentrated in particular directions. The directionality of the longer scale lengths may reflect patterns of horizontal flow concentrated at the base of a lower mantle convection cell. Whatever the exact cause of this directionality, its existence suggests that it may be worthwhile to consider the possibility of long-wavelength anisotropy for *S* waves traversing D″.

Smaller scale heterogeneity (10 km or smaller), distributed throughout the earth, may account for the anomalies observed in *S* and *SKS* polarization on short-period instruments. These anomalies are most likely generated by multiple scattering and mode conversons rather than by changes in ray path or polarization vector of the directly transmitted waves. Longer scale heterogeneity (1000 km or greater), with velocity perturbations of the order of several per cent, cannot produce large anomalies in *S* polarization. Such anomalies may be able to focus or defocus *S* and *SKS* phases traversing the deep mantle, but the per cent velocity perturbations required to produce the observed amplitude fluctuations in the long-period band are as yet unknown.

Without at least perturbing a radially symmetric model with lateral heterogeneity at the longer scale lengths of several 1000 km, a model of D″ has yet to be found that can satisfy all of the body wave data. Lay and Helmberger (1983a, b), for example, have found that data require some heterogeneity in the depth of their D″ discontinuity. Based on *S* waveform data at longer distance ranges and the decay constant of diffracted *P* waves, Schlittenhardt et al. (1985) have shown that if Lay and Helmberger's velocity discontinuity exists, it is unlikely to be a global feature. In order to be consistent with a global data set of waveforms, travel times and slownesses, the properties of this discontinuity can be summarized as follows: (a) if such a discontinuity exits, it varies laterally in depth as well as in the size of its jump; (b) the scale lengths of these variations may be as short as 500–1000 km; (c) the per cent jump in *P* velocity must be one-half or less that in *S* velocity; (d) the relative size of the *P* versus *S* velocity jump suggests a thermal effect, but its sharpness suggests a compositional change.

Although differences in radially symmetric models may be reconciled by appealing to heterogeneity at scale lengths of several 1000 km, such an appeal may neglect potentially large effects of smaller scale heterogeneities. The effects of these smaller scale heterogeneities on long-period body waves have not yet been calculated, nor have data yet been analysed in ways that would highlight their potential effects on long-period *S* waves. A key test would be to examine the three-component particle motion of *S* waves traversing D″ on broad-band, three-component, digitally recorded seismic stations. These data can then be compared with synthetic particle motion calculated in both spherically symmetric models and heterogeneous/anisotropic models. Another important test of the effects of velocity heterogeneity at the smaller scale lengths would be to see how well a model of intrinsic attenuation together with strong constraints on the source-time function can account for the pulse broadening and rise times of *S, SKS* and *ScS* phases traversing the deep mantle. If such a model cannot successfully match the observed waveforms, then it may be possible that the heterogeneities of the mantle are introducing a stochastic dispersion of the type that has been found in numerical modelling of wave propagation in randomly heterogeneous media (e.g. Richards and Menke, 1983; Frankel and Clayton, 1984). The pulse broadening and waveform complexity due to this effect may mimic the type due to closely spaced triplications in a travel-time curve.

Finally, the stability and coherence of the waveform features used to infer any complex structure of D″ should be observed across densely spaced arrays. These experiments can be used to check whether a waveform feature is truly due to deep structure rather than local structure beneath a receiver.

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