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Apparent and intrinsic Q: the one-dimensional case

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Abstract. The propagation of plane waves through statistically layered media is investigated both numerically and with single-scattering theory in the one-dimensional case. Exact apparent or stratigraphic Q, Q_s, is determined from synthetic seismograms with the spectral-ratio method. Maximum velocity (impedance) fluctuations up to 30% ($\sim$40%) are studied; the fluctuations are uniformly distributed with zero mean. In all cases the trend of Q_s as a function of frequency is well described by the analytical Q_s, as determined from single-scattering theory under the assumption of an exponential autocorrelation function of the impedance fluctuations. The frequency dependence of the analytical Q_s^{-1} follows a Debye-peak function, its maximum is $\gamma^2/2$ and corresponds to the wavelength $4\pi a$(γ^2 = variance of relative impedance fluctuation, a = correlation distance). In further numerical calculations intrinsic or anelastic Q, Q_a, is introduced, and it is shown that total attenuation Q^{-1} agrees very well with the sum of apparent and anelastic attenuation, $Q_s^{-1}+Q_a^{-1}$. Finally, a simple, minimum-phase stratigraphic attenuation operator is derived which describes the amplitude decay and the dispersion in a one-dimensional random medium with good accuracy. Stratigraphic attenuation is similar to the anelastic attenuation of a standard linear solid.

Key words: Quality factor – Stratigraphic attenuation – Single-scattering theory

Introduction

The amplitudes of seismic waves are influenced by factors depending on the source, the propagation path and the receiver. O'Doherty and Anstey (1971) have given an often-quoted survey of these factors, in particular as they are of importance in seismic prospecting. Among the many influential factors on the propagation path are attenuation due to scattering, i.e. deviation of energy from the general propagation direction, and attenuation due to intrinsic losses or anelasticity, i.e. conversion of energy into heat. These two effects, and their relation, will be discussed in this paper in the context of a simple 1D model.

Anelasticity is conveniently described by the quality factor Q_a of the rock. Q_a depends, among others, on viscous processes between the rock matrix and liquid inclusions such as pore fluids or melt fractions, and on movements of dislocations through the mineral grains; Q_a can be frequency dependent. An accurate and frequently chosen way to introduce anelasticity into numerical codes for wave propagation is to use a complex wave velocity as a consequence of a complex (and minimum-phase) viscoelastic modulus. For instance, in the case of frequency-independent Q_a this velocity is

$$v_a = v_0\left(1+\frac{1}{\pi Q_a}\ln\frac{\omega}{\omega_r}+\frac{j}{2Q_a}\right), \qquad (1)$$

where ω is the (circular) frequency, ω_r a reference frequency, v_0 the real phase velocity at the reference frequency and j the imaginary unit. As is well known, the imaginary part of v_a describes the absorption, whereas the frequency-dependent real part gives the associated dispersion.

Since scattering has the same effect as anelasticity, namely a reduction of amplitudes, it is also described by a quality factor: the apparent or scattering quality factor Q_s. Q_s depends on the spatial structure of the scattering heterogeneities in the medium, on the size of the velocity and density fluctuations and on frequency or wavelength. Only recently has Q_s been determined successfully from these quantities with the aid of single-scattering theory in the 3D case (Wu, 1982; Sato, 1982a, b, 1984) and in the 2D case (Frankel and Clayton, 1986). The limits of this theory are not yet clear (Hudson and Heritage, 1981). Investigations of the 1D case can help to clarify them.

A 1D model, having variations only in one direction, say the depth (z) direction, has been studied several times for plane waves propagating perpendicularly to the interfaces (Schoenberger and Levin, 1974, 1978; Spencer et al., 1977; Sato, 1979, 1981; Menke, 1983; Richards and Menke, 1983). This model is of practical importance for seismic prospecting where seismic waves often propagate more or less vertically through horizontally layered sediments; Q_s in this case is sometimes called stratigraphic Q. This special model, which produces only strict forward scattering and strict backward scattering, is also studied here. Its advantage is that exact (numerical) results are easily obtained with matrix methods for complicated layering with arbitrary parameter fluctuations. Hence, Q_s can be determined exactly and compared with the result of single-scattering theory. First results of such comparisons in the 2D case are due to Frankel and Clayton (1986); they used finite-difference calculations to produce the seismograms for a random medium.

Our particular configuration consists of two identical half-spaces with an arbitrary number of homogeneous

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layers of total thickness D in between. Velocity, density and thickness of the layers are determined with the aid of uniformly distributed pseudo-random numbers and fluctuate about mean values which for velocity and density agree with the values outside the layer stack. The incident wave in the upper half-space, a P wave, is specified through its time function, $u(z=0, t)$, the transmissivity of the layer stack is determined with a matrix method (e.g. Temme and Müller, 1982), and the complete seismogram $u(D, t)$ in the lower half-space is calculated. The spectra at $z=0$ and $z=D$, $\bar{u}(0, \omega)$ and $\bar{u}(D, \omega)$, are related by the complex, frequency-dependent wave velocity $v(\omega)$ of the random medium:

$$\bar{u}(D, \omega)=\bar{u}(0, \omega)\, e^{-j\omega\frac{D}{v(\omega)}}. \tag{2}$$

From this equation, $v(\omega)$ can be determined by spectral division:

$$v(\omega)=-j\omega D/\ln\frac{\bar{u}(D, \omega)}{\bar{u}(0, \omega)}. \tag{3}$$

If the exponential term in Eq. (2) is written in the form

$$e^{-j\omega\frac{D}{v(\omega)}}=e^{-j\omega D\left[\frac{1}{c(\omega)}-\frac{j}{2c(\omega)Q(\omega)}\right]},$$

which implies a propagation term and a decay term, with the real phase velocity $c(\omega)$ and the real quality factor $Q(\omega)$ of the random medium, these quantities can be determined with the aid of Eq. (3):

$$c(\omega)=1/\mathrm{Re}\left[\frac{1}{v(\omega)}\right]$$

$$Q(\omega)=1/\left\{2c(\omega)\,\mathrm{Im}\left[\frac{1}{v(\omega)}\right]\right\}. \tag{4}$$

The quality factor so determined is an apparent or stratigraphic Q alone, if the medium is elastic. If anelasticity in the layers is assumed, e.g. by making the layer velocities complex according to Eq. (1), Q contains contributions both from anelasticity and from scattering. Calculations for elastic layers thus give Q_s and its dependence on velocity and density fluctuations and on frequency. Calculations for anelastic layers then allow us to compare Q_s and Q_a and, in particular, to see whether the total amplitude losses due to scattering and anelasticity follow the simple law

$$\frac{1}{Q}=\frac{1}{Q_s}+\frac{1}{Q_a}, \tag{5}$$

that is often assumed (e.g. Spencer et al., 1982; Richards and Menke, 1983; Menke and Dubendorff, 1985). In principle, this law holds only if these losses do not occur concurrently, but separately, e.g. when a zone with stratigraphic attenuation is *followed* by a homogeneous anelastic medium. Lerche and Menke (1986) have given a theoretical argument for the validity of Eq. (5) in the case of weak anelastic attenuation (see also Wenzel, 1982).

The purpose of this paper is three-fold. After a few examples of stratigraphic-Q calculations we first derive Q_s from single-scattering theory for the 1D case and compare it with the numerical results; quite good agreement is found for maximum velocity fluctuations from 5% to 30%. Second, we illustrate with a few examples that the superposition law (5) holds with very good approximation for the whole frequency range studied here; similar results have already been obtained by Richards and Menke (1983) who, however, have not considered the frequency dependence. Finally, a stratigraphic attenuation operator is derived which simulates the attenuation and dispersion effects of a random stratification.

Numerical results for stratigraphic Q

The numerical calculations have been performed for the mean values of P velocity and density, $\alpha_0=4000$ m/s and $\rho_0=2.39$ g/cm^3, and for N layers with total thickness $D=1600$ m; N varies from 100–1600. The relative velocity fluctuation

$$\xi(z)=\frac{\delta\alpha(z)}{\alpha_0} \tag{6}$$

has zero mean, a uniform distribution between $-p$ and $+p$ and hence a mean squared value or variance of $p^2/3$ (standard deviation $p/\sqrt{3}$); p varies from 5% to 30%. The relative density fluctuation

$$\chi(z)=\frac{\delta\rho(z)}{\rho_0}=\kappa\,\xi(z) \tag{7}$$

is assumed to be proportional to the velocity fluctuation, which is true if density and velocity are linearly related. We have used the relation $\rho=0.000173\,\alpha+1.695$ (α in m/s, ρ in g/cm^3) which is suitable for sedimentary rocks (Grant and West, 1965, Fig. 7-7); this implies $\kappa=0.290$. The density fluctuations vary from $-\kappa p$ to $+\kappa p$. The layer thickness fluctuates by a somewhat larger amount, $3p$, around the mean value D/N.

Figure 1 shows synthetic seismograms for $N=400$ and variable p. The incident wave (lowest trace) has an almost flat spectrum between 0 and 300 Hz. The layer-stack thickness D corresponds to 60 wavelengths at the central frequency 150 Hz. Thus, in terms of wavelengths the propagation distance in the random medium is rather long. With increasing p, pronounced low-pass filtering develops and seismogram duration increases strongly. For the largest p values, the effective duration is 1–2 s. For analysis we have taken the time interval 0.125 s, shown in Fig. 1, which is not longer than a few times the duration of the main part of the transmitted pulse; a similar restriction would be applied in the analysis of observed data.

The spectral-ratio method described in the introduction [Eqs. (3) and (4)] has been applied to seismograms of the kind shown in Fig. 1. Results for apparent Q are given in Figs. 2 and 3. The solid curves in these figures have quite an oscillatory character which masks to some extent the general trend. A minimum of Q_s is indicated in Fig. 2 at about 30–60 Hz and appears to be independent of p, whereas according to Fig. 3 it depends on N: it is shifted to higher frequencies for decreasing average layer thickness. The general decrease of Q_s with increasing p (Fig. 2) is an expected result. The dashed curves in Figs. 2 and 3 are discussed below.

Stratigraphic Q from single-scattering theory

In the following we give a *simple* derivation of Q_s in the 1D case; the results is identical with a result of Sato (1982b)

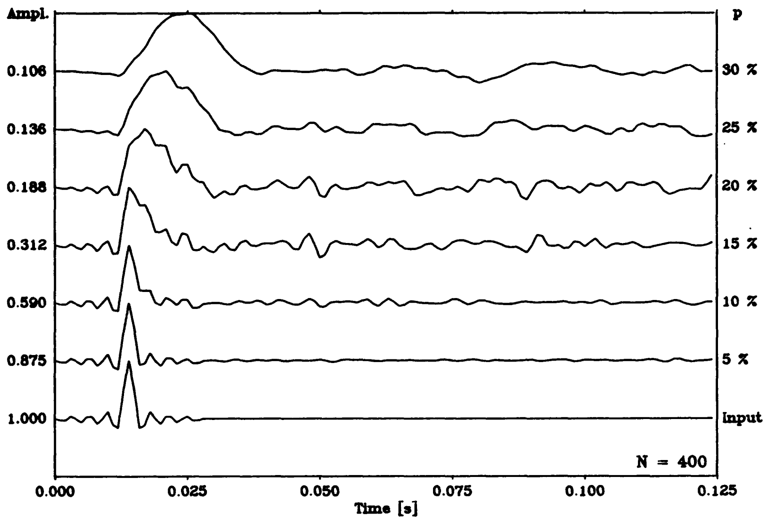

Fig. 1. Synthetic P-wave seismograms, illustrating the low-pass effect of transmission through a random sequence of $N=400$ layers. The frequency band of the input pulse is 0–300 Hz, the thickness of the layer stack is 60 wavelengths at 150 Hz, and the maximum relative velocity fluctuation p varies from 5% to 30%. Seismograms are normalized, with peak amplitudes given on the left

which was derived by the more complicated mean-wave formalism with travel-time corrections (see also Banik et al., 1985a, b). The basic idea is, as in the 3D case (Wu, 1982; Sato, 1982a, b, 1984), to calculate the energy of singly scattered waves, to identify this energy with the energy loss of the transmitted wave, and to relate this energy loss to Q_s.

We consider a particular frequency ω or wavenumber $k=\omega/\alpha_0$ of a plane P wave incident upon the random structure of total thickness D (Fig. 4). This wave has unit amplitude and unit intensity $E=1$. The impedance of the structure is

$$I(z)=I_0+\delta I(z)$$

with the mean value $I_0=\rho_0\,\alpha_0$ and the fluctuation $\delta I(z)$. The reflected wave from the whole layer in the single backscattering (or primary reflection) approximation has the complex amplitude

$$u_r=\int_0^D\left\{\frac{\delta I'(z)\,dz}{2I_0}\,e^{-j\omega\frac{2z}{\alpha_0}}\right\},\tag{8}$$

where the first term in the curly brackets is the reflection coefficient at coordinate z, and the exponential factor takes account of the phase shift due to the two-way travel time $2z/\alpha_0$. Partial integration of Eq. (8), together with the assumption $\delta I(0)=\delta I(D)=0$, yields

$$u_r=jk\int_0^D\eta(z)\,e^{-2jkz}\,dz=jk\,\bar{\eta}(2k),\tag{9}$$

where $\eta(z)$ is the relative impedance fluctuation $\delta I(z)/I_0$ and $\bar{\eta}(k)$ its Fourier transform. $\eta(z)$ is related to the relative velocity and density fluctuation, defined in Eqs. (6) and (7), by

$$\eta(z)=\xi(z)+\kappa(z)=(1+\kappa)\,\xi(z).\tag{10}$$

Equation (9) could also have been found by the Born approximation.

The intensity of the reflected wave and hence the intensity loss of the transmitted wave is $\Delta E=u_r\,u_r^*$, where u_r^* is the complex conjugate of u_r. From (9) one obtains $\Delta E=k^2 D\bar{R}(2k)$, with the Fourier transform

$$\bar{R}(k)=\int_{-\infty}^{+\infty}R(z)\,e^{-jkz}\,dz\tag{11}$$

of the autocorrelation function of $\eta(z)$,

$$R(z)=\frac{1}{D}\int_0^D\eta(z')\,\eta(z'+z)\,dz'.$$

$R(z)$ is dimensionless, and $R(0)$ is the variance of the relative impedance fluctuations.

Apparent or stratigraphic Q is then found by noting that Q_s^{-1} is defined as $1/2\pi$ times the relative intensity loss per wavelength. The relative intensity loss here is $\Delta E/E=\Delta E$, and corresponds to $Dk/2\pi$ wavelengths. One obtains therefore, using Eq. (11) and the evenness of $R(z)$,

$$Q_s^{-1}=\frac{\Delta E}{Dk}=k\bar{R}(2k)=2k\int_0^{\infty}R(z)\cos 2kz\,dz.\tag{12}$$

This result is in agreement with Eq. (38) of Sato (1982b); in the case of constant density there is also agreement with Eq. (45) of Wenzel (1982).

Our derivation of Eq. (12) has not used any statistical argument, since we have considered one particular impedance structure. Since Q_s depends only on the autocorrelation function of the impedance fluctuation, Eq. (12) represents also a whole ensemble of random media, provided that all have the same autocorrelation function.

In the following, we use an exponential autocorrelation function

$$R(z)=\gamma^2\,e^{-|z|/a}\tag{13}$$

with the variance γ^2 of the relative impedance fluctuation and the correlation distance a. This form is a good approximation to the autocorrelation functions or our numerical models, as the examples in Fig. 5 show. Inserting Eq. (13) into Eq. (12) yields:

$$Q_s^{-1}=\gamma^2\frac{2ak}{1+4a^2k^2}.\tag{14}$$

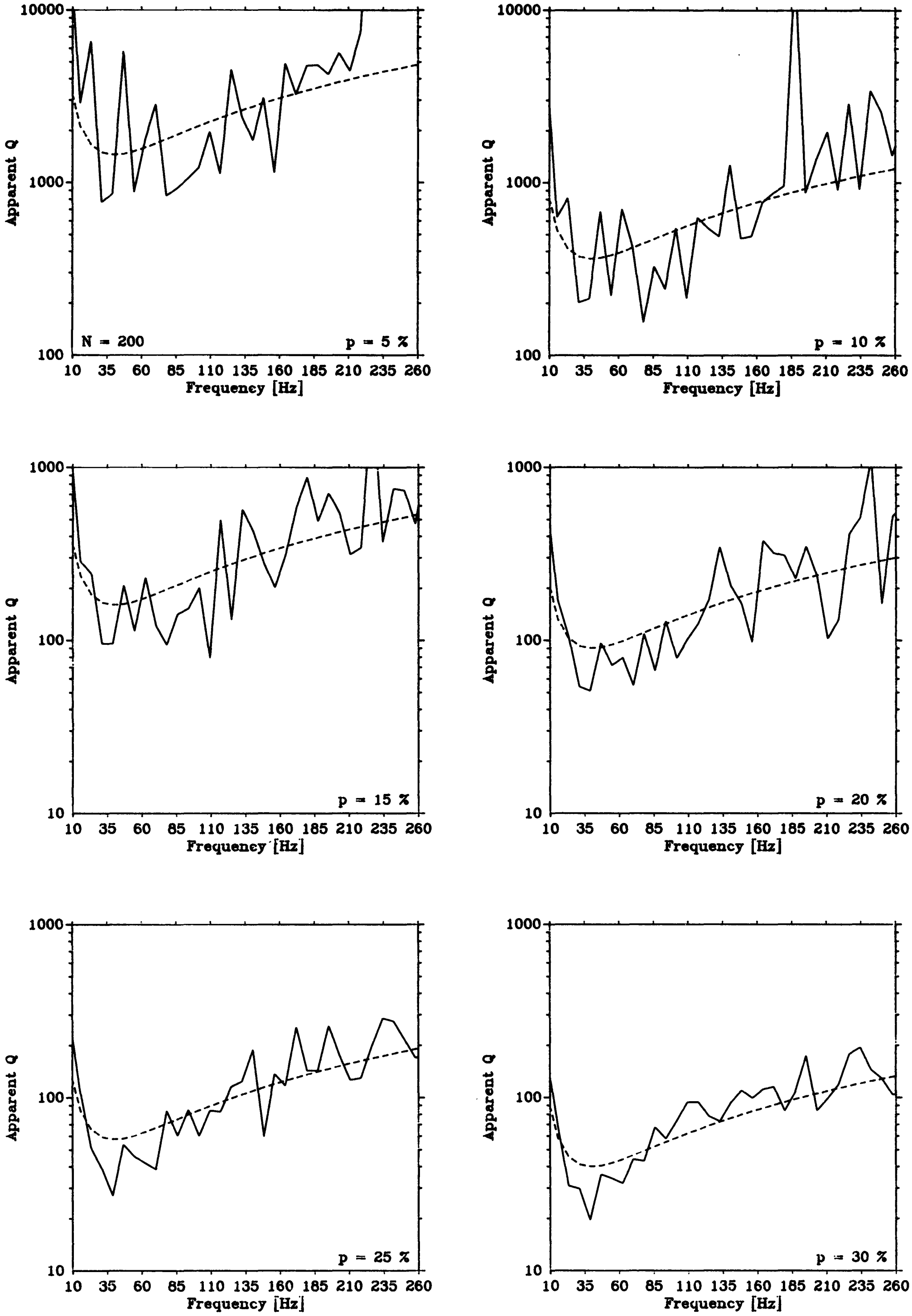

Fig. 2. Stratigraphic or apparent Q (*solid curves*) as a function of frequency for different values of maximum relative velocity fluctuation p in a stack of $N=200$ layers. The maximum relative fluctuation of layer thickness is $3p$, and the total thickness is 60 wavelengths at 150 Hz. Density fluctuation is $\kappa=0.29$ times the velocity fluctuation. The *dashed curves* are results from single-scattering theory and follow from Eq. (14)

This is a classical Debye-peak function which also plays an important role in the phenomenological description of anelasticity; for instance, Q_a^{-1} of a standard linear solid follows this law. The relaxation time τ of the Debye peak (14) is defined by $\omega\tau=2ak$ and is $\tau=2a/\alpha_0$; this is the two-way travel time related to the typical scatterer dimension a. The frequency dependence of stratigraphic attenuation is quite pronounced: the half-width of Q_s^{-1} is only about one decade. For quick estimates of stratigraphic Q one may use

$$Q_s \geq Q_{s_{\min}} = \frac{2}{\gamma^2}. \tag{15}$$

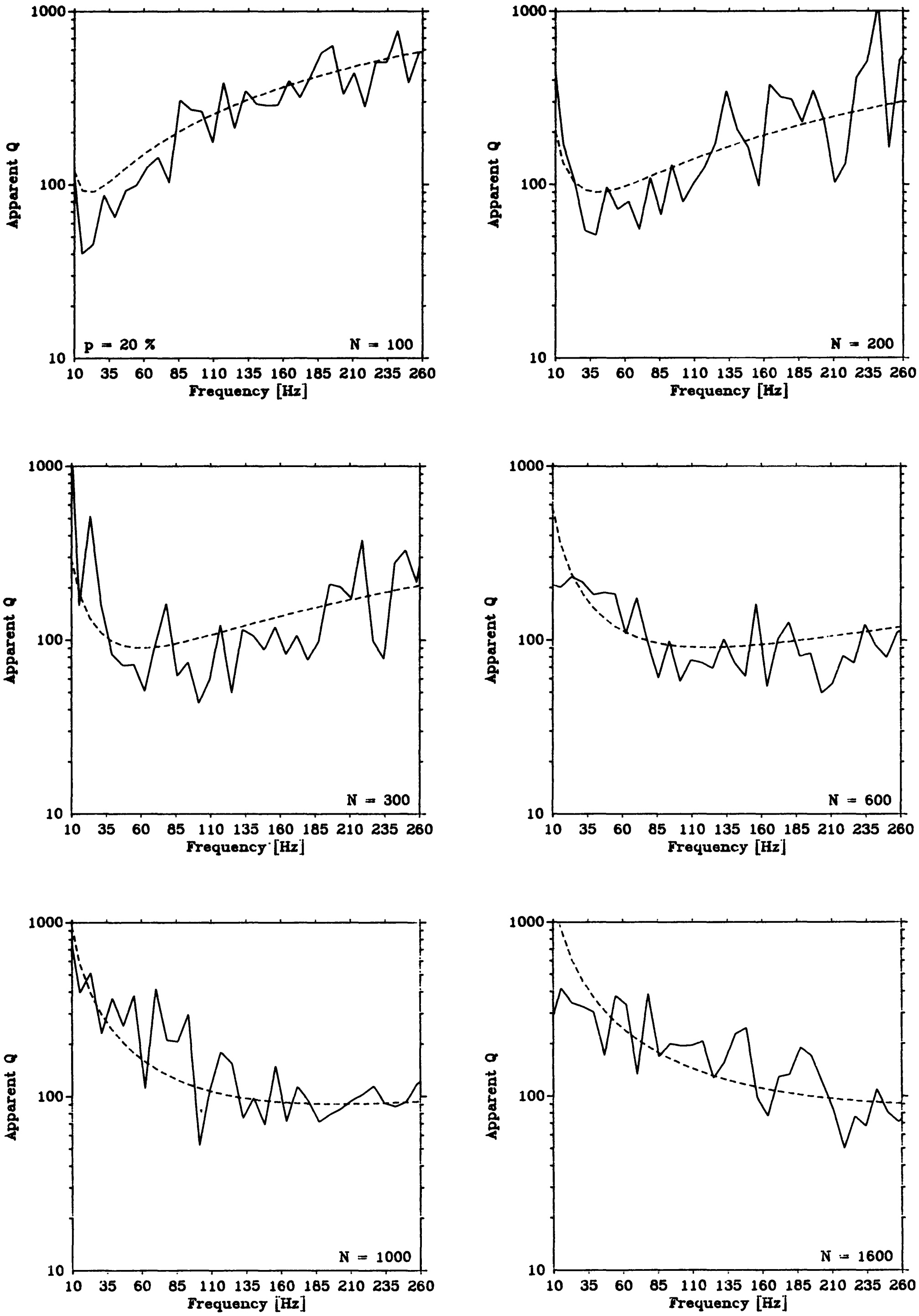

Fig. 3. Stratigraphic or apparent Q (*solid curves*) as a function of frequency for different values of layer number N and for maximum relative velocity fluctuation $p=20\%$. The *dashed curves* follow from Eq. (14). See also Fig. 2 caption

The minimum corresponds to the wavelength $4\pi a$, which is about one order of magnitude larger than the correlation distance.

Theoretical Q_s curves are included in Figs. 2 and 3 as dashed lines. The variance of the impedance fluctuation in these applications is related by

$$\gamma^2=(1+\kappa)^2\frac{p^2}{3}=0.555\,p^2 \tag{16}$$

to the maximum relative velocity fluctuation p [see Eq. (10)], and the correlation distance a is identified with the average layer thickness D/N.

In general, the trend of the numerical Q_s is well represented by the theoretical curves. The agreement is no worse for the larger p values than for the lower p values (Fig. 2). The frequency shift of the minimum of the numerical Q_s with increasing N follows the theoretical prediction (Fig. 3). Therefore, stratigraphic Q is represented by Eq. (14) with

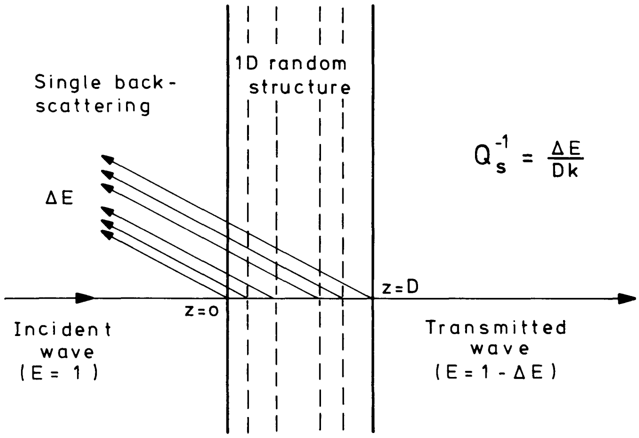

Fig. 4. Illustration of Q_s determination from the intensity loss ΔE due to single back-scattering at a 1 D random structure

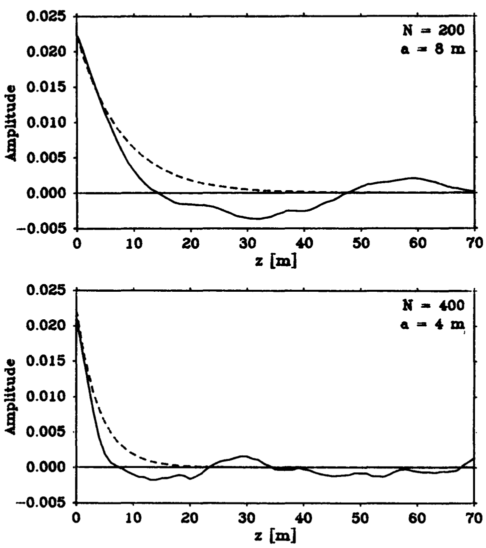

Fig. 5. Autocorrelation functions of the impedance fluctuations of two random structures having $N=200$ and $N=400$ layers, respectively, and a maximum velocity fluctuation $p=20\%$ (*solid curves*). The theoretical curves (*dashed*) are exponential functions, Eq. (13), with the variance $\gamma^2=0.0222$ of the impedance fluctuation according to Eq. (16). The correlation distance a agrees with the average layer thickness

good accuracy over the whole frequency range of seismic prospecting and for a rather broad range of impedance fluctuations.

Superposition of stratigraphic and intrinsic Q

The numerical calculations for Figs. 2 and 3 have been performed for purely elastic layers, and the resulting Q is purely stratigraphic. Allowing for anelasticity in the layers leads to an intrinsic contribution to the total Q. The numerical method for seismogram calculation allows incorporation of anelasticity via complex P-wave velocities according to the simple law (1), i.e. the intrinsic Q factor Q_a is assumed to be frequency independent. The reference circular frequency ω_r corresponds to the upper frequency limit 300 Hz, and v_0 fluctuates around the mean value $\alpha_0=4000$ m/s. Varying the value of Q_a for a particular random structure yields seismograms with different amounts of anelasticity-related low-pass filtering, in addition to the common low-pass filtering due to scattering (Fig. 6).

Seismogram analysis by the spectral-ratio method gives total Q (Fig. 7, left part); in the case $Q_a=\infty$, it represents the stratigraphic Q of the structure: $Q=Q_s$. Then, a test of the superposition hypothesis (5) for the attenuation is possible for the cases $Q_a<\infty$. A typical example is shown in the right part of Fig. 7 for $Q_a=100$. The solid curve labelled $1/Q$ follows from the total-Q curve for $Q_a=100$ (Fig. 7, left part); the solid curve labelled $1/Q_s$ corresponds to the elastic case. The dashed curve is the sum $1/Q_s+1/Q_a$ and represents Eq. (5) exactly. The agreement with the $1/Q$ curve is very good; similar results were found in all other cases studied.

Therefore, the superposition hypothesis (5) has a solid basis as already stated, e.g., by Richards and Menke (1983); certainly, it applies also in cases where Q_a is frequency dependent and where the frequency dependence of Q_s is more pronounced than in Fig. 7.

Stratigraphic attenuation operator

Anelastic attenuation can be described by a complex wave velocity v_a or, alternatively, by an anelastic attenuation operator (usually called a dissipation operator). Equation (1) is an example of v_a, and the corresponding dissipation oper-

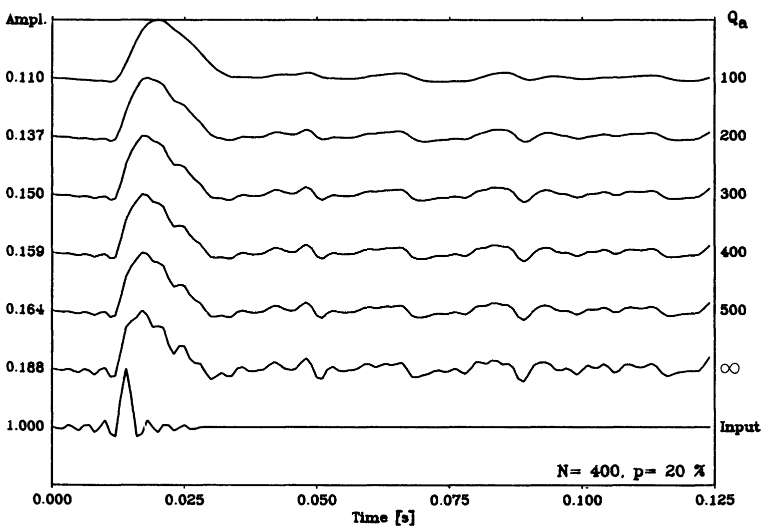

Fig. 6. Synthetic seismograms for a random structure with $N=400$ layers and maximum velocity fluctuation $p=20\%$. Anelasticity in the layers with frequency-independent Q factor Q_a is assumed; Q_a is the same for all layers. The seismogram for $Q_a=\infty$ displays only stratigraphic attenuation

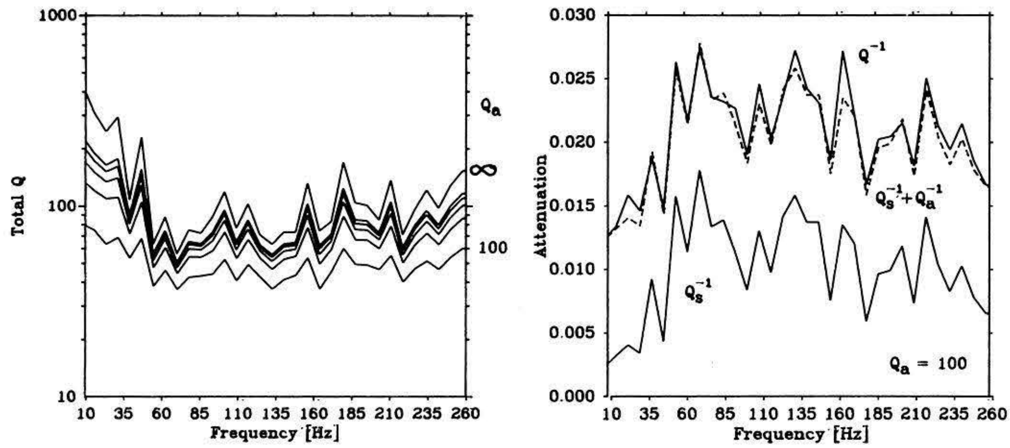

Fig. 7. *Left*: Total Q determined by the spectral-ratio method from the seismograms of Fig. 6. *Right*: Test of Eq. (5) in the case $Q_a = 100$

ator in the frequency domain follows from Eq. (2) through $v = v_a$:

$$A(\omega, T) = \exp\left[-\frac{T\omega}{2Q_a}\left(1 - \frac{2j}{\pi}\ln\frac{\omega}{\omega_r}\right)\right]. \tag{17}$$

$T = D/v_0$ is the travel time. The elastic response of the medium is multiplied by $A(\omega, T)$, and the result is the anelastic response. Equation (17) represents a minimum-phase operator in the time domain, apart from a time shift which depends on the value of the reference frequency ω_r.

Likewise, stratigraphic attenuation can approximately be represented by a complex velocity v_s or a stratigraphic attenuation operator. To find these we start from the fact that the transmission operator of a layer stack, corrected for the phase shift due to the one-way travel time through the stack, is minimum phase (Sherwood and Trorey, 1965). Therefore, as in the case of anelastic attenuation, the phase velocity c, which is very close to the real part of v_s, follows from Q_s^{-1} by a dispersion relation:

$$\frac{1}{c(\omega)} = \frac{1}{\alpha_0}\left\{1 + \frac{1}{2\pi} P \int_{-\infty}^{+\infty} \frac{Q_s^{-1}(\omega')}{\omega' - \omega} d\omega'\right\}.$$

An alternative in the present case, where Q_s^{-1} is given by Eq. (14), is to use directly the dispersion result for a standard linear solid (e.g. Kanamori and Anderson, 1977) to obtain c. In either case

$$v_s = \alpha_0\left[1 - \frac{0.5\,\gamma^2}{1 + \tau^2\omega^2}(1 - j\tau\omega)\right] \tag{18}$$

is found; γ^2 and τ have been defined earlier, and α_0 is the (real) high-frequency limit of v_s. Then Eq. (2) can be used, now with $v = v_s$, to derive the stratigraphic attenuation operator:

$$S(\omega, T) = \exp\left[-\frac{0.5\,\gamma^2\,T\omega}{1 + \tau^2\omega^2}(\tau\omega + j)\right], \tag{19}$$

where $T = D/\alpha_0$ is the travel time at the mean velocity α_0. The meaning of the operator (19) is as follows: the homogeneous-medium response $\bar{u}(0, \omega)\, e^{-j\omega T}$, which is the incident wave at $z = 0$ delayed by T, has to be multiplied by $S(\omega, T)$, and the result is approximately the transmission response of the random structure. The dashed seismograms in Fig. 8 have been determined in this way. The agreement with the exact seismograms is remarkably good, both in the amplitudes and in the long-period trend. Thus, the complex velocity (18) and the operator (19) appear to be well suited to describe the absorptive and dispersive nature of stratigraphic attenuation.

If both anelastic and stratigraphic attenuation is present, the resulting complex wave velocity is obtained from an obvious combination of the velocities (1) and (18), under the assumption $v_0 = \alpha_0$. The resulting attenuation operator is the product of Eqs. (17) and (19).

At this point a comparison with the work of Banik et al. (1985a, b) is in order. Our 1D random model, characterized by the exponential autocorrelation function (13) of the impedance fluctuations, the inverse quality factor (14) and the attenuation operator (19), is identical with the so-called telegraph model of these authors. In particular, our operator (19) agrees in essence with their attenuation operator [Banik et al., 1985b, Eq. (19)]. This is quite satisfactory in view of the grossly different derivations; namely, from a stochastic wave equation and mean-field theory in the Banik et al. approach and from simple energy considerations and an absorption-dispersion pair here.

Discussion and conclusions

The main result of this study is that stratigraphic attenuation is well described by single-scattering theory, leading to Eq. (14) for apparent Q. More empirical Q_s formulas, given by Menke (1983) and Richards and Menke (1983), are less well suited; in general, they do not reproduce the level of Q_s and its frequency dependence.

Equation (14) for Q_s is valid under the assumption of an exponential autocorrelation function of the relative impedance fluctuations of stratigraphy. This form is a good approximation in the case of uniformly distributed impedance fluctuations with mean value zero. In more general cases Eq. (12) has to be applied.

Under the validity of Eq. (14), stratigraphic attenuation is similar to the anelastic attenuation of a standard linear solid. Thus, its frequency dependence is rather pronounced: the main effects are concentrated in a frequency decade around the frequency $(2\pi\tau)^{-1}$, where τ is the two-way travel time related to the correlation distance.

From the success of Eq. (14) in the 1D case we conclude that the corresponding 3D results for Q_s (Sato, 1984) have

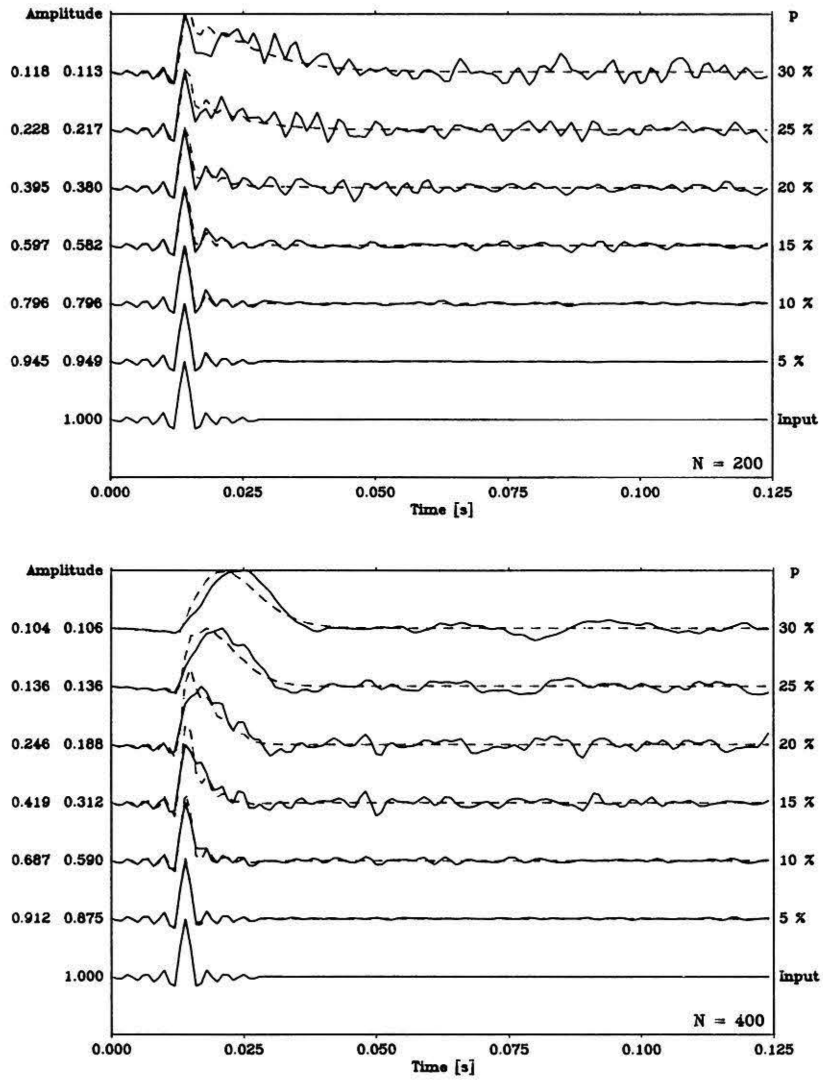

Fig. 8. Exact synthetic seismograms (*solid lines*) and approximations (*dashed lines*), calculated with the stratigraphic attenuation operator (19) for $N=200$ (*top*), $N=400$ (*bottom*), and in each case for variable maximum velocity fluctuation p

a similar range of applicability and that multiple scattering plays no major role for attenuation due to scattering. With results for Q_s available in the 3D case, it should be possible to derive the corresponding complex velocity and scattering attenuation operator as generalizations of Eqs. (18) and (19), respectively.

The parameters that enter Eq. (14) are the variance of the impedance fluctuation and the correlation distance. If these parameters are available, e.g. from sonic logs, the stratigraphic component Q_s of Q can be estimated and the intrinsic component Q_a determined; the latter is considered as an important lithological parameter. Of course, this requires reliable inferences on total Q from seismic data. Since Q_s of a *particular realization* of a random structure can have pronounced oscillations around the statistical average represented by Eq. (14) (Figs. 2 and 3), smoothing of total Q over frequency is required for a reliable estimate of Q_a. Q should be available for at least 1–2 frequency decades.

Besides the sediments of the upper crust, the rocks of the lower crust may possess a stratigraphic Q. Recent seismic investigations (e.g. DEKORP Research Group, 1985; Sandmeier and Wenzel, 1986) give some evidence of more or less horizontal laminae in much of the lower crust down to the crust-mantle boundary. Lamina thicknesses of 100–150 m and maximum velocity fluctuations of $\pm 10\%$ have been suggested by Sandmeier and Wenzel. From Eqs. (15) and (16), with $\kappa \approx 0.7$ for rocks of the lower crust, a minimum Q_s value of about 200 is estimated. It corresponds to frequencies of 3–5 Hz and applies only to steeply propagating waves. This rather low Q_s value indicates that stratigraphy and scattering contribute quite essentially to wave attenuation in the lower crust, at least in the frequency band of explosion seismology. For frequencies from 3–15 Hz, Q_s is in the range from 200 to about 400.

We have concentrated in this paper on those features of stratigraphic attenuation which are successfully explained by single-scattering theory. There are also, however, unexplained characteristics; for instance, differences between the exact (numerical) Q_s and the analytical Q_s in Figs. 2 and 3 which sometimes can exceed 50%. Moreover, the stratigraphic attenuation operator (19) does not explain the high-frequency seismogram reverberations in Fig. 8. These features are influenced or even dominated by multiple scattering, possibly up to high orders. These unexplained features are not always negligible, but their analytical description is more difficult. Single-scattering theory gives a good description of a few basic features of stratigraphic attenuation. This is sufficient for quite a number of applications.

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