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Velocity Variations in Systems of Anisotropic Symmetry

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Abstract. Angular variations of seismic velocities have been observed in the Earth and attributed to some form of anisotropy caused by aligned crystals, orientated cracks and inclusions, and laminated strata. The exact analytical expressions for the velocities in each particular symmetry-system, derived from the Kelvin-Christoffel equations, are complicated functions of the elastic constants and cannot be easily manipulated. This paper examines the form of the velocity variations for the several systems of elastic symmetry; five of these seven symmetry-systems have been suggested for possible Earth structures. We shall demonstrate that the approximate equations of Backus (1965) and Crampin (1977a) are good estimates for the velocity variations in symmetry planes of all symmetry systems, but not in general for off-symmetry planes. These equations are linear in the elastic constants, and provide a convenient link between velocity variations and elastic constants, if used judiciously. The behaviour of shear waves in off-symmetry directions is complicated by pinches, caused by the proximity of shear-wave singularities, where the two shear-waves exchange polarizations. Despite the restrictions to their use, the equations are the fundamental relationship for a number of modelling studies.

Key words: Anisotropic symmetry – Velocity variations – Approximate anisotropic velocities – Shear-wave singularities: Kisses, Intersections and Point singularities.

Introduction

Azimuthal velocity variations of *Pn* waves have now been observed in a variety of tectonic regimes: in lithosphere formed at oceanic ridges (Hess 1964; Raitt et al. 1969; and many others); beneath the Rhinegraben (Bamford 1977); and beneath the Western USA (Bamford et al. 1979). Upper-mantle velocity-anisotropy beneath much of Eurasia is also implied by the polarization of higher-mode surface-waves (Crampin and King 1977), and beneath much of the Pacific Ocean by the polarization of fundamental-mode surface-waves (Kirkwood and Crampin in press 1980b). Azimuthal anisotropy throughout the whole of the oceanic lithosphere beneath the NAZCA plate is suggested by the velocity variations of fundamental-mode surface-waves (Forsyth 1975). Most of these observations are equivalent to measuring the velocity variations in a single plane-section of the anisotropic material, and place little constraint on the choice of symmetry-system or the three dimensional nature of the velocity variations in the upper mantle. This system is usually taken to be some orthorhombic orientation, since crystalline olivine, the supposed major anisotropic constituent, has orthorhombic symmetry.

In the crust, exploration seismologists have long recognised that regular sequences of thin sedimentary beds may simulate a homogeneous transversely-isotropic material, and hence have hexagonal symmetry (Postma 1955; Backus 1962; Levin 1978). However, it appears that very little direct evidence for transverse isotropy has yet been found. Crack anisotropy (Crampin 1978) also suggests a number of potential sources of anisotropy in the crust, with a variety of possible symmetry-systems, although only one has yet been observed (monoclinic symmetry. Crampin et al. 1980).

We see that anisotropy in the Earth is still in a largely descriptive stage of investigation, although the theoretical and numerical development is now quite advanced. Clearly, inversion of in situ anisotropy is important for the information it may contain on material composition, lithology, three-dimensional velocity-structure, and tectonics (by identifying the stress field) (Crampin 1977b; Crampin and Bamford 1977; Crampin et al. 1980; Kirkwood and Crampin in press 1980a and b). However, the larger number of elastic constants to be specified make it more difficult to invert anisotropic than isotropic structures (Crampin 1976; Crampin 1977a; Kirkwood 1978).

One further complication in anisotropic media is that the direction of the ray-, wave-, or group-velocity vector deviates from the phase-propagation vector for both body and surface waves. It is the phase-velocity which appears explicitly in most analytical expressions, whereas it is the group-velocity arrival which is usually observed on seismograms.

The phase-velocities of the three orthogonally-polarized body-waves that propagate in anisotropic media (a quasi *P*-wave, *qP*, and two quasi shear-waves, *qS1* and *qS2*, or *qSH* and *qSV* where appropriate) are solutions of the Kelvin-Christoffel equations (Musgrave 1970; Auld 1973). These solutions are rational functions of the elastic constants and direction cosines, depend on the particular symmetry-system, and are sufficiently complicated to make it difficult to estimate the constants from observations of velocity, even when the particular symmetry-system can be identified. Consequently, we make use of the approximate equations of Backus (1965) and Crampin (1977a), which are much easier to manipulate.

There are many references to anisotropic symmetry in crystallographic literature (Nye 1957; Musgrave 1970; Auld 1973). Perhaps the major reference in geophysics is Backus (1970), which gives a strictly mathematical background to symmetry-systems. The numerical techniques used in the present paper are applicable generally and do not depend on the particular symmetry-system. The

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paper discusses the velocity variations in six of the seven systems of anisotropic symmetry, five of which are possible configurations within the Earth (Table 1), and focuses attention on several aspects of particular importance to seismologists, which are not adequately treated elsewhere:

1. The relevance of the approximate equations of Backus (1965) and Crampin (1977a) to particular symmetry-systems,
2. the importance of the 2θ and 4θ variations with direction for understanding the behaviour of body-waves,
3. the significance of the singularities of shear-wave slowness-sheets for the propagation and polarization of shear-waves (see also Crampin and Yedlin 1980).

Approximate Equations for Variation of Phase-Velocity

Backus (1965) determined approximate equations for the variations of qP velocity over a plane in a weakly anisotropic solid in terms of linear combinations of the elastic constants, and cosines and sines of the angle of azimuth. We shall demonstrate in subsequent sections that these equations are strictly applicable only to velocity variations in planes of mirror symmetry, and, although they may be good approximations in some off-symmetry planes, they are not good in all such planes. Crampin (1977a) derived similar expressions for shear-waves propagating in symmetry planes. These equations of Backus and Crampin provide a simple direct link between the velocities and the elastic constants, and have proved very convenient for estimating anisotropic elastic constants from velocity variations (Crampin and Bamford 1977; Crampin 1978; Crampin et al. 1980).

The azimuthal variations in the velocities of body-waves propagating in the $x_3=0$ plane of mirror symmetry of a weakly anisotropic material can be written (Backus 1965; Crampin 1977a):

$$\begin{aligned}
\rho V_{qP}^2 &= A+B_c\cos 2\theta+B_s\sin 2\theta+C_c\cos 4\theta+C_s\sin 4\theta,\\
\rho V_{qSH}^2 &= D+E_c\cos 4\theta+E_s\sin 4\theta, \text{ and}\\
\rho V_{qSV}^2 &= F+G_c\cos 2\theta+G_s\sin 2\theta, \qquad (1)
\end{aligned}$$

where
$$\begin{aligned}
A &= \{3(c_{1111}+c_{2222})+2(c_{1122}+2c_{1212})\}/8,\\
B_c &= (c_{1111}-c_{2222})/2,\\
B_s &= (c_{2111}+c_{1222}),\\
C_c &= \{c_{1111}+c_{2222}-2(c_{1122}+2c_{1212})\}/8,\\
C_s &= (c_{2111}-c_{1222})/2,\\
D &= \{c_{1111}+c_{2222}-2(c_{1122}-2c_{1212})\}/8\\
E_c &= -C_c,\\
E_s &= -C_s,\\
F &= (c_{1313}+c_{2323})/2,\\
G_c &= (c_{1313}-c_{2323})/2,\\
G_s &= c_{2313},
\end{aligned}$$

ρ is the density, V_{qP} is the velocity of the qP wave, V_{qSH} and V_{qSV} are the velocities of the quasi shear-waves with polarizations parallel (qSH), and perpendicular (qSV) to the plane of variation, c_{jkmn} are the moduli of the elastic tensor rotated so that the x_3-axis is normal to the plane of variation, and θ is the azimuth of propagation measured from the x_1-axis. The x_1, x_2, and x_3 axes are not necessarily principal axes. If θ is measured from a direction of sagittal symmetry ($x_2=0$, a plane of mirror symmetry), the coefficients of the sine terms are identically zero in Eq. (1) leaving the *reduced equations* in $\cos 2\theta$ and $\cos 4\theta$.

The shear-waves have polarizations strictly parallel and perpendicular to the plane of variation only if the plane possesses mirror symmetry (this restriction, although obvious, has not been stated explicitly in previous publications). The polarizations of the shear-waves ($qS1$ and $qS2$) are not parallel or perpendicular to generally orientated planes of variation, and the coefficients of the expansions in these cases are not simple combinations of elastic constants. The expansion for $qS1$ and $qS2$, even in solids with weak anisotropy, cannot generally be expressed in terms of trigonometric functions of 2θ and 4θ, but also require 6θ and higher terms for their description. The approximations are the first five terms of the Fourier Series expansion of the velocity variations – the complete expansions for general orientations contain an infinite number of terms. We find that the various restrictions on the application of these approximations to modelling studies present few problems in practice, because most symmetry-systems have sufficient symmetry planes for the equations to be easily applied.

Equations (1) are correct, with the restrictions given above, to the first order of the differences between anisotropic and isotropic elastic constants; the following figures demonstrate that the equations are good approximations even for strong anisotropy. It should be noted that the goodness of fit of the equations to any anisotropic velocity variation depends on the particular symmetries and the particular elastic constants and cannot be generalized in terms of, say, the fit being good to a certain percentage of velocity anisotropy.

Velocity Surfaces, Wave Surfaces, and Shear-Wave Singularities

We display the variations of phase-velocity with direction in various structures for the several systems of anisotropic symmetry. These curves are sections of what is sometimes called the velocity surface. However, this velocity surface is not what is usually observed in field or laboratory experiments. In anisotropic media, energy propagates with a component (usually small) parallel to the wave front. Thus the energy travels along a ray at an angle to the propagation vector and propagates with the group-velocity. The surface traced by this energy radiating along rays from a point source in a given time is known as the wave surface, or ray surface (Postma 1955; Musgrave 1970). This surface is the envelope of the wave fronts (Synge 1957). Sections of the wave surface cut by symmetry planes take a particularly simple form: the wave or group-velocity is

$$V_w=(V^2+(dV/d\theta)^2)^{1/2},$$
in a direction
$$\phi=\tan^{-1}[(V\sin\theta+(dV/d\theta)\cos\theta)/(V\cos\theta-(dV/d\theta)\sin\theta)], \qquad (2)$$

where $V(\theta)$ is the velocity in a direction θ in a symmetry plane (Postma 1955, first applied these equations to transversely-isotropic symmetry, but they are applicable generally, to symmetry planes in any system of anisotropic symmetry). The wave surface is very close to the velocity surface when the anisotropy is weak, and near symmetry directions in stronger anisotropy. In geophysics, where anisotropy is likely to weak (observed to be less than 8% in the upper-mantle although possibly stronger in some crustal rocks), velocity surfaces will generally be good approximations to wave surfaces.

Sections of the wave surfaces and velocity surfaces for structures with hexagonal symmetry are compared in Fig. 3 below.

These hexagonal examples have much weaker velocity-anisotropies (a maximum of 10% for qSH in zinc oxide, and 20% for qP in the cracked structure) than those in the remaining figures, and the wave surfaces and velocity surfaces are very close together. The structures with large anisotropies in the other figures have wave surfaces (not shown) which are substantially different from the velocity surfaces and frequently display cusps. However, we are using the structures to demonstrate the form of the velocity variations and the fit of the approximate equations for particular symmetry conditions, and these forms will be retained in weaker concentrations, where the wave and velocity surfaces do not differ significantly.

Crampin and Yedlin (1980) draw attention to the singularities of shear-wave velocities, where there are coincident roots. These are most easily considered in the slowness surfaces; there are three types of singularity; kiss singularities, where the two sheets touch tangentially, either convexly or concavely, but do not intersect; intersection singularities, where the two sheets intersect each other (possible only with hexagonal symmetry); and point singularities, where the two sheets have a common point at the vertex of two cones on their surfaces. Intersections are comparatively straightforward and do not cause any particular complications in either propagation or interpretation. Point singularities are places where the inner and outer surfaces exchange polarizations at a point. For variations in planes which pass near, but not through, such a point the waves exchange polarities at a pinch without coming into contact. When the pinch is tight, the approximations (1) for shear-waves are particular inappropriate. Pinches may cause considerable complications in some types of shear-wave propagation Crampin and Yedlin (1980).

Velocity Variations in Symmetry-Systems

The classes of elastic symmetry are usually divided into the seven named symmetry-systems listed in Table 1, which are described by the fourth-rank elastic-tensors of Fig. 1 (Nye 1957, noting the errors in Nye's specification for trigonal symmetry). These named systems encompass the full range of symmetry structures possible in homogeneous anisotropic elastic solids, and possible applications in the Earth are listed for five of the seven systems in Table 1. The triclinic system is omitted from the discussion: its description requires 21 independent elastic constants (Nye 1957), and the velocity variations are too general to be usefully summarized (conveniently, triclinic symmetry has not yet been suggested for any Earth structures).

The following figures show the velocity variations in symmetry planes and the orthogonal corner $x=0$, $y=0$, and $z=0$ (which we specify as x-cut, y-cut, and z-cut) for a range of anisotropic symmetry-structures. Since the amplitude and sign of each variation depends on the sum and difference of various combinations of elastic constants, these amplitudes and signs can vary widely from material to material even within the same symmetry system. The materials illustrated were chosen to display particular variations; further examples of crack systems relevant to the Earth can be found in Crampin (1978). The figures should be examined in relation to the other parameters of symmetry systems given in Tables 1 and 2, and in Fig. 1.

The velocity variations show a variety of different forms. The only general constraint is the near equality, but opposite sign, of the amplitudes of the 4θ variations of the squared qP and qSH velocities in symmetry planes, as expected from (1).

Table 1. Symmetry systems (referred to principal axes)

Symmetry system	Number of independent elastic constants	Number and specification of symmetry planes	Possible symmetry structures in the Earth
Cubic	3	9 { 3 identical: x-, y-, and z-cuts (Fig. 2a); 6 identical: planes joining opposite sides of cube (diagonal-cuts) (Fig. 2b) }	Orthogonal triplanar-systems of cracks with equal crack-densities (Fig. 5, Crampin 1978)
Hexagonal[a]	5	{ z-cut (Figs. 3b, and 3d); All planes through axis of cylindrical symmetry: z-axis (Figs. 3a and c) }	Lithologic alignments (Oil shale, Kaarsberg 1968); thin sedimentary sequences (Postma 1955; Backus 1962; Levin 1978); orientations of olivine in upper mantle (Francis 1969)
Trigonal	6(7)[b]	3 { 3 identical: sides of triangular prism; x-, $((1, \sqrt{3},0))$-, and $((1, \bar{\sqrt{3}},0))$-cuts (Fig. 4a) }	None known
Tetragonal	6(7)[b]	5 { 2 identical: sides of square prism; x-, and y-cuts (Fig. 5a); z-cut (Fig. 5b); 2 identical: planes joining edges of prism; $((1, 1, 0))$-, and $((1, \bar{1}, 0))$-cuts (Fig. 5c) }	Orthogonal biplanar systems of cracks with equal crack-densities (Fig. 4, Crampin 1978)
Orthorhombic	9	3 { x-cut (Fig. 6a); y-cut (Fig. 6b); z-cut (Fig. 6c) }	Aligned olivine-crystals in the upper mantle (Hess 1964; Avé Lallemant and Carter 1970; Crampin and Bamford 1977)
Monoclinic	13	1 { z-cut (Fig. 7c) }	Biplanar cracks with unequal crack-densities (Crampin et al. 1980)
Triclinic	21	None	None known

[a] Systems with hexagonal symmetry are transversely isotropic when the symmetry axis is vertical

[b] The name of the system refers to two possible configurations of constants. The identification of symmetry planes and possible Earth structures refer to the configuration with fewer constants

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ISOTROPIC            CUBIC                HEXAGONAL

a c c . . .          a c c . . .          a c d . . .
c a c . . .          c a c . . .          c a d . . .
c c a . . .          c c a . . .          d d b . . .
. . . x . .          . . . e . .          . . . e . .
. . . . x .          . . . . e .          . . . . e .
. . . . . x          . . . . . e          . . . . . x
where                                     where
x = (a - c)/2                             x = (a - c)/2

TRIGONAL (1)         TRIGONAL (2)*        TETRAGONAL (1)

a c d f . .          a c d f g .          a c d . . .
c a d -f . .         c a d -f -g .        c a d . . .
d d b . . .          d d b . . .          d d b . . .
f -f . e . .         f -f . e . z         . . . e . .
. . . . e y          g -g . . e y         . . . . e .
. . . . y x          . . . z y x          . . . . . f
where                where
x = (a - c)/2,       x = (a - c)/2,
and y = f            y = f, and z = -g

TETRAGONAL (2)*      ORTHORHOMBIC         MONOCLINIC

a c d . . g          a d e . . .          a d e . j .
c a d . . -g         d b f . . .          d b f . k .
d d b . . .          e f c . . .          e f c . l .
. . . e . .          . . . g . .          . . . g . m
. . . . e .          . . . . h .          j k l . h .
g -g . . . f         . . . . . i          . . . m . i
```

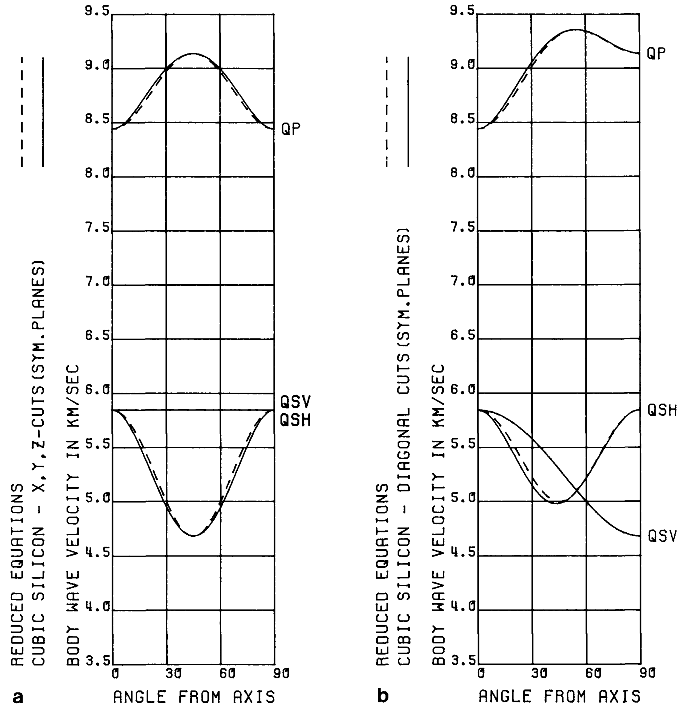
Fig. 1. The form of the elastic tensors for the various systems of elastic symmetry, with the conventional choice of axes. The letters *a–m* represent independent quantities in each tensor, unless otherwise specified. The tensor for the triclinic system is not shown. The starred tensors are more complicated varieties of the unstarred tensors and will not be discussed in this paper

In order to make the results more easily intelligible to those unfamiliar with crystallographic notation, we specifiy, where necessary, the direction of wave propagation and orientation of planes, not in terms of Miller indices, but in terms of the direction cosines of the normal referred to the conventional Cartesian coordinate system of the elastic tensor. Thus, the plane specified, within double parentheses, as the ((*a, b, c*))-cut is the plane normal to the line with direction cosines proportional to *a, b,* and *c*.

The velocity variations are drawn in rectangular rather than polar coordinates, which would have preserved the direction of propagation, because rectangular coordinates show details of the variations more clearly. The solid lines in the figures are the exact velocities calculated by the eigenvalue techniques of Crampin (1977a), and the dashed lines are the approximate velocity variations obtained by substituting the appropriate elastic constants into (1).

Cubic

The velocity variations in the two types of symmetry plane of cubic silicon, shown in Fig. 2, are typical of cubic structures.



a b

Fig. 2a and b. *Cubic symmetry.* Comparison of velocity variations of the approximate equations with the exact phase-velocities of silicon (elastic constants from McSkimin 1953), as a function of angle from a coordinate axis, in the symmetry planes: **a** *x*-, *y*-, and *z*-cuts; and **b** the diagonal cuts. In this and all following figures, the *solid lines* are the exact velocity variations, and the *dashed lines* are from the approximate Eq. (1)

Table 2. Singularities of the shear-wave slowness-sheets. The figures in brackets represent alternative but less-common configurations. The figures with asterisks represent point singularities on axes where symmetry planes intersect (Crampin and Yedlin 1980)

Symmetry system	Kiss singularities	Intersection singularities	Point singularities
Cubic	6	0	8
Hexagonal	2	2, (0)	0
Trigonal	0	0	2*+6, (2*+18), etc.
Tetragonal	2	0	8
Orthorhombic	0	0	4, (12, as in Fig. 6), etc.
Monoclinic	0	0	8, etc.

The velocity variations in the symmetry planes are very simple and the approximations (1) are very close. The simplicity is deceptive. The shear-wave slowness-sheets have more singularities than most other symmetry-systems (Table 2: there are six kisses, two on each of the principal axes, and eight point singularities, one in each solid quadrant at the intersection of the three diagonal symmetry-planes. The shear-wave approximate equations (1) are not good for off-symmetry directions, when there are so many point singularities. The equations for *P*-waves are also not good approximations in many off-symmetry directions: the ((1, 1, 1))-cut has small 6θ variations which clearly cannot be modelled by the 2θ and 4θ variations of (1).

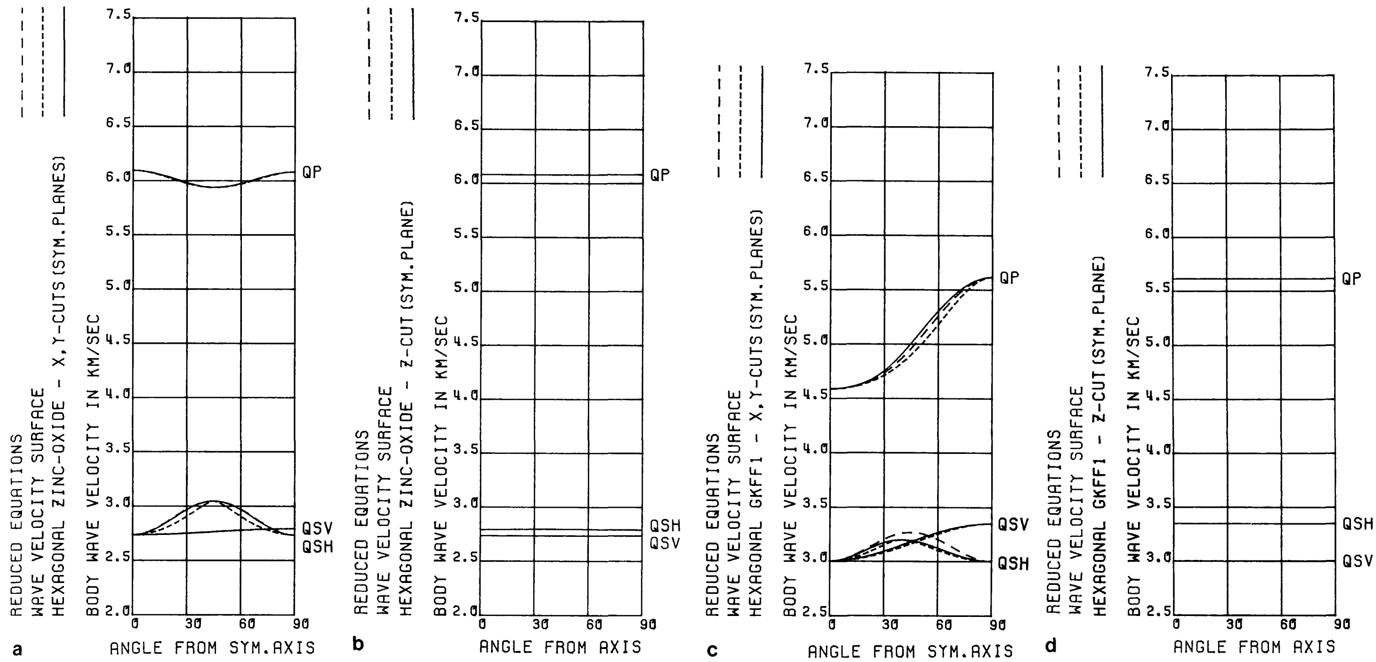


Fig. 3a–d. *Hexagonal symmetry.* Comparison of approximate velocities, with phase and wave velocities of zinc oxide (Bateman 1962), in the symmetry planes: **a** through the axis of symmetry; and **b** perpendicular to the axis of symmetry. Velocity variations of a structure GKFF1 with small dry parallel-cracks (Crampin 1978), in the symmetry planes: **c** through the axis of symmetry (the normal to the cracks); and **d** perpendicular to the axis of symmetry. The *fine-dashed lines* are sections of the wave velocity-surface calculated by Postma's (1955) equations

Hexagonal

Figure 3 shows the velocity variations in crystalline zinc-oxide, where the *P*-wave variation has a largely 4θ variation, and in dry parallel cracks (GKFF1, Crampin 1978), where the *P*-wave has a largely 2θ variation. Most hexagonal structures have shear-wave slowness-sheets intersecting as they do in both structures in Fig. 3. However, this is not always the case; randomly oriented cracks with co-planar normals may have shear-wave slowness-sheets with only two kiss singularities.

The reduced equations are good approximations to the phase-velocities, and the variations of qP and qSV in any plane through the axis determine all five elastic constants. Figure 3 also compares the wave-and velocity-surfaces. As discussed previously, there is very little difference between the two velocities for these comparatively weak anisotropies.

Earth structures in which the velocities of body-waves are invarient in horizontal directions are called transversely isotropic by seismologists. Structures with hexagonal symmetry are transversely isotropic perpendicular to the symmetry axis, and the term "transverse isotropy" is sometimes used as if it were synonymous with hexagonal symmetry. Strictly speaking, however, transverse isotropic refers only to cylindrical symmetry, and structures will possess hexagonal symmetry only if the average properties are constant over depth intervals greater than a seismic wavelength. Postma (1955) and Backus (1962) present techniques for deriving the hexagonal elastic constants of structures made up of regular laminations of thin isotropic beds.

Trigonal

Apart from the constraints on the 4θ variations of qP and qSH in the symmetry planes, wide variations of sign and amplitude are possible for different trigonal structures.

The approximate Eq. (1) model the velocity variations of alpha-quartz reasonably well in the symmetry planes (Fig. 4a), although they cannot model the rapid changes in direction of qP and qSH at some 75° from the z axis. The y- and z-cuts are not planes of symmetry, and the shear-waves display pinches due to the proximity of point intersections on the planes of symmetry. Consequently, the shear-wave Eq. (1) are poor approximations in these and other off-symmetry planes. The z-cut demonstrates the lack of generality of the Backus (1965) *P*-wave approximations.

Tetragonal

The approximate Eq. (1) follow the main trends of the velocity variations in the three types of symmetry plane of tetragonal rutile (Fig. 5) but are not particularly close estimates, because rutile has large velocity-anisotropy (50% for qSH wave in the z-cut) with a marked 8θ contribution. Apart from the constraints on the 4θ variations, the signs and amplitudes of the variations may vary widely between different tetragonal structures. The shear-wave equations are often poor approximations in the off-symmetry directions due to the proximity of point singularities.

Orthorhombic

A comparison of the approximate velocities and the phase-velocities in the three symmetry planes of orthopyroxene-bronzite are shown in Fig. 6. Except for the constraints on the 4θ variations of qP and qSH, all combinations of sign and amplitude variation are possible for structures with orthorhombic symmetry. The reduced equations are good approximations in all three symmetry planes. A variety of possible configurations of shear-wave singular-

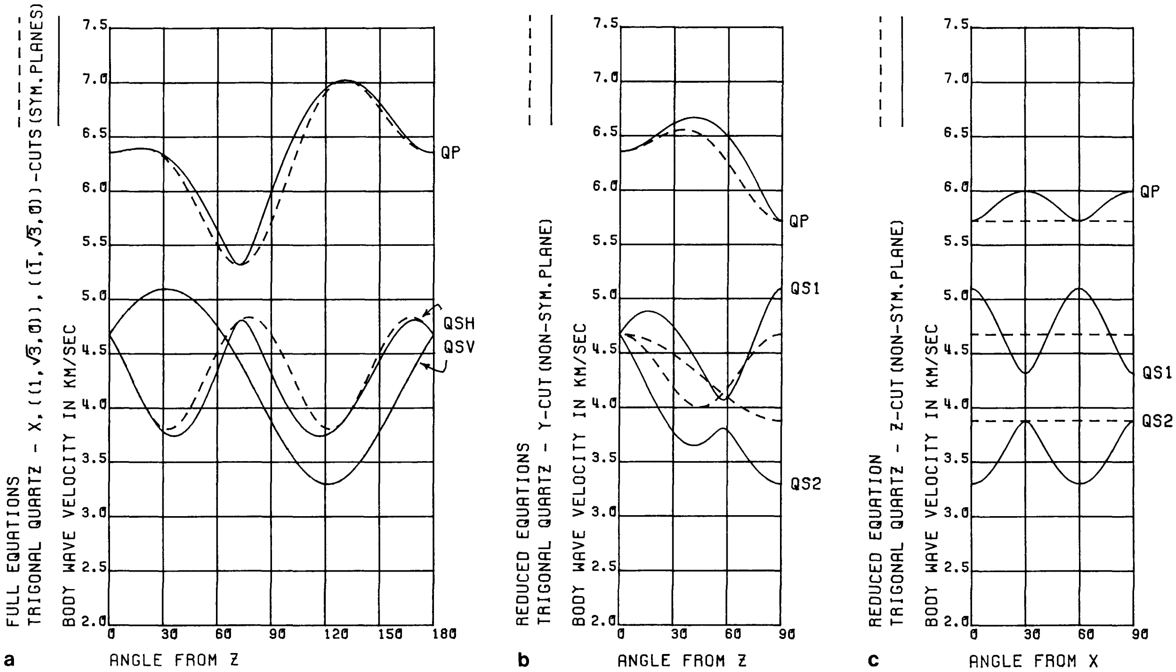


Fig. 4a–c. *Trigonal symmetry.* Comparison of approximate velocities with phase velocities of alpha-quartz (Bechman 1958), in the planes: **a** *x*-, ((1, $\sqrt{3}$,0))-, and (($\bar{1}$, $\sqrt{3}$,0))-cut symmetry planes; **b** *y*-cut, not a symmetry plane; and **c** *z*-cut, not a symmetry plane

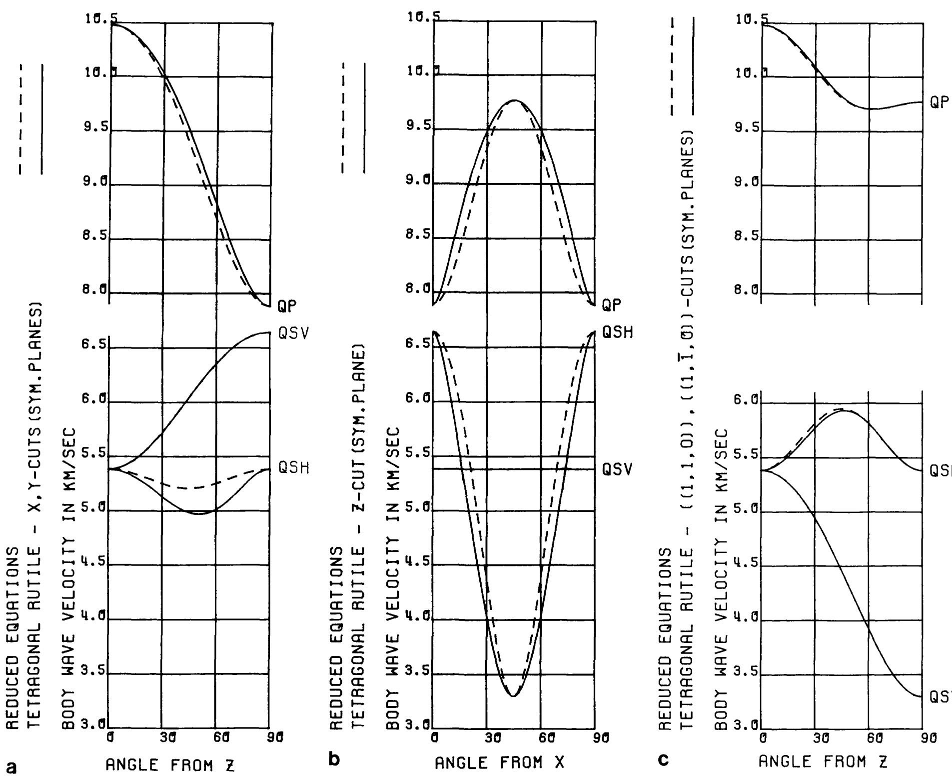


Fig. 5a–c. *Tetragonal symmetry.* Comparison of approximate velocities with phase velocities of rutile (National Lead 1967), in the symmetry planes: **a** *x*-, and *y*-cuts; **b** *z*-cut; and **c** ((1, 1, 0))-, and ((1, $\bar{1}$, 0))-cuts

ities in other orthorhombic materials make the equations poor approximations in many off-symmetry planes.

Olivine and many of the possible upper-mantle pyroxenes take orthorhombic symmetry with comparatively minor differences of sign and amplitude of the velocity variations. We note here that the sign of the 4θ variation of the *P*-wave in *z*-cut olivine is positive (Crampin 1976), whereas the corresponding sign in orthopyroxene is negative in Fig. 6c. Crampin and Bamford (1977) suggested that the sign and amplitude of the 4θ variations in observed *qP* velocity-anisotropy may be important as a structural discriminant. Crampin and Bamford could fit observed oceanic velocity-anisotropy very well with mixtures of olivine and isotropic media, whereas the velocity-anisotropy beneath the continental Rhinegraben displayed negative 4θ variations, and could not be directly fitted by olivine mixtures. It is possible that the negative sign of the observed velocity-anisotropy may be due to a higher pyrox-

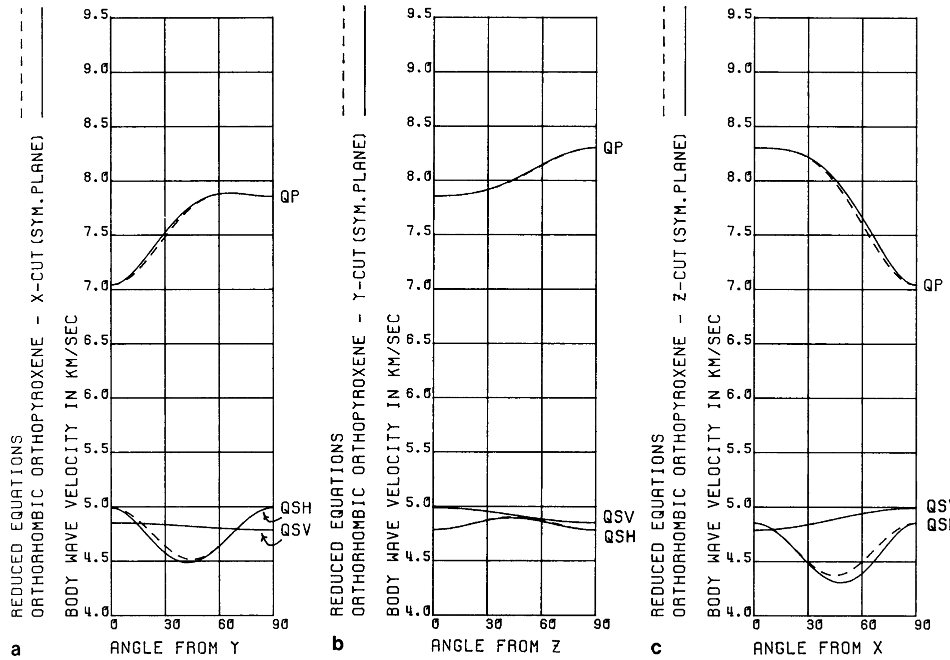


Fig. 6a–c. *Orthorhombic symmetry.* Comparison of approximate velicities with phase velocities of orthopyroxene–bronzite (Kumazawa 1969), in the symmetry planes: **a** x-cut; **b** y-cut; and **c** z-cut

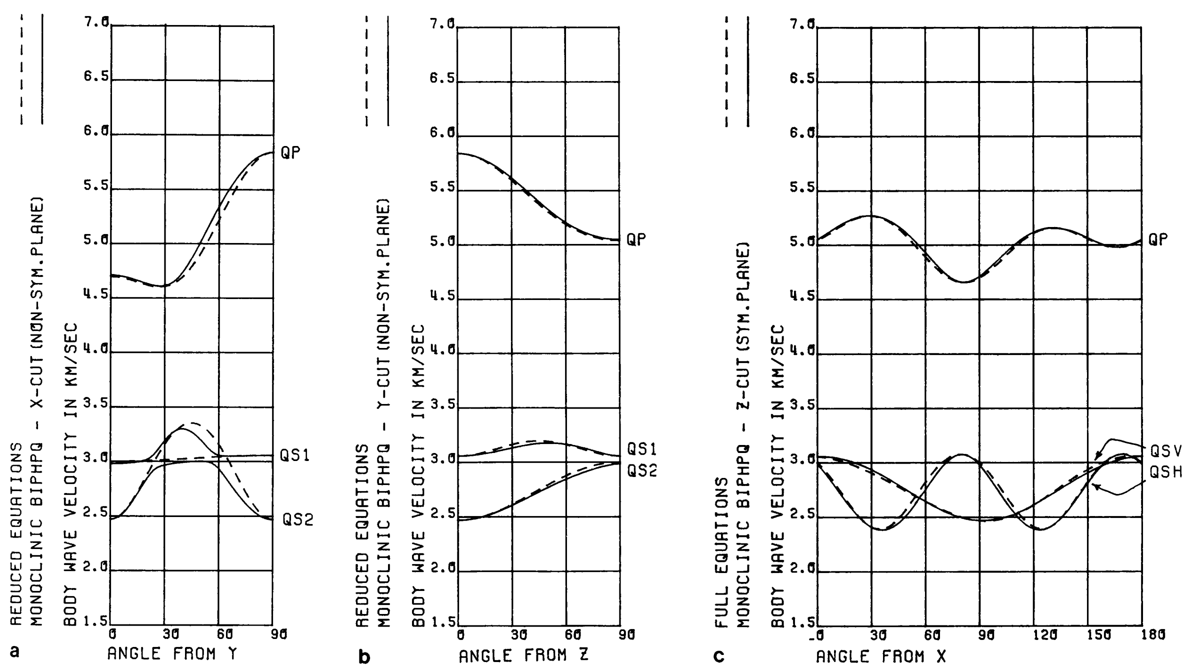


Fig. 7a–c. *Monoclinic symmetry.* Comparison of approximate velocities with phase velocities of BIPHQP, a structure with a biplanar system of cracks (Crampin et al. 1980), in the planes: **a** x-cut, not a symmetry plane; **b** y-cut, not a symmetry plane; and **c** z-cut symmetry plane

ene content in the upper-mantle beneath continents, although, as the velocities of pyroxene are generally lower than the usual sub-Moho velocities, the pyroxene would have to be mixed with higher-velocity isotropic media to produce typical upper-mantle velocities.

Monoclinic

The approximate velocities and phase-velocities for the monoclinic structure BIPHPQ observed by Crampin et al. (1980) are shown in Figure 7. The shear-waves in off-symmetry planes do not in general have polarizations parallel or perpendicular to the planes, however, by chance, the shear wave polarizations in the y-cut for this particular system are nearly parallel and perpendicular to the plane. Except for the constraints on the 4θ variations of qP and qSH in the symmetry plane, there are wide variations of velocity and shear-wave singularity configurations possible in monoclinic structures. The approximate equations are very good approximations in the symmetry plane (z-cut) and the y-cut in Fig. 7, but not in the x-cut, which demonstrates shear-wave pinching: the $qS1$ wave has nearly SV polarization, except between the pinches at 20° and 60°, where the polarization is nearly SH; similarly, $qS2$ has SH motion except for SV polarization between 20° and 60°.

BIPHPQ is derived from an observed P-wave velocity anisotropy in Carboniferous limestone (Bamford and Nunn 1979) due to a biplanar system of joints and fractures. The assumption of cracks enables the *P*-wave anisotropy to be inverted to give the full parameters of the cracks and the equivalent anisotropic structure BIPHQP (Crampin et al. 1980).

Conclusions

The velocities in Figs. 2–7 display wide variations in amplitude and phase for different symmetry-systems. Wide variations are possible for different materials within the same symmetry-system, particularly in the behaviour of shear-waves for variations in off-symmetry planes, where the two shear-waves may pinch together and exchange polarization characteristics due to the proximity of singularities in nearby planes.

The approximate expressions of Backus (1965) for *P*-waves, and Crampin (1977a) for shear-waves, are good estimates for the velocity variations in symmetry planes, and provide a direct link between velocities and constants in these planes. In off-symmetry planes, the approximate equations may be very poor estimates of both *P* and shear-wave velocities. However, despite these limitations to the use of approximate equations, they are very valuable in practice for transforming from velocities to constants (Crampin 1978; Crampin and Bamford 1977; Crampin et al. 1980). Systems of symmetry have a number of symmetry planes (Table 1), and the equations may be used by judicious choice of origin and plane of variation.

It is interesting to note that it is difficult to place the systems of anisotropic symmetry in any sequential order. It seems that any particular parameter used for ordering – number of elastic constants, number of planes of symmetry, number of shear-wave singularities – leads to a different order. Each symmetry system is unique and has unique features.

Anisotropic velocity variations take a number of significantly different forms, which are difficult to classify in any simple way. The percentage of velocity variation $((V_{\max}-V_{\min})/V_{\max})$, usually of the *P*-wave, in any given plane is sometimes called the coefficient of anisotropy. It is clearly a very uninformative description: it contains no information about the symmetry, the velocities of the other body-waves in the given plane, or the velocities of any waves in other planes, and we suggest it is not used in future.

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